Zurek-Kibble Domain Structures: The Dynamics of Spontaneous Vortex Formation in Annular Josephson Tunnel Junctions

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Phase transitions create a domain structure with defects, which has been argued by Zurek and Kibble (ZK) to depend in a characteristic way on the quench rate. We present an experiment to measure the ZK scaling exponent σ. Using long symmetric Josephson tunnel junctions, for which the predicted index is $\sigma = 0.25$, we find $\sigma = 0.27 \pm 0.05$. Further, we agree with the ZK prediction for the overall normalization.

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Because phase transitions take place in a finite time, causality guarantees that correlation lengths remain finite. Order parameter fields become frustrated, and defects arise so as to mediate the correlated regions with different ground states. Since defects are, in principle, observable, they provide an excellent experimental signature for the way in which transitions are implemented.

For condensed matter systems, which include the long annular Josephson tunnel junctions (JTJs) that we discuss below, Zurek [1,2] suggested that causality alone is sufficient to determine the initial density of defects arising in a continuous transition. In this he paralleled proposals made by Kibble [3] in the context of quantum field theory models of the early Universe.

As applied to JTJs, the idea is as follows. Consider a thin linear JTJ with critical temperature T_c, cooled through that temperature so that, if $T(t)$ is the temperature at time t, then $T(0) = T_c$. $\dot{T}(0) = -T_c/\tau_Q$ defines the quench time τ_Q. Suppose, at time t, that $c(t) = c[T(t)]$ is the Swihart velocity [4,5], vanishing at $t = 0$, and that $\xi_{\rm ad}(t) = \xi_{\rm ad}[T(t)]$ is the adiabatic healing length [the Josephson length $\lambda_J[T(t)]$], diverging at $t = 0$. The first time that fluxons (or Josephson vortices), the defects of linear JTJs, can appear is at time $\bar{t}$, when $|\dot{\xi}_{\rm ad}(t)| \simeq c(t)$.

For the case in hand, $\bar{t}$ has the form $\bar{t} = \sqrt{\tau_Q \tau_0}$, where $\tau_0 \ll \tau_Q$ is the relaxation time of the longest wavelength modes. Details are given in papers [6,7] by two of us (R. M. and R. R.). As a result, $\tau_Q \gg \bar{t} \gg \tau_0$. If $\xi_{\rm ad}(t) \sim \xi_0(t/\tau_Q)^{-\nu}$ for $t \sim 0$, where ξ_0 is simply related to $\xi_{\rm ad}(T = 0)$, the cold fluxon size, then the initial domain size and fluxon separation is predicted to be

$$\bar{\xi} \sim \xi_{\rm ad}(\bar{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\sigma} \gg \xi_0, \qquad (1)$$

where $\sigma = \nu/2$. We term σ the Zurek-Kibble (ZK) characteristic index.

The arguments are not specific to JTJs. Prior to our experiment, five other condensed matter experiments had been performed to test the prediction (1) for the separation of defects at their production, two experiments [8,9] on superfluid ^{3}He, two [10,11] on superfluid ^{4}He, and one [12] on high temperature superconductors. In addition, an experiment [13] on JTJs by two of us (R. M. and J. M.) was compatible with (1), although it had not been performed with a test of (1) in mind.

Before discussing our new experiment a few comments are in order. The experiments [8,9] on superfluid ^{3}He-B rely on the fact that, when it is bombarded with slow neutrons, $n + {}^3\mathrm{He} \rightarrow p + {}^3\mathrm{H} + 760$ keV. The energy released in such a collision leads to a hot spot which, when cooled by its environment below T_c, leaves behind a tangle of vortices (the topological defects in this system). τ_Q is fixed by the nuclear process that breaks up the ^{3}He atom. With only a single data point conflating both normalization and σ, it is not possible to confirm the predicted value $\sigma = 1/4$. However, both experiments are highly compatible with (1), agreeing to a factor of a few in each case.

In principle, the ^{4}He experiments [10,11], which use a pressure quench with a varying time scale τ_Q to implement the transition, allow for a more complete test. Yet again, vortices are the relevant defects. In practice, the most reliable experiment [11] sees no vortices. This is not necessarily a sign of failure in that it has been suggested [14] that the vortices decay too fast to be seen. This is irrespective [15] of whether a pressure quench, which preserves high thermal fluctuations, would of itself lead to somewhat different predictions. In this context, the vortices seen in an earlier ^{4}He experiment [10] were most likely an artifact of the experimental setup.

The fifth experiment [12], on high-T_C superconductors, measures total flux through a surface carried by the Abrikosov vortices. The vortex separation of (1) can be converted into a prediction for the flux, but no flux is seen in contradiction with this prediction, despite the phase

separation that leads to the result being demonstrated elsewhere [16]. In this case there is no obvious explanation of the null result, despite later work [17] that takes the effect of gauge fields into account fully.

In summary, these early experiments (including [13]) have either provided *one* data point for (1), or have been null. Subsequently, two experiments of a very different type have been performed that permit varying quench rates and so an estimate for σ. The most recent [18] involves the Bénard-Marangoni conduction-convection transition. The defects here are not associated with the line zeros of an order parameter field, and the viscosity-dependent σ does not match the ZK prediction, most likely for that reason. The more relevant experiment [19] is carried out in a nonlinear optical system, with complex beam phase the order parameter. Increasing the light intensity (the control parameter in this case) leads to pattern formation (defects) at a critical value. The predicted $\sigma = 1/4$ is recovered to good accuracy, but agreement with normalization is not stated.

Our experiment, whose methodology and results we outline below, is also one in which, by varying τ_Q, we can measure and compare σ with its theoretical value, as well as confirming overall scale.

In its essence, we quench a long annular JTJ through its critical temperature and count such fluxons or Josephson vortices as they appear. In its idealized form the annular JTJ consists of two dimensionally identical annuli of superconductors of narrow width, superimposed upon one another, separated by an insulating barrier in the same plane. In practice, it is sufficient for the lower superconductor to be a planar substrate upon which the other annulus sits. The effective theory for fluxons in JTJs [5] is the sine-Gordon model with respect to the field $\phi = \phi_1 - \phi_2$, the difference in the phases of the complex order parameter fields in the separate superconductors. The Josephson vortices are then the sine-Gordon kinks.

The JTJs in our experiment are *symmetric,* by which is meant that the electrodes are made of identical superconducting material with the same energy gaps and the same T_c. This is confirmed by seeing that (a) there is no logarithmic singularity in the voltage-current characteristic at finite voltages and (b) the temperature dependence of the critical current is linear as T approaches T_c. The outcome is that [6,7] $\nu = 1/2$. Therefore, at the time of their formation the separation of fluxons is expected to be given by (1) with $\sigma = 1/4$.

In terms of the parameters of the JTJs, the Josephson length at temperature T is [5]

$$\xi_{\mathrm{ad}}(T) = \lambda_J(T) = \sqrt{\frac{\hbar}{2e\mu_0 d_s J_c(T)}},$$

where $J_c(T)$ is the Josephson current density at temperature T. Typically $\lambda_J(0)$ is in the 10–100 μm range and was equal to 7 μm for the sample discussed below. In Ref. [7] ξ_0 has been inferred as

$$\xi_0 = \sqrt{\frac{\hbar}{2e\mu_0 d_s \alpha J_c(0)}}.$$

The parameter $3 \lesssim \alpha \lesssim 5$ is given in terms of the superconductor gap energy $\Delta(T)$ and T_c. As for τ_0, it is given as $\tau_0 = \xi_0/c_0$, where c_0 defines the behavior $c(t) = c_0(t/\tau_Q)^{1/2}$ of the Swihart velocity for the system near $T = T_c$. If the thickness of the two superconducting electrodes differs, the effective thickness d_s is the harmonic mean of the individual thicknesses [5].

Our samples are high quality, 500 μm long, 3 μm wide, $\mathrm{Nb/Al - Al_{ox}/Nb}$ JTJs fabricated on 0.5 mm thick silicon substrates using the trilayer technique in which the junction is realized in the window opened in a SiO insulator layer. Details of the fabrication process can be found in Ref. [20]. For all samples the high quality has been inferred by a measure of the I-V characteristic at $T = 4.2$ K. In fact, the subgap current I_{sg} at 2 mV was small compared to the current rise ΔI_g in the quasiparticle current at the gap voltage V_g, typically $\Delta I_g > 35 I_{sg}$. The gap voltage was as large as $V_g = 2.76$ mV and the maximum critical current I_c was larger than $0.55\Delta I_g$ for the overlap-type junction. Furthermore, the application of a strong enough external magnetic field in the barrier plane completely suppressed any Josephson structures indicating the absence of electrical shorts in the barrier. At an order of magnitude level, we have $\tau_0 \sim 0.1$ ps and $\xi_0 \sim 1$ μm. As a result $\bar{\xi} \sim 1$ mm for $\tau_Q \sim 1$ s. We will be more specific later.

Our JTJs have a critical temperature $T_c = 8.95$ K, whereas the individual superconductors have a critical temperature of 9.1 K. Even at our fastest quench the conductors are superconducting, by which is meant that the Cooper-pair order parameter field has achieved its final magnitude, 1 ms before the JTJ can develop fluxons. This is necessary for (1) to be valid without modification [6,7], since only then is ϕ the relevant order parameter.

In order to vary the quenching time over the widest possible range, we have realized the experimental setup shown schematically in Fig. 1. The annular JTJ samples are fabricated on a chip (shown endways), mounted to a Cu block by a thermally insulating Teflon sheet. The entire system is enclosed in a vacuum-tight can immersed in the liquid He bath. In all cases the heat is removed from the system by He exchange gas, using a manual pump. By varying the pressure of the gas we can modify the rate of cooling of the sample. On the other side of the block is a 50 Ω carbon resistor, which enables us to heat and cool the JTJs on a relatively long time scale (τ_Q from about 1 to 10 s) depending on the He exchange gas pressure inside the can. With $\sigma = 1/4$, we need to vary τ_Q by at least 2 orders of magnitude. This is not possible just by heating the block alone (even if it were smaller). To extend the range of τ_Q we mounted a small, pulse-driven, surface 100 Ω resistor on the same side. This permits much smaller thermal cycles (0.07 to 0.2 s). These two completely different

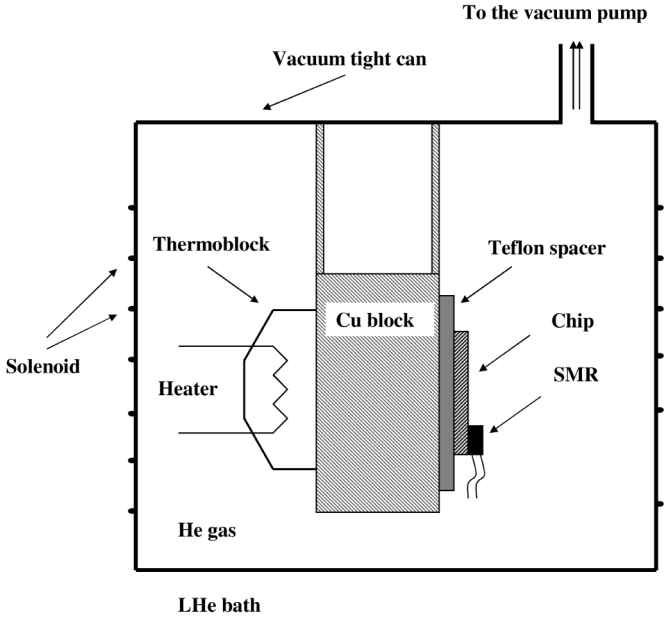

FIG. 1. Sketch (dimensions are not to scale) of the cryogenic insert developed to perform the junction thermal cycles with a time scale changing over a broad range.The JTJs are fabricated within the chip (shown endways), to which the surface mounted resistor (SMR) is attached. The whole is surrounded with liquid He.

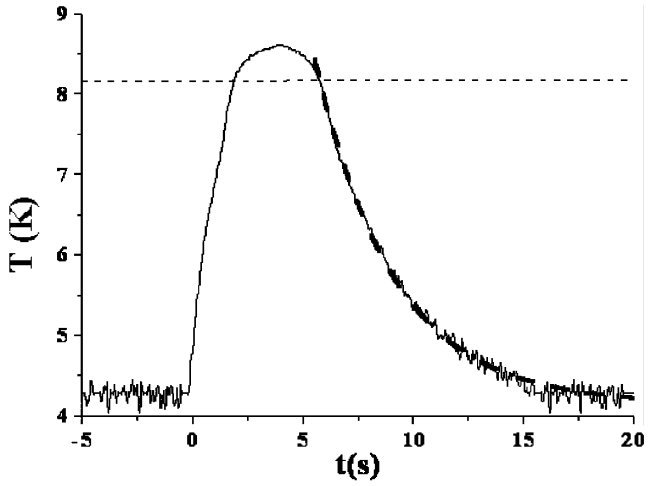

FIG. 2. Time dependence of the junction temperature during a ''slow'' thermal cycle from 10 to 4.2 K, as inferred from the junction gap voltage. Only below the horizontal dashed line does the continuous curve describe the temperature. The thick dashed line is the best fitting curve to (2), to be extrapolated to T_c.

quenching techniques provide time scale ranges that do not overlap, leaving a gap between 0.2 and 1 s, that would require a third quenching technique to be filled.

The whole system is then enclosed in a μ-metal shielded cryostat. The temperature of the JTJ is monitored by measuring the junction gap voltage, which is proportional to the known superconductor gap energy $\Delta[T(t)]$. All quenches were taken from 10 to 4.2 K through $T_c = 8.95$ K. By making use of the Thouless equation [21], it is possible to infer the temperature of the JTJ from the gap voltage in the range 8.2 to 4.2 K, extremely accurately at the upper end of the range, and with fluctuations of a few percent at the lower. Whether for slow or fast cooling an excellent fit to the temperature of the JTJ in this range, for initial temperature $T_{\rm in} = 10$ K and final temperature $T_{\rm fin} = 4.2$ K, is given by the thermal relaxation equation

$$T(t) = T_{\rm in} + (T_{\rm in} - T_{\rm fin})\, e^{-(t-t_0)/\tau}, \qquad (2)$$

where τ is the relaxation time which sets the cooling time scale. See Fig. 2 for an example. This equation is then used to extrapolate $dT(t)/dt$ to the vicinity of T_C and yields $\tau_Q \simeq 1.7\tau$.

On cooling the system in this way, we expect fluxons to appear from the inhomogeneity of ϕ at the transition, according to (1). In the absence of any current through the barrier or applied external magnetic field the fluxons are in indifferent equilibrium as far as the barrier is homogeneous and pin-hole free. (In reality, there is a small pinning effect so that, after a short transient, the fluxons are static.) To make them visible, and countable, we apply a bias current, whereby they move as magnetic dipoles under the resulting Lorentz force, at speed less than the Swihart velocity. According to Josephson theory N traveling fluxons (and antifluxons) with speed v develop a voltage $V = N\Phi_0 v/C$ across the junction, where $\Phi_0 = h/2e$ is the flux in a (Josephson) fluxon and C is the annulus circumference. This voltage can be measured, and the fluxon number determined. A detailed description of the experiment and the data will be given elsewhere [22].

Many samples have been measured, but only one had such a large critical current density [and sufficiently small $\lambda_J(T)$] that only 3000 thermal cycles were enough to get reliable statistics, and it is this sample that we discuss now. However, we stress that, within the less good statistics of the other samples, none gave results that were incompatible with (1).

The symmetric annular JTJ with which the experiment was performed had a circumference $C = 500$ μm and width $\Delta r = 4$ μm. The effective superconductor thickness was $d_s \approx 250$ nm. At the final temperature $T_{\rm fin} = 4.2$ K, the critical current density was $J_c(T_{\rm fin}) = 3050$ A/cm^2, the quality factor was $V_m = 49$ mV, and the Josephson length was $\lambda_J(T_{\rm fin}) = 6.9$ μm. The velocity c_0 is calculated to be $c_0 \approx 2.2 \times 10^7$ m/s. From this, we infer that $\xi_0 \approx 3.8$ μm and $\tau_0 \approx 0.17$ ps. It follows that $\bar{\xi} \approx 5.9$ mm for $\tau_Q = 1$ s, several times the circumference of the JTJ.

As a result, the likelihood of finding a single fluxon is small. We estimate the probability of finding a fluxon in a single quench to be

$$P_1 \simeq \frac{C}{\bar{\xi}} = \frac{C}{\xi_0}\left(\frac{\tau_Q}{\tau_0}\right)^{-\sigma}, \qquad (3)$$

where, from (1), $\sigma = 0.25$.

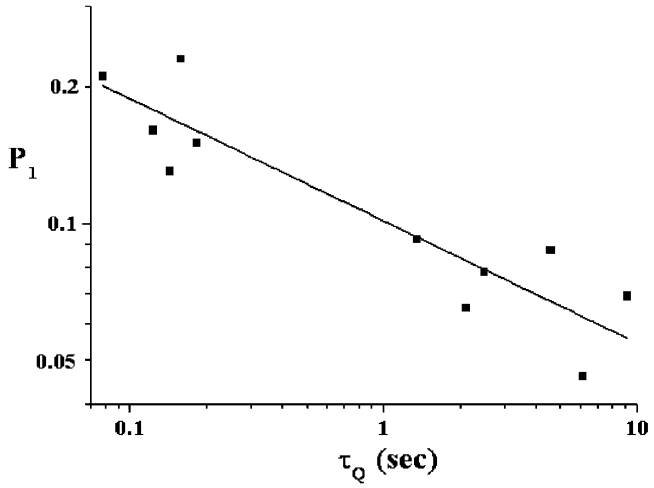

FIG. 3. Log-log plot of the measured probability P_1 to trap one fluxon versus the quenching time τ_Q. Each point corresponds to 300 thermal cycles. The solid line (slope $b = 0.27$) is the best linear fit, in good agreement with the 0.25 value expected for symmetric JTJs. Errors in τ_Q are a few percent, and systematic errors in P_1 due to the neglect of fluxon-antifluxon pairs are $O(P_1^2)$, again a few percent.

In Fig. 3 we show a log-log plot for the measured P_1 as a function of τ_Q, changed by varying the exchange gas pressure and by using both the fast and slow quenching techniques. Each data point corresponds to 300 quenches. Although the data do not distinguish between a single fluxon and a fluxon plus an antifluxon pair, the likelihood of the latter is sufficiently small that it can be ignored. Similarly, the data do not distinguish between no fluxons and a fluxon-antifluxon pair, with similar errors. We observe that the points are quite scattered, meaning that the data are statistically poor. Further, for the reasons given earlier there is a gap between fast and slow quenches. Nonetheless, we have clear evidence that (i) the trapping of a fluxon occurs on a purely statistical basis for identical conditions of each thermal cycle and (ii) the probability to trap one fluxon is larger when the transition is performed at a faster speed (smaller quenching time) in accordance with the causality principle. This complements our qualitative results from other samples with smaller $J_c(0)$ [for which the statistics is too poor to permit a fit to (1)] that, at fixed τ_Q, the probability of finding a fluxon decreases with increasing ξ_0.

Regardless of the data spread, to test (3) we attempted to fit the data with an allometric function $P_1 = a\,\tau_Q^{-b}$, with a and b being free fitting parameters. We found, for the coefficient a, the best fitting value of $0.1 \pm 10\%$ (taking τ_Q in seconds). This is in excellent agreement with the predicted value of $C\tau_0^{1/4}/\xi_0 \approx 0.08\ \mathrm{s}^{1/4}$, given the fact that we expect agreement in overall normalization only to somewhat better than an order of magnitude. At this level this agreement survives any systematic error that could exist between the different cooling mechanisms. After the problems with the experiments discussed in [10–12] to find (reliable) defects at expected densities, if at all, our experiment shows that the ZK estimate remains sensible. Further, the best fitting curve, shown by the solid line in Fig. 3, has a slope $b = 0.27 \pm 0.05$, in remarkable agreement with its predicted value of 0.25.

The ZK scenario needs further testing with JTJs for which there is a greater likelihood of observing fluxons. In [6,7] we observed that this is the case for significantly nonsymmetric JTJs, for which the value of σ is $\sigma = 1/7$; a further experiment, with markedly nonsymmetric JTJs, is being planned.

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