Colloidal Dynamics on Disordered Substrates

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Using Langevin simulations, we examine driven colloids interacting with quenched disorder. For weak substrates the colloids form an ordered state and depin elastically. For increasing substrate strength, we find a sharp crossover to inhomogeneous depinning and a substantial increase in the depinning force, analogous to the peak effect in superconductors. The velocity versus driving force curve shows criticality at depinning, with a change in scaling exponent occurring at the order to disorder crossover. Upon application of a sudden pulse of driving force, pronounced transients appear in the disordered regime which are due to the formation of long-lived colloidal flow channels.

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Colloidal crystals are an ideal system in which to study the general problem of ordering and dynamics in 2D [1–4], since the particle size permits direct imaging of the particle locations and motion. A considerable amount of work has been conducted on the melting of 2D colloidal crystals in the absence of a substrate [1,2]. In addition, a number of experimental and theoretical studies have considered colloidal crystallization and melting in 2D systems with periodic 1D [3] and 2D substrates [4,5], where a rich variety of crystalline states can be stabilized.

Colloid crystals are also ideal for studying the ordering and dynamics of an elastic media interacting with *random* substrates, a problem that is relevant to a wide variety of systems, such as superconducting vortices, Wigner crystals, and charge density waves (CDWs). Open issues include the nature of the dynamical response to applied forces, as well as whether an order to disorder transition occurs as the strength of the random substrate increases. Recently, Carpentier and Le Doussal have theoretically investigated the effects of quenched disorder on the order and melting of 2D lattices and find a sharp crossover from the ordered Bragg glass (where defects are absent) to a disordered or molten state [6]. They predict that the depinning threshold increases at this crossover due to the softening of the lattice, which allows the particles to better adjust to the substrate. A similar mechanism could account for the peak effect observed in vortex matter in superconductors [7–12], in which the depinning threshold rises dramatically when the applied magnetic field is increased. In low temperature superconductors, where the fairly stiff vortices can be considered as effectively 2D, recent small angle neutron scattering experiments have shown that the peak effect is associated with a sharp disordering or melting transition [13].

In addition to static properties, the dynamics of elastic media interacting with quenched disorder in 2D is a topic of intense study. In the disordered region, the driven system may break up into pinned and flowing regions, as observed in experiments [14] and simulations [15,16] of superconducting vortices. Conversely, for weak substrate disorder, the elastic media is defect free and undergoes elastic depinning, in which the particles keep the same neighbors as they move. Fisher predicted that elastic depinning would show criticality [17] and that the velocity vs force curves would scale as $v = (f - f_c)^\beta$, where f_c is the depinning threshold force. This scaling has been studied extensively in 2D CDW systems where $\beta = 2/3$ [18,19]. It is, however, not known whether this exponent occurs in other systems undergoing elastic flow. Another intriguing dynamical phenomenon is the pronounced transient behavior exhibited by vortices under a sudden applied current pulse at magnetic fields near the peak effect regime [9,10]. Because of surface barrier effects, it is not clear whether these transient effects arise from the plasticity of the vortex dynamics or from contamination of the vortex lattice by disorder from the sample edges [12]. Recently, Pertsinidis and Ling [5] have studied colloids in 2D driven by an electric field and interacting with a disordered substrate. They observe plastic depinning with filamentary or riverlike flow of colloids and a velocity-force curve scaling with $\beta = 2.2$, as well as elastic depinning of an ordered colloidal lattice with β around 0.5. Under a pulsed drive the system shows very long time transients that fit to a stretched exponential.

Motivated by the recent colloidal experiments as well as the pulse drive experiments in vortex matter, we have conducted Langevin simulations of colloidal particles interacting via a Yukawa potential in 2D systems with random disorder. In simulation, the strength of the disorder can be carefully tuned, which is difficult to achieve in experiments. In addition, the initial conditions of the colloidal arrangements are easily controlled, whereas in experiments, defects generated in the colloidal lattice during preparation may become frozen in by the disorder. We find that for weak substrates the colloids form an ordered triangular array which depins elastically without the generation of defects. For increased substrate strength, there is a sharp crossover to a disordered phase where the colloids depin plastically into riverlike structures. This crossover is accompanied by a sharp increase in the depinning

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threshold, analogous to the peak effect phenomenon in superconductors. We find scaling of the velocity vs applied drive with an exponent of $\beta = 0.67$ in the elastic regime, in agreement with studies in 2D CDWs. In the plastic regime we find $\beta = 1.94$, close to the experimentally observed value [5]. In the disordered region, long time transients that fit to a stretched exponential occur in response to a sudden applied drive pulse, as also observed in experiments.

The colloids are simulated using Langevin dynamics in 2D [2] and interact via a Yukawa or screened Coulomb interaction potential $V(r_{ij}) = (Q^2/|\mathbf{r}_i - \mathbf{r}_j|)\exp(-\kappa|\mathbf{r}_i - \mathbf{r}_j|)$. Here Q is the charge of the particles, $1/\kappa$ is the screening length, and $\mathbf{r}_{i(j)}$ is the position of particle $i(j)$. Length is measured in units of the lattice constant a_0, and the screening length is $\kappa = 2/a_0$. The quenched disorder is modeled as randomly placed parabolic traps with radius $r_p < a_0$ and a maximum force f_p. The equation of motion for colloid i is $d\mathbf{r}_i/dt = \mathbf{f}_{ij} + \mathbf{f}_p + \mathbf{f}_T + \mathbf{f}_d$. Here $\mathbf{f}_{ij} = -\sum_{j\neq i}^{N_c} \nabla_i V(r_{ij})$ is the interaction force from the other colloids, $\mathbf{f}_p$ is the pinning force, $\mathbf{f}_T$ is a randomly fluctuating force due to thermal kicks, and $\mathbf{f}_d$ is the force due to an applied drive. We start the system at a temperature above the melting temperature $T > T_m$ and gradually cool to $T/T_m = 0.4$. The driving force is increased from zero by small increments and the velocity is averaged for 5×10^4 time steps at each increment, with typical simulations running for 10^7 time steps. In this model we do not take into account hydrodynamic effects or long-range attractions between colloids. This colloidal model differs from vortex simulations in the form of the particle-particle interaction. For vortices interacting logarithmically, the shear modulus is much smaller than the compression modulus [20], making filamentary or plastic flow at depinning likely. To our knowledge, no simulation of vortex matter in 2D has observed an order to disorder transition as a function of pinning strength.

In Fig. 1(a) we show the depinning force f_c vs substrate strength f_p from a series of simulations. For $f_p < 0.18$ the depinning force increases as a power law, $f_c \propto f_p^{-1.9\pm0.1}$. To compare the depinning force to the order in the system, in Fig. 1(b) we show the percentage of defects or non-sixfold coordinated particles P_d as calculated from a Delaunay triangulation. This measure indicates that the colloidal crystal is in an ordered state ($P_d = 0.0$) for $f_p < 0.18$ and that there is a crossover to a disordered state ($P_d \neq 0$) at $f_p = 0.18$. In Fig. 1(c) we show a representative Delaunay triangulation for the ordered state where there are no defects but small distortions in the particle positions can be seen, and in Fig. 1(d) we show the disordered state where defects are present. The crossover to the disordered state coincides with a rapid *increase* in the depinning force as seen in Fig. 1(a) and in the inset in Fig. 1(a), which shows a peak in df_c/df_p at the crossover. This behavior is consistent with the recent experiments in superconductors, which find an increase in the pinning at the peak effect with a simultaneous disordering of the lattice [13]. For $f_p > 0.2$, the depinning scales as $f_c \propto f_p$,

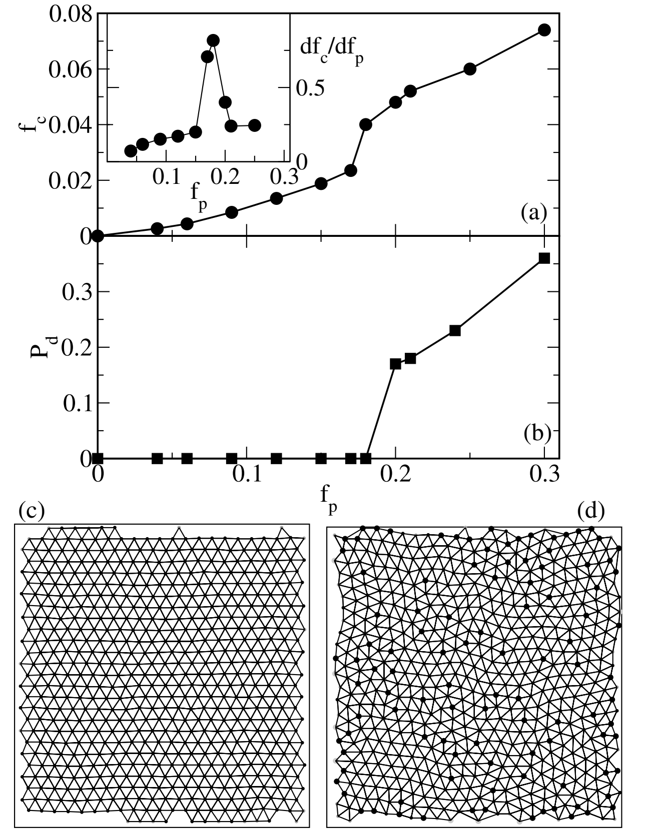

FIG. 1. (a) Depinning force f_c vs pinning strength f_p. Inset: Corresponding df_c/df_p. (b) Percentage of non-sixfold coordinated colloids, P_d. (c) Delaunay triangulation of colloid positions at depinning for $f_p = 0.12$; (d) $f_p = 0.25$. Filled circles indicate non-sixfold coordinated particles.

as expected for the single particle pinning regime. The sudden increase in the depinning force results from the fact that the defected colloid lattice is much softer than the ordered lattice, allowing the colloids to adjust their positions to accommodate to the optimal pinning sites. We have also investigated this transition for different colloidal densities and disorder strengths. For increasing T, the order to disorder transition is shifted to lower values of f_p. For increasing system sizes, the order-disorder crossover shifts only a small amount before saturating, while the sharpness of the transition persists with increased system size.

It is beyond the scope of this Letter to determine whether the order to disorder crossover is a first order transition. Although the sharpness suggests a possible first order transition, Carpentier and Le Doussal show that for 2D systems with quenched disorder, a sharp disordering crossover, rather than transition, occurs [6]. In addition, a first order transition is not expected since the Bragg glass in 2D has been shown to have dislocations on large scales at all temperatures. The distance between these dislocations can be arbitrarily large [21].

In Fig. 2 we show that the order-disorder crossover coincides with the onset of plastic flow above depinning. In Fig. 2(b) the elastic colloid flow is shown for $f_p = 0.12$

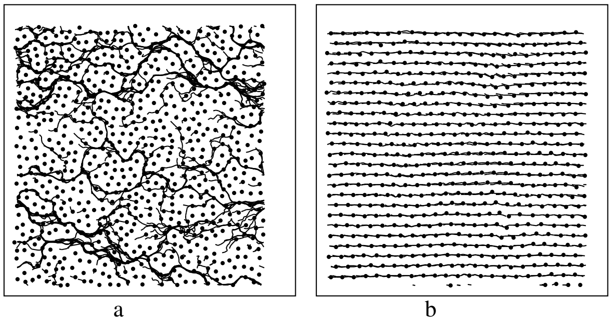

FIG. 2. Colloid positions (black dots) and trajectories (lines) for (a) plastic flow regime ($f_p = 0.25$) and (b) elastic flow regime ($f_p = 0.12$).

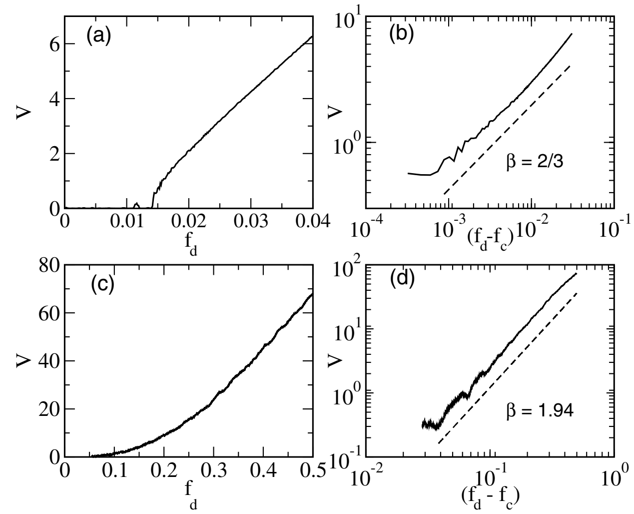

FIG. 3. Velocity v vs applied drive f_d for (a) elastic regime $f_p = 0.08$. (b) Log-log plot of v vs $(f_d - f_c)$ from (a); line indicates fit to $\beta = 2/3$. (c) v vs f_d for plastic depinning $f_p = 0.25$. (d) Log-log plot of v vs $(f_d - f_c)$ from (c); line indicates fit to $\beta = 1.94$.

above depinning ($f_d/f_c = 1.1$). Here each colloid keeps the same neighbors as it moves. In Fig. 2(a) the inhomogeneous or plastic colloidal flow is shown for $f_d/f_c = 1.1$ for $f_p = 0.25$. Here only a portion of the colloids are moving at any one time, the colloid velocities are bimodally distributed, and the motion occurs in channels or rivers between pinned regions. In addition, the channels seen in Fig. 2(a) are not static but change over time, so that any one colloid is only temporarily trapped in a pinning site. These features of the plastic flow are in agreement with observations in colloidal experiments [5] and in vortex simulations of the strongly pinned regime [15,16]. Elastic depinning of the colloids occurs through elastic flow similar to that in Fig. 2(b).

In order to correlate the different types of flow observed in Fig. 2 with properties of bulk measurements, we show in Fig. 3 the scaling of the velocity vs driving force. For elastic depinning in the ordered regime [Fig. 3(a)], $v - f_d$ is fit to $v = (f_d - f_c)^\beta$ with $\beta = 0.66 \pm 0.02$, as illustrated in Fig. 3(b). These results are in good agreement with theoretical predictions [18] and simulation results [19] for elastic depinning of 2D CDWs. In contrast, in driven 2D vortex matter, Bhattacharaya and Higgins [8] found an exponent of $\beta = 1.2$ below the peak effect where elastic flow is expected to occur. This may be due to the effects of surface barriers disordering the lattice. Colloid experiments on elastic depinning [5] find $\beta < 1.0$, with a best fit to $\beta = 0.5$. We point out that for an infinite size system, true elastic depinning is not expected since dislocations should appear at large scales [21]. In addition, Coppersmith argued that rare pinning regions will lead to phase slips or plasticity for 2D systems with random disorder [22]. Both the simulation and the experiments are at a finite size, so in the elastic regime the distance between dislocations may be larger than the system size. In Figs. 3(c) and 3(d) the $v - f_d$ scaling for the plastic regime shows $\beta = 1.94 \pm 0.03$, close to the value of 2.2 found in the colloid experiments [5]. For larger system sizes, we find that the scaling region is expanded but the exponent is unchanged. The question of whether there is a universal exponent for plastic depinning remains open. Other studies in the plastic flow regime found $\beta = 2.0$ for electron flow simulations in metallic dots [23] and $\beta = 2.22$ for vortex flow in Josephson-junction arrays.

The velocity-force curves for both regimes are nonhysteretic. It is interesting to compare our results to experimental results for CDWs that in some cases find [24] *discontinuous* and hysteretic depinning transitions, which are believed to be due to phase slips or plasticity. Similar behavior appears in vortex simulations with periodic pinning and an incommensurate vortex lattice [25]. It would be very interesting to investigate the colloidal depinning for systems with periodic or anisotropic pinning to shed light on the type of dynamics that occurs during sharp and hysteretic depinning.

In Fig. 4 we show the response of colloids prepared in an ordered state to the application of a sudden pulse of driving force of different strengths in the plastic flow regime. Since the pulse strength is chosen to be below the depinning threshold value f_c, the initial colloid velocity is high and then gradually decreases. We find that a simple functional form cannot be fit to the curves. Instead, we use a stretched exponential fit as performed in experiments [5]: $v(t) = v_0 \exp[-(t/t_0)^\alpha] + v_1$. The values of t_0 and α depend on the magnitude of the drive. For the parameters investigated here, α falls between 0.08 and 0.4, in agreement with experiment. A similar stretched exponential decay was also found in vortex matter for the transient response to pulses [10]. We find that, in the long time limit, the colloid flow occurs only through a few long-lived channels. In the elastic regime, the decay of v is much faster and fits to an initial pure exponential with the velocities going to zero. In the elastic regime, the colloids move less than a lattice constant after a pulse is applied, whereas in the plastic

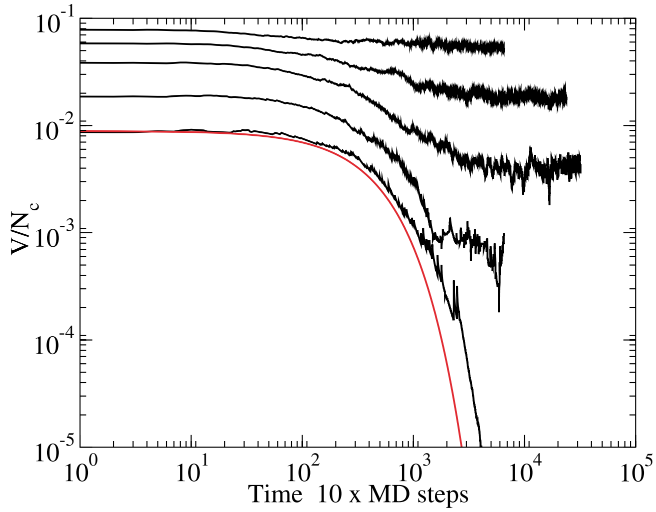

FIG. 4 (color online). Average transient velocity per colloid V/N_c vs time in the disordered region for $f_p = 0.25$ at a sudden applied drive of (from top to bottom) $f_d/f_c = 1.2, 1.0, 0.8, 0.6$, and 0.3. A stretched exponential $A \exp[-(t/t_0)^\alpha] + v_1$ can be fit to all the curves. Bottom curve: a stretched exponential fit for the $f_d/f_c = 0.3$ case, with $V/N_c = 0.0089 \exp[-(t/400)^1] + 0$.

regime, colloids in the moving channels can travel the entire length of the system. For increased system sizes, the transient times are enhanced in the plastic flow regime but are unchanged in the elastic regime. The long-lived transients in the plastic regime are responsible for the very slow velocity-force sweep necessary to measure an accurate depinning threshold. This sweep-rate dependence is also consistent with the experimentally observed sweep-rate dependent critical currents in the peak regime [11], where slow rates produce larger measured critical currents.

To summarize, we investigated the behavior of 2D colloids interacting with random disorder using Langevin simulations. For weak disorder the colloids form an ordered lattice which depins elastically and shows critical scaling in the velocity vs force curves, with $\beta = 0.67$, in agreement with studies of 2D CDWs. For increasing disorder strength, we find a sharp crossover to a disordered state, accompanied by a sharp increase in the depinning force, analogous to the peak effect observed for vortex matter in superconductors. In the disordered region, the colloids depin inhomogeneously into fluctuating channels and the $v - f$ curve scaling gives $\beta = 1.94$, in agreement with experiments. In the disordered flow regime, pronounced transients occur in response to a sudden pulse, with the late time dynamics determined by a few long-lived channels. Our results are in good agreement with recent experiments.

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