Zener Tunneling Between Landau Orbits in a High-Mobility Two-Dimensional Electron Gas

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Magnetotransport in a laterally confined two-dimensional electron gas (2DEG) can exhibit modified scattering channels owing to a tilted Hall potential. Transitions of electrons between Landau levels with shifted guiding centers can be accomplished through a Zener tunneling mechanism, and make a significant contribution to the magnetoresistance. A remarkable oscillation effect in weak field magnetoresistance has been observed in high-mobility 2DEGs in GaAs-AlGa$_{0.3}$As$_{0.7}$ heterostructures, and can be well explained by the Zener mechanism.

Scattering and dissipation are central issues in quantum transport in electronic systems. Of particular interest are the peculiar phenomena in a quantum Hall (QH) system, realized when a two-dimensional electron gas (2DEG) is subject to a strong magnetic field, B. In the integer quantum Hall effect (IQHE) regime, the Hall plateau is formed and the longitudinal transport is dissipationless [1]. It has been known for quite some time that, when a substantial current is passed through the 2DEG, the IQHE tends to break down. Above a certain critical current density, the quantized Hall plateau disappears and the transport in this regime becomes dissipative. Such phenomena have been observed in confined QH systems, such as Hall bar [2] or Corbino geometries [3].

The mechanism leading to the breakdown effect is still under debate. Among the possible explanations, the Zener tunneling mechanism was originally proposed by Tsui *et al.* [4]. In essence, if a sufficient Hall field, E_y, is established in a laterally confined QH system, the degeneracy of the Landau orbits is lifted. Zener tunneling can occur between the occupied Landau orbits below the Fermi level, E_F, and the empty orbits above the E_F, separated at a distance equivalent to the cyclotron diameter, $2R_c$. The condition for energy conservation is satisfied whenever $2R_c e E_y = l\hbar\omega_c$, where $l = 1, 2, 3\ldots$ integers, $\omega_c = eB/m^*$ is the cyclotron frequency, and m^* is the electron effective mass. By this mechanism, a critical current density of $j \sim 60$ A/m is needed to initiate the IQHE breakdown in GaAs, at a typical magnetic field of $B \sim 10$ T [4]. In the breakdown effect of IQHE, such tunneling events are likely to take place near the edges of the QH system where a high field can build up.

Quite surprisingly, we have observed the Zener tunneling effect in a different regime. Specifically, our effect takes place in the bulk of a 2DEG subjected to a weak magnetic field. Remarkable magnetoresistance oscillations are observed in high-mobility Hall bar specimens. The period of oscillation, $\Delta(1/B)$, is tunable by a dc bias current, J_{dc}: $\Delta(1/B) \propto n_e^{1/2}/J_{dc}$, where n_e is the electron

sheet density. This observation confirms a general selection rule for electron transport in a weak magnetic field, namely, $\Delta k_x \approx 2k_F$ for momentum transfer between the initial and the final Landau levels, where x labels the direction of the current and $2k_F$ is the Fermi wave vector of the 2DEG at zero magnetic field. This selection rule in momentum space translates into $\Delta Y \approx 2R_c$ in real space, where ΔY is the guiding center shift. Such a modulation of the scattering phase space is responsible for an oscillatory magnetoresistance, equivalent to a geometrical resonance [5] without a potential modulation.

We report in this Letter the observations for the new oscillations, and propose a simple model which can account for all the major features. The roles played by the short-range scatterers, in this effect, and possible experiments to explore Zener tunneling between composite fermion (CF) orbits in a half-filled Landau level [6] will be briefly discussed.

Our samples were cleaved from a wafer of a high-mobility GaAs-Al$_{0.3}$Ga$_{0.7}$As heterostructure grown by molecular-beam epitaxy, having an electron density $n_e \approx 2 \times 10^{11}$ cm^{-2} and a mobility $\mu \approx 3 \times 10^6$ cm^2/V s at a temperature $T = 4.2$ K. Such parameters were obtained after a brief illumination from a light-emitting diode. The distance between the electrons and the Si δ-doping layer is $d_s \approx 70$ nm. Four Hall bar specimens of width $w = 200, 100, 50,$ and 20 μm were processed by photolithography and wet etching. The 50 μm specimen has a NiCr front gate so that its electron density can be tuned between 1.9 and 4.0×10^{11} cm^{-2}. The experiments were performed in a sorption-pumped ^{3}He cryostat equipped with a superconducting magnet.

In principle, the Zener tunneling effect can be detected in standard magnetoresistance, $R_{xx}(B)$. However, in order to increase the sensitivity, a differential resistance r_{xx} was measured in the following fashion. A constant dc current I_{dc} was passed through the Hall bar, along with a small (100 nA) low frequency ($f = 23$ Hz) modulation current, i_{ac}. The differential magnetoresistance at the given dc bias,

$r_{xx} = (\partial V / \partial I)_{I_{dc}} = v_{ac}/i_{ac}$, was then recorded by a lock-in amplifier at the modulation frequency. A schematic circuit for the electrical measurement is shown in the inset of Fig. 1.

Our central finding concerns the strong oscillation in r_{xx} in the weak magnetic field region, $B < 4$ kG. In Fig. 1, we show such features from a 50 μm Hall bar measured at $T = 0.33$ K, for $I_{dc} = 0$, 10, 20, 30, 40, and 50 μA, respectively. For a zero dc bias current, the trace shows well-resolved Shubnikov–de Haas (SdH) oscillations for $B > 0.5$ kG. New oscillations emerge when a finite I_{dc} is applied to the specimen. Up to three orders of peaks can be clearly seen from the traces. Furthermore, the peaks shift towards higher B with increasing I_{dc}. The weakening of the SdH effect can be attributed partially to electronic heating by the dc bias. In contrast to the SdH oscillation whose amplitude diminishes quickly at increasing temperatures, the new oscillations persist to a temperature up to $T = 4$ K.

It is apparent that the new oscillations, observed here in r_{xx}, are roughly periodic in $1/B$. However, as can be shown, the exact resonance condition for Zener tunneling should correspond to peaks in its derivative, viz., in $\partial r_{xx}/\partial |B|$ traces [7]. To illustrate this point, we show in Fig. 2(a) the $\partial r_{xx}/\partial |B|$ trace which is obtained by numerical differentiation performed on the $r_{xx}(B)$ trace from a 50 μm sample with a bias $I_{dc} = 30$ μA. The inset, $1/B_l$ vs l, confirms that the oscillation in $\partial r_{xx}/\partial |B|$ is strictly periodic in $1/B$.

Remarkably, the oscillation period is tunable by the bias current I_{dc}. In Fig. 2(b), we plot the maximum positions B_l (1, 2, 3), obtained from $\partial r_{xx}/\partial |B|$, against the current density $J_{dc} = I_{dc}/w$, for the four samples. Roughly, all data collapse according to
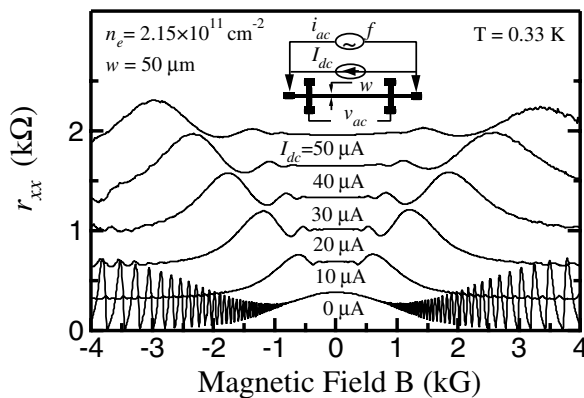
$$B_l \propto J_{dc}/l. \qquad (1)$$

Such oscillations arise owing to a new scattering channel opened up by a tilted Hall potential. To begin with, we consider a 2DEG system under crossed electric and magnetic fields, depicted in Fig. 3. The electric field is a Hall field along the y direction, $E_y = v_d B$, induced by a dc current density $J_{dc} = n_e e v_d$, where v_d is the drift velocity of the electrons.

The 2DEG is quantized into a series of Landau levels and has a wave function [8,9]

$$|NY\rangle = L_x^{-1/2} e^{ik_x x} \phi_N(y - Y), \qquad (2)$$

where N is the index of Landau levels, and $\phi_N(y - Y)$ is an oscillatory wave function centered at $Y = -l_B^2(k_x - m^* v_d/\hbar)$, with $l_B = \sqrt{\hbar/eB}$ the magnetic length.

The energy levels are given by

$$E_{NY} = (N + \tfrac{1}{2})\hbar \omega_c - eE_y Y + \tfrac{1}{2} m^* v_d^2. \qquad (3)$$

Because of the potential of the Hall field, the degeneracy of Landau levels with respect to the guiding center Y is lifted and the Landau levels are tilted spatially along the y direction with a slope given by eE_y. Since the density of electrons is largely homogenous within the sample, the distribution function of electrons should not depend on Y [9], which means the Fermi level is also tilted along the y direction with the same slope as that of the Landau levels [10]. As a result, all the states within one Landau level with different guiding center Y are equally occupied.

In the presence of elastic scattering, an electron may transfer a momentum $q_x = k_x - k_x'$ to a scatterer, which is equivalent to a hopping (shifting of the guiding center) in the y direction at a distance $\Delta Y = Y' - Y = l_B^2 q_x$. This hopping gives a current density j_y [9,11], hence, a conductivity

$$\sigma_{yy} = \frac{j_y}{E_y} = \frac{1}{E_y} \frac{e}{2L_x L_y} \sum_{\mu\mu'} W_{\mu\mu'}(Y' - Y)(f_\mu - f_{\mu'}), \qquad (4)$$
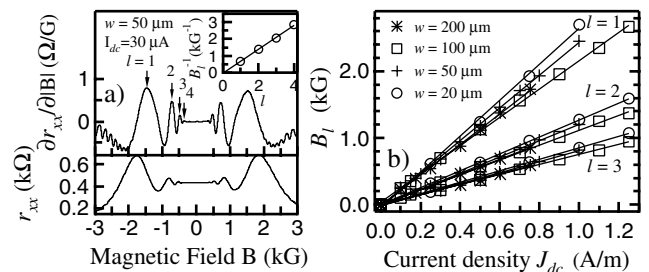

FIG. 1. The measured differential magnetoresistance traces at various dc current I_{dc} are shown for a 50 μm Hall bar (for clarity, the traces are shifted vertically in steps of 0.3 kΩ). Up to three orders of oscillations are clearly seen from the traces, and the oscillations are roughly periodic in $1/B$. The inset is a diagram for the electrical measurement.

FIG. 2. (a) The differential magnetoresistance r_{xx} at $I_{dc} = 30$ μA for the 50 μm Hall bar is illustrated together with its derivative $\partial r_{xx}/\partial |B|$. The inset shows the precise $1/B$ periodicity extracted from the $\partial r_{xx}/\partial |B|$ trace. (b) The oscillation maximum B_l ($l = 1, 2, 3$) versus current density J_{dc} are shown as a fan diagram for the Hall bars with different widths $w = 200$, 100, 50, and 20 μm (the electron densities, in units of 10^{11} cm^{-2}, are 2.08, 1.94, 2.15, and 2.14), respectively. A linear fit precisely reveals the relation $B_l \propto J_{dc}/l$, and the slope is nearly the same for all the samples.
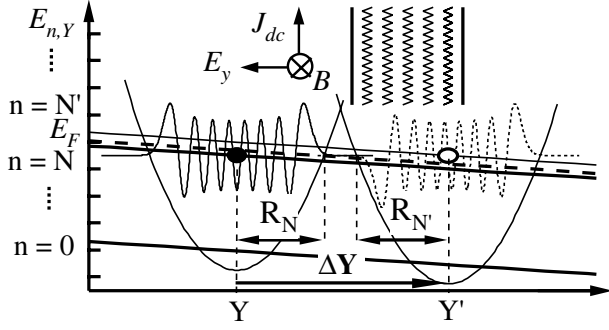
FIG. 3. The Landau levels are spatially tilted along the direction of the electric field (y direction), and the Fermi level has the same slope as the Landau levels. Elastic scattering causes the electron hopping between different Landau levels with a distance given by $\Delta Y = Y' - Y = l\hbar\omega_c/eE_y$. The vanishing of overlap between the oscillatory wave functions when $\Delta Y > R_N + R_{N'} \approx 2R_c$ means the maximum hopping distance allowed is about $2R_c$. The configuration of the crossed electric and magnetic fields is also shown.

where $\mu = (NY)$, $\mu' = (N'Y')$, L_x and L_y are the dimensions of the 2DEG, $f_\mu = 1/[e^{(E_\mu - E_F)/k_BT} - 1]$ the Fermi distribution of the electrons, and $W_{\mu\mu'}$ is the transition rate from the initial state $|\mu\rangle$ to final state $|\mu'\rangle$ in the Born approximation [12]:

$$W_{\mu\mu'} = \frac{2\pi}{\hbar}\frac{n_i}{L_xL_y}\sum_{q_x,q_y}|V(q)|^2|\langle\mu'|e^{i\vec{q}\cdot\vec{r}}|\mu\rangle|^2\delta(E_\mu - E_{\mu'})$$
$$= \frac{n_i}{\hbar L_x l_B}\delta(E_\mu - E_{\mu'})\int dQ_y\left|V\left(\frac{Q}{l_B}\right)\right|^2 J_N^l(Q),$$
$$(5)$$

where $Q \equiv ql_B = \sqrt{Q_x^2 + Q_y^2}$, $Q_x \equiv \Delta Y/l_B$, n_i is the density of the random scatterers, $V(q)$ is the effective Fourier component of the scattering potential seen by the 2DEG, and [13]

$$J_N^l(Q) = \left|\int e^{iq_y y}\phi_N(y - Y)\phi_{N+l}(y - Y')\right|^2$$
$$= \frac{N!}{(N + l)!}\left(\frac{Q^2}{2}\right)^l e^{-Q^2/2}\left[L_N^l\left(\frac{Q^2}{2}\right)\right]^2, \quad (6)$$

with $l = N' - N$ the index difference between involved Landau levels and $L_N^l(x)$ the generalized Laguerre polynomial.

In Eq. (5), the $\delta(E_\mu - E_{\mu'})$ accounts for the conservation of energy, which gives $eE_y\Delta Y = l\hbar\omega_c$. This means an electron hopping along the y direction should cause a transition between Landau levels, and the hopping distance is determined by

$$\Delta Y_l = \frac{l\hbar\omega_c}{eE_y} = l\frac{\hbar}{m^*v_d} = l\frac{e\hbar n_e}{m^*J_{dc}}, \quad (7)$$

which does not depend on the magnetic field, and is fixed for a given current density J_{dc}. The selection rule of Eq. (7)

is unusual, in that it stems only from the two-dimensional nature of the electrons.

The resistivity along the x direction (assume $\mu B \gg 1$) is $\rho_{xx} = \sigma_{yy}/(\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2) \approx \rho_{xy}^2\sigma_{yy}$. By working out σ_{yy}, finally we get

$$\rho_{xx} = \frac{h}{e^2}\frac{(2\pi)^3 n_i m^{*4}v_d^2}{h^6 n_e^2}\sum_{Nl}(f_N - f_{N+l})P_{Nl}(Q_{xl}), \quad (8)$$

with $Q_l \equiv \sqrt{Q_{xl}^2 + Q_y^2}$, $Q_{xl} \equiv \Delta Y_l/l_B$, and

$$P_{Nl}(Q_{xl}) = \frac{Q_{xl}^5}{l^4}\int dQ_y\left|V\left(\frac{Q_l Q_{xl}}{\Delta Y_l}\right)\right|^2 J_N^l(Q_l). \quad (9)$$

The function $P_{Nl}(Q_{xl})$ can be numerically evaluated for a given $V(q)$. We simply assume a constant $V(q)$, which is good for short-range scattering, to calculate this function. The results show that P_{Nl} has a dominant maximum at the point

$$Q_{xl} = \gamma\sqrt{2N + 1} \quad \text{with } \gamma \approx 2.0. \quad (10)$$

We need to consider only small l because $P_{Nl}(Q_{xl})$ is a monotonically decreasing function of l, thus, the term $f_N - f_{N+l}$ in Eq. (8) means the transition should occur at the vicinity of the Fermi level, i.e., we have $N \approx N_F$, where N_F is the Landau level index at the Fermi level. Therefore, Eq. (10) is equivalent to

$$\Delta Y_l = Q_{xl}l_B = \gamma R_c \approx 2R_c, \quad (11)$$

where $R_c = \sqrt{2N_F + 1}l_B = l_B^2\sqrt{2\pi n_e}$. Comparing with Eq. (7), the condition Eq. (11) leads to

$$B_l = \gamma\frac{\sqrt{2\pi}m^*}{e^2}\frac{1}{\sqrt{n_e}}\frac{J_{dc}}{l}, \quad (12)$$

which explains well the result of Eq. (1).

From the slopes of the fan diagram in Fig. 2(b), we obtain $\gamma = 1.72, 1.63, 1.88$, and 2.05 for the Hall bars with width $w = 200, 100, 50$, and $20 \ \mu$m, respectively. Such values, determined experimentally, are close enough to the theoretical value $\gamma \approx 2.0$, indicating the validity of our model even in a quantitative sense. Moreover, we measured the density dependence of the oscillation maxima, and plot the results in Fig. 4. It is clearly shown that peak positions $\propto 1/\sqrt{n_e}$.

The resonance condition Eq. (11) can be interpreted semiclassically as following. From Eq. (4), the conductivity σ_{yy} is proportional to the transition rate $W_{\mu\mu'}$ between two Landau levels near the Fermi level. The transition rate drastically goes to zero when $\Delta Y > R_N + R_{N'} \approx 2R_c$ because within this region there is almost no overlap between the oscillatory wave functions, as shown in Fig. 3. Thus, the furthest distance the electron can hop is around $(\Delta Y)_{max} \approx 2R_c$. Note that σ_{yy} is proportional to ΔY, so naturally a conductivity peak appears at $\Delta Y = (\Delta Y)_{max} \approx 2R_c$.
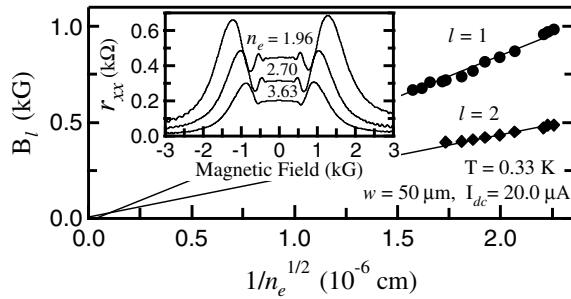
FIG. 4.   The density dependence measurement shows that the maximum positions are scaled as $1/\sqrt{n_e}$. The inset shows measured differential magnetoresistance traces at selected electron densities (in units of 10^{11} cm^{-2}).

The hopping at a distance $2R_c$ along the y direction is equivalent to a momentum transfer along the x direction $\Delta k_x = 2R_c l_B^2 = 2k_F$. It is interesting to point out that a similar momentum transfer mechanism has been used to account for the magnetoacoustic phonon resonance of a 2DEG [13].

In a high-mobility 2DEG, the elastic scatterers for electrons are mainly ionized impurities in the remote doping layer, residual background ionized impurities throughout the material, interface roughness, and neutral impurities in the GaAs well [14]. The remote ionized impurity scattering is long ranged, and in momentum space its potential is exponentially confined into a narrow range with a characteristic momentum $q_s \sim 1/d_s$ [15]. In our samples $d_s = 70$ nm, equivalent to a $q_s \sim 0.014$ nm^{-1} which is much less than $2k_F \approx 0.22$ nm^{-1}; therefore the remote ionized impurities are not likely to contribute to the oscillations. The other three mechanisms mentioned above are short ranged, and therefore, in principle, could contribute to the oscillations we are discussing. The scattering length of the interface roughness and neutral scatters are both at atomic scale, so their potential seen by the 2DEG in momentum space are all almost constant within the scale of $2k_F$. Our numerical result of Eq. (11) is modeled on this fact. It is usually assumed that the remote ionized impurities are the main sources of scattering for 2DEGs in GaAs/AlGaAs heterostructures. However, for high-mobility samples with a wide spacer, the residual impurities and interface roughness become important [16]. Indeed, according to a theoretical study [17], the negative magnetoresistance as shown in the $I_{dc} = 0$ μA trace of Fig. 1 is a strong evidence for the significance of such short-range scatterers in our samples.

In conclusion, we have observed a novel type of magnetoresistance oscillations in a laterally confined high-mobility 2DEG, which can be attributed to the spatial hopping of electrons between tilted Landau levels under a current-induced Hall field. Strong short-range scattering is needed for this class of oscillations to occur.

We comment on the possible application of such measurements to the composite fermion regime in a half-filled Landau level [6], where the Fermi surface effects have been well established but many interesting questions remain open. Because of the selection rules of both momentum and energy, the cyclotron energy (hence, the effective mass) of the CF state could be explored by Zener resonance. From the Zener tunneling mechanism point of view, since such oscillations require a finite momentum transfer, observation of this effect with CF would allow a look into their short-range interactions. Furthermore, in the CF regime, additional scattering mechanisms such as fluctuations of the effective magnetic field would be interesting to investigate.

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[7] The resonance condition is Eq. (12). From a Fourier analysis point of view, it gives an oscillatory resistivity $\rho_{xx} \propto \cos(2\pi l)$, where $l \propto J_{dc}/B$. Accordingly, the voltage $V \propto \rho_{xx}J_{dc} \propto J_{dc}\cos(2\pi l)$ and $r_{xx} \equiv \partial V/\partial I \propto \partial V/\partial J_{dc}$. It is easy to show that r_{xx} has a phase shift $\sim 90°$ with respect to ρ_{xx}, which can be largely restored by $\partial r_{xx}/\partial |B|$.
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