# New Spin-Orbit-Induced Universality Class in the Integer Quantum Hall Regime

Yshai Avishai and Yigal Meir

*Department of Physics and Ilse Katz Center for Meso- and Nanoscale Science and Technology, Ben Gurion University, Beer Sheva, Israel*

(Received 31 July 2001; published 29 July 2002)

Using heuristic arguments and numerical simulations it is argued that the critical exponent $\nu$ describing the localization length divergence at the integer quantum-Hall transition is modified in the presence of spin-orbit scattering with short-range correlations. The exponent is very close to $\nu = 4/3$, the percolation correlation length exponent, consistent with the prediction of a semiclassical argument. In addition, a band of weakly localized states is conjectured.

DOI: 10.1103/PhysRevLett.89.076602 PACS numbers: 72.20.My, 73.40.Kp, 73.50.Jt

Spin-orbit scattering (SOS) causes pronounced effects in disordered systems. In three dimensions (3D), it changes the universality class of the metal-insulator transition [1], while for a system of noninteracting electrons in 2D, it leads to a metal-insulator transition, which does not exist in its absence [2]. In the weakly localized regime, SOS changes localization into antilocalization [3], reversing the sign of the magnetoresistance, while in the strongly localized regime it increases the localization length (e.g., by a factor of 4 in quasi-1D systems), thus affecting the resistance by orders of magnitude [4]. The change in the universality class manifested in level statistics also suppresses the conductance fluctuations in the weakly localized regime (again by a factor of 4) [5].

In spite of these remarkable effects, there are just a few studies of SOS in the integer quantum-Hall (IQH) regime [6]. One possible reason is that in the presence of a strong magnetic field, SOS is not expected to change the symmetry of the Hamiltonian, and thus may not affect universal quantities. A counterexample, however, exists in the strongly localized regime, where SOS increases the localization length even in the presence of a strong magnetic field. Spin mixing induced by *random magnetic field* was studied in Refs. [7,8] (as a specific model for SOS), and [9]. The main conclusion is that random Zeeman term causes splitting of the spin-degenerate IQH transition, but does not change its universality class. The critical exponent for this kind of disorder then remains about $2.35 \pm 0.02$, the accepted numerical value for the IQH transition [10,11].

In this work we study an electronic system in the IQH regime, subject to random scalar and SOS potentials. Semiclassical (SC) considerations anticipate a change in the critical behavior of the IQH transition for short correlation length of the SOS potential. These arguments are then corroborated by numerical calculations.

Recall the SC approach to the IQH system in 2D [12] which was later extended to treat the IQH in layered three-dimensional systems [13]. In this description, electrons at energy $E$ follow skipping orbit trajectories around potential hills or valleys, and there is a critical energy $E_c$ where the trajectory percolates through the entire system [14]. Away from $E_c$, the electron is confined on a cluster of typical size (correlation length) $\xi_p$. Near threshold, $\xi_p \sim |E_c - E|^{-\nu_p}$, where $\nu_p = 4/3$ is the two-dimensional percolation exponent. As one approaches $E_c$, clusters approach each other near saddle points of the potential energy landscape. While, classically, the electron cannot move from one cluster to another, quantum mechanically it can tunnel through the potential barrier. If $E$ is close enough to $E_c$, the potential barrier is close to an inverted parabola and the tunneling probability is then proportional to $\exp[-(E_c - E)]$. The number of such saddle points through which tunneling occurs in a system of length $L$ is typically $L/\xi_p$. Since the transmission coefficient is multiplicative, the conductance $\sigma$ (or the tunneling probability) through the whole system is

$$\sigma \sim [e^{-(E_c - E)}]^{L/\xi_p} \equiv e^{-L/\xi}, \tag{1}$$

with $\xi \sim (E_c - E)^{-\nu}$ and $\nu = \nu_p + 1 = 7/3$. The numerical estimate $\nu = 2.35 \pm 0.02$ [11], which is somewhat supported by experimental data [15], is in excellent agreement with the result of the above argument (especially in view of its crudeness).

Following Ref. [13], this argument can be generalized to include SOS. If the spin-dependent part of the Hamiltonian is slowly varying, one can carry out a local gauge transformation, so that the spin points in the direction of the local effective random magnetic field $B(x, y)$ (generated by the SOS potential) [7]. In the adiabatic limit, where the spin-dependent potentials vary slowly in space, the problem separates into two independent IQH ones with different critical energies, split by twice the typical magnetic field $B_{\text{eff}} \equiv |\langle B(x, y)\rangle|$. Nonadiabaticity (short-range correlations) leads to mixing between these two effective spin-directions. Consequently, one may repeat the above argument recalling that in this case $E_c$ is shifted away by $B_{\text{eff}}$ from the potential energy of the saddle point [13]. The conductance $\sigma_{so}$ then behaves as

$$\sigma_{so} \sim [e^{-B_{\text{eff}}}]^{L/\xi_p} \equiv e^{-L/\xi_{so}}, \tag{2}$$

with $\xi_{so} \sim (E_c - E)^{-\nu}$ and $\nu = \nu_p = 4/3$.

  

This SC argument then predicts that the localization length critical exponent is equal to the two-dimensional classical percolation exponent. The physical reasoning behind the reduced localization exponent is simple: since the potential landscape for the opposite spin-directions is different, then, due to the random effective magnetic field, an electron approaching a saddle point may "prefer" to flip its spin (rather than tunnel through the saddle point), and then continue to propagate semiclassically [16]. The probability for such a Zener tunneling [17] depends on the local potential gradient, and is exponentially close to unity for strongly varying potentials [18]. In fact, since the tunneling probability at the saddle-point energy is equal to $1/2$ [19], one may expect that for short-correlated potentials there will be an *energy domain* where this spin-flip mechanism is dominant, leading to classical pecolation. Thus, beside affecting the critical exponent, SOS might shift the critical energy to higher values and defines a finite spectral interval of classically extended states.

We next check these SC based predictions within a specific, physically relevant model. Consider noninteracting electrons (charge $-e$, mass $m$, and spin operator $\mathbf{S}$) in *three* dimensions ($-L/2 \le x, y \le L/2$, $-\infty < z < \infty$) subject to a magnetic field $\mathbf{B} = \nabla \times \mathbf{A} = B\hat{\mathbf{z}}$ [with $\mathbf{A} = (-By, 0, 0)$], a random potential $V_R(x, y, z)$, and an additional potential $u(z) = u(-z)$ that confines the electrons near $z = 0$. Replacing $\mathbf{p}$ by $\mathbf{\Pi} = \mathbf{p} + \frac{e}{c}\mathbf{A}$ in the Dirac Hamiltonian for the large spinor component, we get,

$$H = \frac{1}{2m}\Pi^2 + V + \frac{1}{(2mc)^2}(\boldsymbol{\sigma}\cdot\mathbf{\Pi})V(\boldsymbol{\sigma}\cdot\mathbf{\Pi}) + gS_zB, \tag{3}$$

with $V = V_R(x, y, z) + u(z)$. The nonrandom part, $H_0 \equiv \frac{1}{2m}\Pi^2 + u(z) + gS_zB$ is diagonalized first, with eigenfunctions $\Psi_{nk\sigma}(xyz) = Z_0(z)\langle xy|nk\rangle \otimes |\sigma\rangle$. Here $Z_0(z)$ is the normalized ground state of the operator $p_z^2/2m + u(z)$ (a real *even* nodeless function of $z$), $\langle xy|nk\rangle = L_{nk}(xy)$ is an $n$th Landau level function of momentum $k$, and $|\sigma\rangle$ is a two component spin function. The number of degenerate Landau functions $L_{nk}(xy)$ is $\frac{L^2}{2\pi\ell^2}$ where $\ell$ is the magnetic length. Next we concentrate on the case where $\Psi_{0k\uparrow}(xyz)$ is degenerate with $\Psi_{1k\downarrow}(xyz)$, and SOS is highly relevant. (Such a Landau level crossing has been recently realized experimentally in AlAs samples [20,21].) Projection on the subspace spanned by these two degenerate Landau levels is justified when the Landau level and the Zeeman splittings are much larger than $V_R$. The projected Hamiltonian is then $H_{ij} = \langle\Psi_i|V_R + \frac{1}{(2mc)^2}\boldsymbol{\sigma}\cdot\mathbf{\Pi}V\boldsymbol{\sigma}\cdot\mathbf{\Pi}|\Psi_j\rangle$ with $i, j = 1, 2 = (0\uparrow), (1, \downarrow)$. Expanding $V_R(x, y, z)$ near $z = 0$, and defining $U_0(x, y) \equiv V_R(x, y, 0)$, $V_{so}(x, y) \equiv V_R'(x, y, 0)\int Z_0(z)zZ_0'(z)dz$ implies

$$\begin{aligned} H_{11} &= \langle 0k|U|0k'\rangle, \qquad H_{22} = \langle 1k|U|1k'\rangle, \\ H_{12} &= H_{21}^{\dagger} = i(\langle 0k|V_{so}|0k'\rangle + \langle 1k|V_{so}|1k'\rangle), \end{aligned} \tag{4}$$

where $U$ contains, in addition to $U_0$, all the spin-diagonal terms appearing in the Dirac Hamiltonian. The scalar potential $U$ thus plays the role of an effective random magnetic field, while the SOS potential allows for random spin flips.

The statistical properties of the potentials are determined by their strength $\alpha$ and correlation distance $\lambda$ (henceforth lengths and energies are expressed in units of $\ell$ and $\hbar\omega_c$), e.g., $\langle V_{so}(x, y)V_{so}(x', y')\rangle = \alpha_{so}^2 f(x - x')f(y - y')$, with $f(x) = (2\pi\lambda_{so}^2)^{-1/2}e^{-x^2/2\lambda_{so}^2}$. A similar relation holds for the scalar potential $U(x, y)$. Holding $\alpha = 1$ for $U$ and varying $\lambda = \lambda_{so}$ (as both were generated by the same potential $V_R$) and $\alpha_{so}$, we generate and diagonalize an ensemble of random Hamiltonians (4) for squares of different sizes $L = 40$, 60, and 80. The localization length of an eigenstate $\Psi$ is determined by [22]

$$\xi_L^2[\Psi] \propto \int y^2|\Psi(x, y)|^2dxdy - \left(\int y|\Psi(x, y)|^2dxdy\right)^2. \tag{5}$$

By dividing the energy spectrum into bins and averaging over $N(L)$ disorder realizations [$N(40) = 1000$, $N(60) = 300$, $N(80) = 150$], we are able to obtain the energy dependence of $\xi_L(E)$. In Fig. 1 we plot $\xi_L(E)/L$ for two values of $\lambda$, for $L = 80$ and $\alpha_{so} = 2$.

The immediate conclusions drawn from the figure are (i) $\xi_L(E)$, which in the absence of SOS was maximal at $E = 0$ (see inset), now has a maximum at two energies, $E = \pm E_c$. This leads to a splitting of the IQH transition. (ii) $\xi_L(E)$ increases as $\lambda$ becomes smaller. (iii) For small $\lambda$, and for $-E_c < E < E_c$, $\xi_L(E)$ is nearly constant and of the order of the system size, in accordance with the SC arguments.

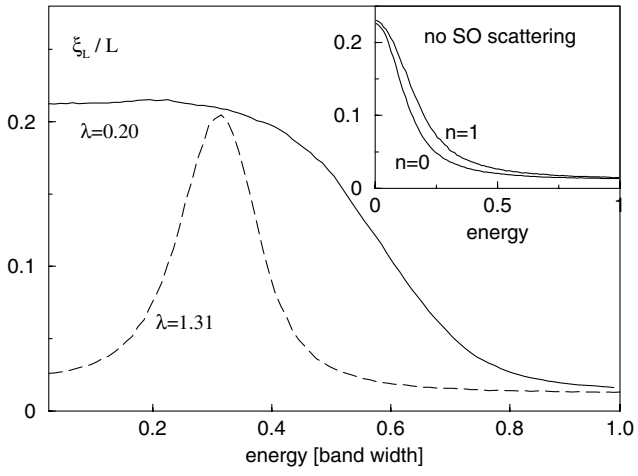


FIG. 1. The ratio $\xi_L/L$ for $L = 80$, as a function of energy, for two values of $\lambda$, the potential correlation length, and for SOS potential strength $\alpha_{so} = 2$. SOS leads to a splitting of the quantum-Hall transition. Lower values of $\lambda$ cause an increase in the localization length. In the inset we show $\xi_L/L$ in the absence of SOS in the lowest two Landau levels. $\xi_L$ is maximal at $E = 0$, the critical energy in this case.

In order to determine whether this variation of $\xi_L(E)$ is due, indeed, to a different critical behavior, we carry out the usual scaling analysis—evaluate $\xi_L(E)$ for different $L$'s, and collapse all the data onto a single plot after scaling the system size by $\xi$ (the $L \to \infty$ localization length), by setting $L/\xi_L(E) = F[L/\xi(E)]$. In Fig. 2 we plot $L/\xi_L(E)$ for different $L$'s, for $\lambda = 1.725$. At the critical energy $E = E_c$, $L/\xi_L(E)$ does not change, and thus $\xi$ diverges as $E \to E_c$. The scaling of all curves onto a single plot (Fig. 2, inset) determines $\xi(E)$, and by fitting $\xi(E) \sim (E - E_c)^{-\nu}$, we obtain the critical exponent $\nu$.

Figure 3(a) depicts the derived best values of $\nu$ as a function of $\lambda$ [23] (for $\alpha_{so}/\alpha = 2$). For small $\lambda$ the critical exponent is indeed very close to the expected SC value $\nu = \nu_p = 4/3$. When $\lambda$ increases, $\nu$ eventually increases and approaches the regular IQH value (open squares) consistent with the SC picture. Concomitant with the decrease in $\nu$ with decreasing $\lambda$, there is a shift of $E_c$ to higher values, again in agreement with the SC analysis. Figure 3 also depicts our results for the exponent $\nu$ in the absence of SOS, where no change from the IQH value is noticed (filled circles), and for a system where only the SOS correlation length, $\lambda_{so}$, has been changed (keeping $\lambda = 1$), for $\alpha_{so} = 1$ (diamonds) and $\alpha_{so} = 2$ (circles), demonstrating that the change in critical behavior is solely due to the change in $\lambda_{so}$. In order to minimize systematic errors we plot in Fig. 3(b) the difference $\nu - \nu_{QH}$ demonstrating, as expected, a crossover from a value of $-1$ to 0 as $\lambda$ increases. Our results also indicate that, unlike the SC prediction, eigenstates with energies between $\pm E_c$ are still localized even for short-correlated potentials. This is most probably due to quantum effects that tend to localize states in 2D.

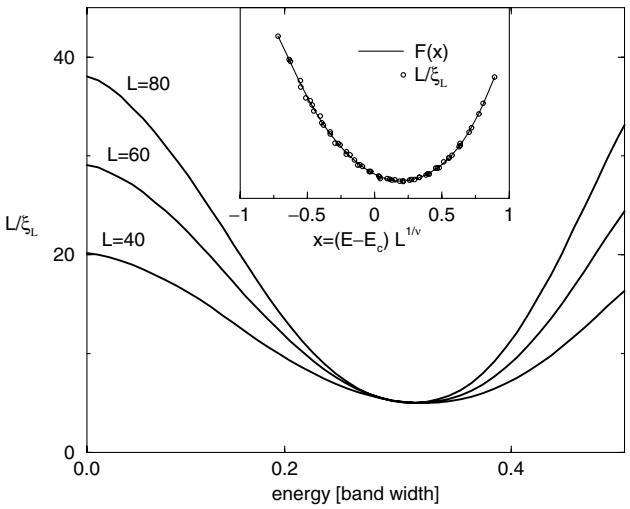


FIG. 2. The ratio $L/\xi_L$ as a function of energy for different values of $L$, for $\alpha_{so} = 2$ and $\lambda = 1.725$. The value of $E$ where the curves meet determines the critical energy. By scaling all three curves near the critical energy (inset), one finds $\xi(E)$, from which the critical exponent is determined (here $\nu = 2.16$).

Thus, for short-range correlations, calculations suggest the existence of a band of weakly localized states.

It is interesting to compare our results with previous works. Lee [7] and Hanna *et al.* [8] studied a Hamiltonian with a spin-dependent term $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$, in which $\mathbf{H}(\mathbf{r})$ is a random field that couples to the electron spin $\mathbf{S}$. Their conclusion is that, at least for random field which varies smoothly in space, the quantum-Hall transition splits, but the critical behavior remains unchanged. Indeed we checked that within the $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$ model, even if the correlation length of the random magnetic field decreases, the critical behavior remains quantum-Hall-like ($\nu \simeq 2.35$). The difference between that model and ours might seem surprising, since the model we use (4) looks very much like a random field. However, the distinction becomes clearer in Fig. 4, where we plot the density of states and $\xi_L(E)/L$ for the two models. While for the $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$ model the density of states splits into two peaks, indicating a splitting of the spin-degenerate Landau level into two independent Landau levels, in our model the density of states remains flat around $E = 0$ (even though the critical points move away from it), and thus the two effective spin-directions are still strongly mixed. While the $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$ model has only a unitary symmetry, our model, however, still obeys the full symmetry one expects for SOS in the presence of a magnetic field – time-reversal symmetry followed by reversing the (uniform) magnetic field. Thus, it may be that the two models belong to different universality classes.

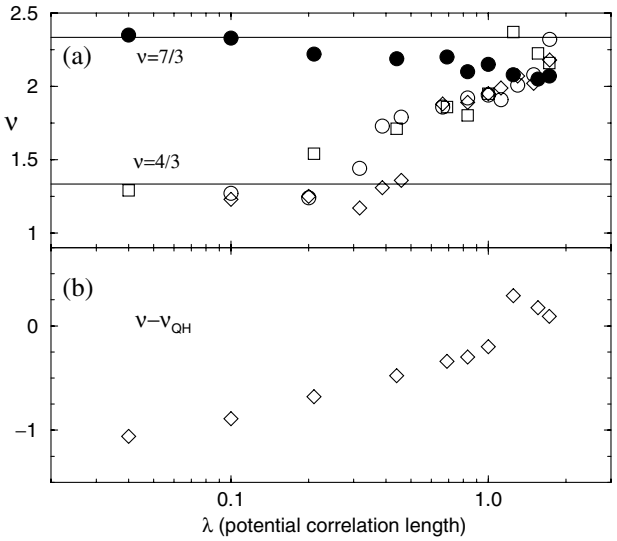


FIG. 3. (a) The derived values of $\nu$, the localization length critical exponent, as a function of $\lambda$, the potential correlation length, without spin-orbit scattering (filled circles) and with spin-orbit scattering (empty symbols, see text). For small $\lambda$ the exponent is close to $\nu_p = 4/3$, the percolation correlation length exponent, in agreement with the prediction of the SC theory, with a crossover to the regular IQH transition ($\nu \simeq 2.35$) for large $\lambda$. (b) The difference $\nu - \nu_{\mathrm{QH}}$ demonstrating a crossover from $-1$ to 0 as a function of the correlation length.

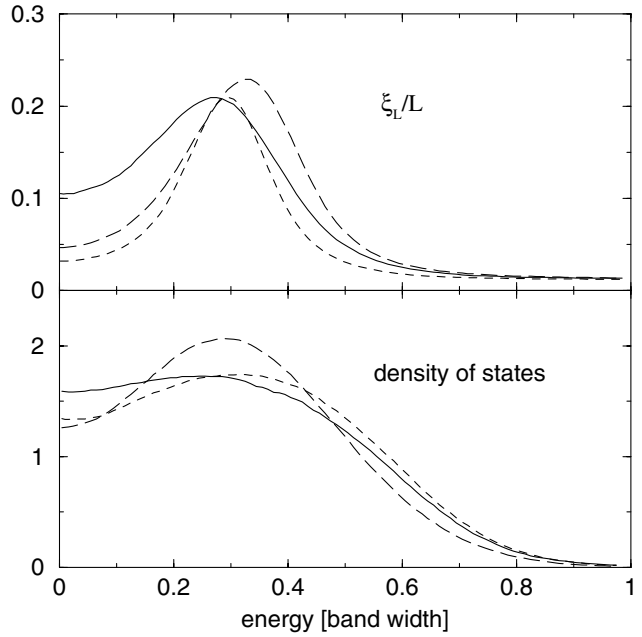


FIG. 4. Comparison of our model [Eq. (4), solid curves] to the $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$ model in the first (dotted curves) and the second (dashed curves) Landau levels: $\xi_L(E)/L$ and density of states for $L = 80$ and $\lambda = 1$. While the critical energies in both models split from $E = 0$, the density of states in our model remains flat at $E = 0$, in contrast with the split density of states for the $\mathbf{H}(\mathbf{r}) \cdot \mathbf{S}$ model, allowing for stronger mixing of the spins and a possible change in the critical behavior.

To conclude, we have presented arguments and demonstrated numerically that spin-orbit scattering in the integer quantum-Hall regime may alter the critical behavior for potentials with short-range correlations. The calculated critical exponent agrees very well with the percolation correlation length exponent, a value predicted by semiclassical arguments. It would be interesting to study the crossover between the two limits and to understand the phase diagram of this model, including the role of Zeeman splitting. In addition, the relevance of our model to the intriguing experimental observations at the crossing of Landau Levels [20] will be explored.

This work is supported by DIP, BSF, and ISF funds.

[1] For a review, see, e.g., N. F. Mott, *Metal-Insulator Transitions* (Taylor & Francis, London, 1990).
[2] S. N. Evangelou and T. Ziman, J. Phys. C **20**, L235 (1987); A. Kawabata, J. Phys. Soc. Jpn. **57**, 1717 (1988).
[3] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980); G. Bergmann, Phys. Rev. B **28**, 515 (1983); See also Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989).
[4] J. L. Pichard *et al.*, Phys. Rev. Lett. **65**, 1812 (1990); Y. Meir *et al.*, Phys. Rev. Lett. **66**, 1517 (1991); E. Medina and M. Kardar, Phys. Rev. Lett. **66**, 3187 (1991); Y. Meir and O. Entin-Wohlman, Phys. Rev. Lett. **70**, 1988 (1993).
[5] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)]; N. Zanon and J.-L. Pichard, J. Phys. (Paris) **49**, 907 (1988).
[6] For a review, see, e.g., M. Stone, *The Quantum Hall Effect* (World Scientific, Singapore, 1992).
[7] D. K. K. Lee, Phys. Rev. B **50**, 7743 (1994).
[8] C. B. Hanna *et al.*, Phys. Rev. B **52**, 5221 (1995).
[9] V. Kagalovsky *et al.*, Phys. Rev. B **55**, 7761 (1997).
[10] T. Ando and H. Aoki, J. Phys. Soc. Jpn. **54**, 2238 (1985).
[11] For a review of the numerical work, see B. Huckestein, Rev. Mod. Phys. **67**, 357 (1995).
[12] G. V. Mil'nikov and I. M. Sokolov, JETP Lett. **48**, 536 (1988).
[13] Y. Meir, Phys. Rev. B **58**, R1762 (1998).
[14] S. Luryi and R. F. Kazarinov, Phys. Rev. B **27**, 1386 (1983); S. A. Trugman, Phys. Rev. B **27**, 7539 (1983); R. Mehr and A. Aharony, Phys. Rev. B **37**, 6349 (1988).
[15] H. P. Wei *et al.*, Phys. Rev. Lett. **61**, 1294 (1988); H. P. Wei *et al.*, Surf. Sci. **229**, 34 (1990); S. Koch *et al.*, Phys. Rev. Lett. **67**, 883 (1991); F. Hohls, U. Zeitler, and R. J. Haug, Phys. Rev. Lett. **88**, 036802 (2002). See, however, V. T. Dolgopolov, G. V. Kravchenko, and A. A. Shashkin, Phys. Rev. B **46**, 13 303 (1992) and A. A. Shashkin, V. T. Dolgopolov, and G. V. Kravchenko, Phys. Rev. B **49**, 14 486 (1994), who report $\nu \simeq 1$; aee also D. Shahar *et al.*, Solid State Commun. **107**, 19 (1998), who see no critical behavior.
[16] D. G. Polyakov and M. E. Raikh, Phys. Rev. Lett. **75**, 1368 (1995).
[17] Zener tunneling between different Landau levels due to short-range potential has been experimentally observed by C. L. Yang *et al.*, cond-mat/0204488.
[18] A. Gramada and M. E. Raikh, Phys. Rev. B **56**, 3965 (1997).
[19] H. A. Fertig and B. I. Halperin, Phys. Rev. B **36**, 7969 (1987).
[20] E. P. De Poortere *et al.*, Science **290**, 1546 (2000).
[21] Such crossings have been experimentally demonstrated for composite fermions, with filling factors around $\nu = 3/2$ [R. R. Du *et al.*, Phys. Rev. Lett. **75**, 3926 (1995)], and at $\nu = 2/3$ and $\nu = 2/5$ [S. Kronmüller *et al.*, Phys. Rev. Lett. **81**, 2526 (1998); S. Kronmüller*et al.*, Phys. Rev. Lett.82, 4070 (1999); J. H. Smet *et al.*, Phys. Rev. Lett. **86**, 2412 (2000)], though the relevance of our model to composite fermions is questionable.
[22] *Finite Size Scaling*, edited by J. L. Cardy (North-Holland, Amsterdam, 1988); *Finite Size Scaling and Numerical Simulations of Statistical Systems*, edited by V. Privman (World Scientific, Singapore, 1990).
[23] The statistical error is ~7%. The error due to the fit to $F(x)$ was estimated by the $\chi^2$. The errors in Fig. 3 are the sum of these two sources, and is estimated to be $\pm 0.2$.