Comment on "Solution of the Relativistic Dirac-Morse Problem"

In a recent letter Alhaidari claims to have formulated and solved the Dirac-Morse problem [1]. He starts with the Hamiltonian (in a slightly different notation)

$$H = \boldsymbol{\alpha} \cdot [\mathbf{p} - i\beta \hat{\mathbf{r}} W(r)] + \beta M + V(r), \tag{1}$$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$. If we separate variables following [2], we get the radial equation

$$\left[-i\rho_2 \frac{d}{dr} + \rho_1 \left(W + \frac{\kappa}{r} \right) - E + V + M\rho_3 \right] \Phi = 0, \tag{2}$$

where $\Phi = \binom{G_{\ell j}(r)}{F_{\ell j}(r)}$, the ρ_i are the Pauli matrices, and $\kappa = \pm(j + \frac{1}{2})$ for $\ell = j \pm \frac{1}{2}$, which corresponds to Alhaidari's equation (1) if the quantum numbers ℓ and j are omitted.

There is no reason for the functions $V(r)$ and $W(r)$ which appear in the Hamiltonian to depend on the angular quantum numbers, which make their appearance only when we separate variables to solve the Dirac equation. Hence his choice of the constraint (in our notation)

$$W(r) = \frac{1}{S} V(r) - \frac{\kappa}{r} \tag{3}$$

with both V and W nonzero and S a constant cannot be satisfied, since κ varies when different values of ℓ and j are considered. Alhaidari could have avoided this mathematical contradiction by taking the Hamiltonian to be

$$H = \boldsymbol{\alpha} \cdot \left[\mathbf{p} - i\beta \hat{\mathbf{r}} \left(W(r) + \frac{K}{r} \right) \right] + \beta M + V(r), \tag{4}$$

where $K = \gamma^0(1 + \boldsymbol{\Sigma} \cdot \mathbf{L})$ is the Dirac operator, which leads to the radial equation

$$\left(-i\rho_2 \frac{d}{dr} + W\rho_1 - E + V + M\rho_3 \right) \Phi = 0. \tag{5}$$

Applying the transformation $\Phi = e^{-i\rho_2 \eta} \hat{\Phi}$ we get

$$\left[-i\rho_2 \frac{d}{dr} - (E - V) + \rho_1 (W \cos 2\eta - M \sin 2\eta) + \rho_3 (W \sin 2\eta + M \cos 2\eta) \right] \hat{\Phi} = 0. \tag{6}$$

Choosing $W = \frac{V}{\sin 2\eta}$, we get Eqs. (4) and (5) of Alhaidari for $G_{\ell j}$ and $F_{\ell j}$ leading to energy levels degenerate in l, j, m which is physically uninteresting. In the nonrelativistic formulation [3] the radial equation for the Morse potential does contain the centrifugal barrier contribution for nonzero values of ℓ.

Thus the only alternative is to choose a value of κ, for $\ell = 0$ we have $\kappa = -1$, so that Eqs. (4) and (5) of Alhaidari refer only to this state.

Even then it should be noted that the unitary transformation is not essential. It does not appear if we start with the Hamiltonian (1) with $V = 0$. Then Eq. (2) gives the F in terms of G and the second order equation for G for $\kappa = -1$ is

$$\left[-\frac{d^2}{dr^2} + \left(W - \frac{1}{r} \right)^2 - \frac{d}{dr} \left(W - \frac{1}{r} \right) - E^2 + M^2 \right] G = 0. \tag{7}$$

The choice $W - \frac{1}{r} = A - Be^{-\lambda r}$ gives the S-wave Morse equation with an additional A^2 term in the potential. Although the relativistic spectrum is different from that obtained by Alhaidari, the same nonrelativistic limit is obtained.

We note that the problems pointed out above also plague the applications mentioned by the author in the Erratum [1].

In conclusion, we do not think that the relativistic Morse potential problem has been correctly formulated and solved. On the other hand, if the problem is treated in the 1 + 1 dimension, no contradictions appear. This appears to be consistent with the fact that in nonrelativistic quantum mechanics one has a relationship between the radial functions of the Coulomb and oscillator problems and the wave functions of the one dimensional Morse problem.

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