Theoretical Prediction for the Ground State of ^{10}He with the Method of Analytic Continuation in the Coupling Constant

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Using the method of analytic continuation in the coupling constant with a ^{8}He + n + n model, we investigated the ground state of ^{10}He. In addition to a solution with the two valence neutrons in p states ($[p_{1/2}p_{1/2}]_{0+}$), we obtained a solution with the valence neutrons in s states ($[s_{1/2}s_{1/2}]_{0+}$) as the ground state. Experimentally, such a state has not yet been observed. This newly predicted state of ^{10}He with the main component of $[s_{1/2}s_{1/2}]_{0+}$ corresponds to the ground state of ^{11}Li with a halo structure.

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Recently, neutron-rich nuclei have been studied with much attention [1]. Among them, ^{10}He has been one of key nuclei in the studies because it has the maximal neutron ratio to the proton (neutron:proton = 4:1) until now and a double closed shell structure ($Z = 2$, $N = 8$). Furthermore, it has very important information on ^{11}Li of a typical Borromean system [2], in which any two-body subsystems (^{9}Li + n, n + n) are not bound, but the total three-body system (^{9}Li + n + n) is bound. ^{10}He is considered to have a ^{8}He + n + n resonant structure because it decays to ^{8}He and two neutrons [3,4]. However, the nucleus that contains one more proton, ^{11}Li, is bound. This binding property of ^{11}Li, which differs from ^{10}He, originates from a $p_{3/2}$-orbital proton in the ^{9}Li core. Then, a study of both nuclei, ^{10}He and ^{11}Li, can make a deep understanding of the Borromean mechanism changing from a barely unbound state (^{8}He + n + n) to a loosely bound state (^{9}Li + n + n).

The observed resonance energy of ^{10}He is $E_r = 1.2 \pm 0.3$ MeV, and the decay width is $\Gamma < 1.2$ MeV [3] ($S_{2n} = -1.07$ MeV [4]). For the ^{8}He + n subsystem, the p-wave resonance is observed at $E_r = 1.16$ MeV [5,6]. Then, because of the presence of the p orbit, it is natural to think that a state with the main component of $[p_{1/2}p_{1/2}]_{0+}$ is located at the low-energy region in ^{10}He. Also, theoretical calculations with the ^{8}He + n + n model, in which the ^{8}He + n interaction is determined by fitting the observed p state [5,6], predict a low-lying resonance [7,8]. Furthermore, very recently, the s-wave ground state is observed as a virtual state in ^{9}He (^{8}He + n) [9]. The energy of this newly observed ground state is about 1 MeV lower than the observed p state. Therefore, for ^{10}He, we also expect that there is another low-lying state having the main component of $[s_{1/2}s_{1/2}]_{0+}$.

For the ^{9}Li + n system, which is a subsystem of ^{9}Li + n + n, the situation is very similar to the ^{8}He + n system. The s-wave ground state is observed as a virtual state and the p states are observed as excited resonances. With a ^{9}Li + n + n model, Thompson and Zhukov pointed out the importance of the virtual state in the ^{9}Li + n subsystem to explain the observed properties of ^{11}Li, such as the momentum distribution [10]. Using the ^{9}Li-n potential, which has a virtual state, we also studied the ^{9}Li + n + n system [11]. The calculated ground state has a large component of $[s_{1/2}s_{1/2}]_{0+}$, which is similar to one in Ref. [10]. Furthermore, we predict a pairing-type three-body resonance with the main component of $[p_{1/2}p_{1/2}]_{0+}$, $E_x = 0.91$ MeV, $\Gamma = 0.18$ MeV [11]. This is easily explained as a picture of the pairing partner of the ground state. Since the s-orbital energy and the p-orbital energy are very near in the ^{9}Li + n subsystem, the $[p_{1/2}p_{1/2}]_{0+}$ configuration and the $[s_{1/2}s_{1/2}]_{0+}$ configuration couple to each other in ^{11}Li. The predicted excited state of ^{11}Li (the main component of $[p_{1/2}p_{1/2}]_{0+}$) would correspond to the observed state of ^{10}He [12].

Because of a difficulty in solving the unbound three-body problem, a quantitative study for ^{10}He is rare [7,8,13] in comparison with that for the neighboring nucleus, ^{11}Li. As a practical method of solving the unbound three-body problem, the complex scaling method (CSM) [14] was applied in studies of neutron-rich nuclei [15,16]. It is a very useful method to study the many-body unbound state. We have also applied it to the core + n + n system (see the review paper [17]).

As another method of solving the unbound three-body problem, the method of analytic continuation in the coupling constant (ACCC) was proposed in the 1970s [18]. However, it has not been used very much. The main reason is that it requires much computational power to solve a realistic three-body problem [18]. However, owing to a recent development of computational power and computational techniques, we do not have any difficulty in using the ACCC [19]. We can thus use a preferable method, the CSM or the ACCC, though the CSM may be better in many cases because of the small computational time and the accuracy of the solution.

The ACCC has very strong merit in comparison with the CSM in the study of resonances of broad widths and even solutions at the third quadrant on the complex energy plane. In the CSM, we transform the coordinate variable in the Hamiltonian as $r \rightarrow re^{i\theta}(p \rightarrow pe^{-i\theta})$. In nuclear physics, we often use a Gaussian-type potential because

of the simplicity. The Gaussian is transformed as $e^{-r^2 e^{2i\theta}} = e^{-r^2\cos(2\theta)} e^{-ir^2\sin(2\theta)}$. When θ is larger than $\frac{\pi}{4}$, this potential diverges. Thus, we cannot obtain the solution. We do not have this limitation in the ACCC, and we can obtain a wide range of solutions on the complex energy plane. Especially, as pointed out in Ref. [19], it is a very useful method to study the three-body S wave. Since the three-body S wave does not have a large potential barrier, it has a large decay width, and a theoretical treatment is difficult. Using a simple schematic model of the Borromean system, Tanaka, Suzuki, Varga, and Lovas studied the property of such a three-body S wave [19]. The present 0^+ state of ^{10}He is a typical situation.

In this study, we used the ACCC in order to study the ground state of ^{10}He. Since the conventional method is not sufficient for analyses of a three-body S wave, we have a theoretical possibility of predicting new states by using the ACCC. The predicted ground state of ^{10}He in this paper is one such state. Also, the presence of this state is a key to understanding the halo structure of ^{11}Li, whose mechanism has been one of the important problems in nuclear physics.

Here, we briefly describe the ACCC. The details are given in Ref. [20]. We consider an unbound state with Hamiltonian H. In the ACCC, we introduce a parameter λ (a coupling constant) in the Hamiltonian as $H(\lambda) = H_0 + \lambda V$. We denote the attractive part of the potential as V. Then, $H(\lambda = 1)$ is the original Hamiltonian. By increasing λ, we can obtain a bound-state solution, because V is the attractive part of the potential. For a two-body system, it is known that the square root of the energy behaves as $k_l(\lambda) \sim \sqrt{\lambda - \lambda_0}$ for $l \neq 0$ and $k_0(\lambda) \sim (\lambda - \lambda_0)$ for $l = 0$ around the branching coupling constant λ_0 [20]. Here, l is the relative angular momentum. In the case of $l \neq 0$, λ_0 is easily obtained as a coupling constant which gives the threshold energy [$E(\lambda_0) = 0$, $k_l(\lambda_0) = 0$]. In the case of $l = 0$, it is known that $k_0(\lambda_0)$ is not zero [$k_0(\lambda_0) = i\chi_0$, $\chi_0 < 0$]. For a three-body system, it is discussed by Tanaka *et al.* that the branching energy of the S wave for the Borromean system is nearly equal to zero due to the presence of an effective barrier of the three-body system [19]. We also numerically checked it for the present ^{8}He + n + n system by using a self-consistent method which is given in Ref. [20]. The calculated result shows that the branching energy $E(\lambda_0)$ is nearly equal to zero within the numerical error. Using the obtained momentum of k as a function of λ in the bound-state region, we carried out analytical continuation to the unbound region with the Padé approximation. The Padé approximation is given as

$$k_l^{MN}(x) = i\,\frac{c_0 + c_1 x + c_2 x^2 + \cdots + c_M x^M}{d_0 + d_1 x + d_2 x^2 + \cdots + d_N x^N}, \quad (1)$$

where $x = \sqrt{\lambda - \lambda_0}$. For the bound-state region, since $x = \sqrt{\lambda - \lambda_0} > 0$, $k_l^{MN}(x)$ is purely imaginary on the positive axis. For the unbound region, since $x = \sqrt{\lambda - \lambda_0}$ is imaginary, $k_l^{MN}(x)$ is complex.

We employ the ^{8}He + n + n Hamiltonian, which is given in Ref. [8]:

$$H = \sum_{i=1}^{2}\left[\frac{1}{2\mu}\boldsymbol{p}_i^2 + V^F_{\text{core}-\text{n}}(\boldsymbol{\eta}_i) + V^{\text{pseud}}(\boldsymbol{\eta})\right] + V_{nn}(|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2|) + \frac{1}{\mu_{\text{core}}}\,\boldsymbol{p}_1 \cdot \boldsymbol{p}_2\,. \quad (2)$$

The details of each term are given in Ref. [8]. As a neutron-neutron interaction, V_{nn}, we use the Minnesota potential [21]. The ^{8}He-n interaction, $V^F_{\text{core}-n}$, is constructed microscopically by using the effective nucleon-nucleon interaction of the modified Hasegawa Nagata potential (MHN) [22]. In the present calculation, we introduced the parameter λ to the mid-range attractive part of the ^{8}He-n potential as $V^F_{\text{core}-n}(\eta) = V^{(1)}(\eta) + \lambda V^{(2)}(\eta) + V^{(3)}(\eta)$. In a previous paper, we already introduced the parameter δ, which has the relation $\lambda = 1 + \delta$, in order to reproduce the observed p-wave spectrum of ^{9}He ($\delta = 0.102$) [8]. Thus, hereafter, we use δ as a substitute for λ.

We used the variational method with the so-called Hybrid-TV model [23]. Using this model, we could accurately treat both the core-n and n-n correlations within a small basis dimension, as discussed in Refs. [16,23]. The variational function for the Hybrid-TV model is expressed by the superposition of the V-type function and the T-type function as

$$\Psi_{JM} = \Phi_{JM}(V) + \Phi_{JM}(T). \quad (3)$$

The details are given in Refs. [8,16].

In Fig. 1, we give the calculated complex energy as a function of the potential strength parameter, δ, within a

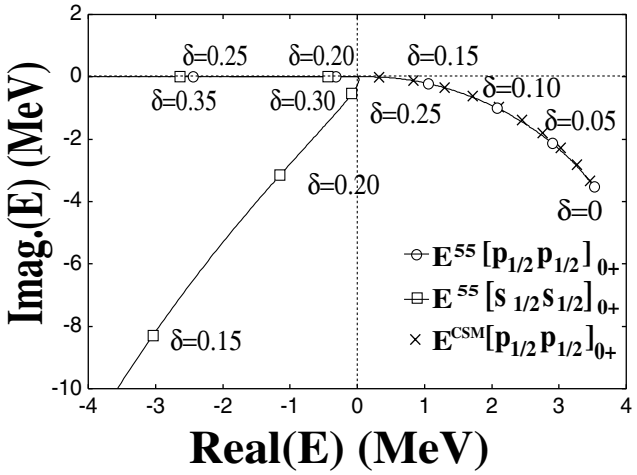

FIG. 1. Trajectory of the complex energy by changing the potential strength parameter (δ). The trajectory of the solid curve with circles is that of two neutrons in the $p_{1/2}$ orbit. The trajectory of the solid curve with squares is that of two neutrons in the $s_{1/2}$ orbit. The crosses indicate the calculated complex energy for two neutrons in the $p_{1/2}$ orbit with the CSM ($\delta = 0, 0.02, \ldots, 0.18$).

single channel. Here, we show a plot of $E^{55}(\delta)$ ($=E_r - i\Gamma/2$) with the 55 Padé approximation ($M = 5$, $N = 5$), where the energy was calculated with the relation of $E = \frac{(\hbar k)^2}{2M}$. The trajectory of the solid curve with circles is that of two neutrons in the $p_{1/2}$ orbit ($[p_{1/2}p_{1/2}]_{0+}$). We also plot the calculated complex energy of the CSM (crosses: $\delta = 0, 0.02, \ldots, 0.18$). The extrapolated energy of the ACCC (the solid curve with circles) exhibits a good correspondence with a more accurate value, which was calculated with the CSM. For example, in the case of $\delta = 0.102$, $E^{55} = 2.05 - i0.97$ MeV, and $E^{\mathrm{CSM}} = 2.067 - i0.925$ MeV [8].

The trajectory of the solid curve with squares is that of two neutrons in the $s_{1/2}$ orbit, $[s_{1/2}s_{1/2}]_{0+}$. The solution is not given in the case of the CSM, because we cannot solve it due to the divergent property of the complex scaled Gaussian potential, as mentioned above. This is the main reason why we used the ACCC in the present analyses. For a value smaller than $\delta = 0.27$, ^{10}He is not bound. Experimentally, the bound state of ^{10}He is not observed, which means that the strength of the ^{8}He-n potential should be smaller than $\delta = 0.27$.

Considering the $^8\mathrm{He} + n$ subsystem, δ is more limited. Experimentally, the s state of the $^8\mathrm{He} + n$ system is observed as a virtual state. In the present model, the $^8\mathrm{He} + n$ system becomes a virtual state for a value smaller than $\delta = 0.291$. For much smaller values of δ, since the energy of the virtual pole is far from the threshold energy, its observation becomes very difficult [24]. Therefore, we cannot accept a solution which is far from the threshold as a physically meaningful solution. It is reasonable to think that δ is larger than ~ 0.24 ($E = -0.74$ MeV in Table I). We can thus expect $\delta = \sim 0.24$–0.27. In the case of a typical value of $\delta = 0.26$, it becomes a state very near to the threshold energy, $E = -0.001 - i0.271$ MeV. The solution with the main component of $[p_{1/2}p_{1/2}]_{0+}$, e.g., $E = 2.067 - i0.925$ MeV [8], produces much higher energy than those of the permitted solutions of $[s_{1/2}s_{1/2}]_{0+}$ around $\delta = \sim 0.24$–0.27. It is natural to think that the main configuration of the ground state should be $[s_{1/2}s_{1/2}]_{0+}$. It should be mentioned that all solutions of $[p_{1/2}p_{1/2}]_{0+}$, as easily seen from trajectories in Fig. 1, have a larger energy than those of $[s_{1/2}s_{1/2}]_{0+}$. If we simply define the ground state as the lowest energy state, the solution of $[p_{1/2}p_{1/2}]_{0+}$ cannot be the ground state. The problem is whether the ground state having the main component of $[s_{1/2}s_{1/2}]_{0+}$ will be observed or not.

In the fourth column in Table I, we give the 0^+ solutions of the lowest energy in which almost all possible configurations are coupled in the $^8\mathrm{He} + n + n$ model (Hybrid TV model): $[s_{1/2}s_{1/2}]_0 + [p_{1/2}p_{1/2}]_0 + [d_{5/2}d_{5/2}]_0 + [d_{3/2}d_{3/2}]_0 + [f_{7/2}f_{7/2}]_0 + [f_{5/2}f_{5/2}]_0 + [g_{9/2}g_{9/2}]_0 + [g_{7/2}g_{7/2}]_0 + [l = L = 0, S = 0]_0$. Here, L is the relative angular momentum between ^{8}He and the center of the two valence neutrons, l is the relative angular momentum between the two valence neutrons, and S is the total spin of the two valence neutrons. This model state contains all $(l_j)^2$ two valence neutrons configurations up to the $g_{7/2}$ orbit. And it contains explicitly the 0^+ pairing correlation between the two valence neutrons as the T-type configuration of $[l = L = 0, S = 0]_0$. Here, for the p-wave potential, the potential strength ($\delta_p = 0.102$) is fixed in order to reproduce the p-wave spectrum of the $^8\mathrm{He} + n$ subsystem. As can be seen in Table I (fourth column), the calculated ground state energy of ^{10}He is very near to the $^8\mathrm{He} + n + n$ threshold energy for the above-mentioned parameter $\delta = \sim 0.24$–0.27.

In Fig. 2, the observed resonance in ^{10}He [3] (first column) and in ^{11}Li (second column) [12] are given. Also, the calculated 0^+ states in ^{10}He (third column) and the calculated $3/2^-$ states in ^{11}Li [11] (fourth column) are given. For ^{10}He, the potential strength, $\delta = 0.25$, is employed. The ground state energy is $E_r = 0.05$ MeV and the decay width is $\Gamma = 0.21$ MeV. This configuration of the ground state of ^{10}He would correspond to the ground state of ^{11}Li. For the excited 0^+ state, we use the CSM because of the accuracy of the solution and the computational time. The calculated resonance energy is $E_r = 1.68$ MeV and the decay width is $\Gamma = 1.12$ MeV. This excited state of ^{10}He would correspond to our predicted state [11] of ^{11}Li. For ^{10}He, the observed resonance has been shown to have mainly the component of $[p_{1/2}p_{1/2}]_{0+}$. Also, for ^{11}Li, the observed ground state has been shown to have the main component of $[s_{1/2}s_{1/2}]_{0+}$. Thus, an observation of the

TABLE I. Calculated energies of ^{9}He and ^{10}He.

δ	$E^{55}(^9\mathrm{He})$ (MeV) $s_{1/2}$	$E^{55}(^{10}\mathrm{He})$ (MeV) $[s_{1/2}s_{1/2}]_0$	$E^{55}(^{10}\mathrm{He})$ (MeV) Hybrid-TV
0.30	−0.00 (bound)	−0.44 (bound)	−0.76 (bound)
0.29	−0.00 (virtual)	−0.17 (bound)	−0.44 (bound)
0.28	−0.00 (virtual)	−0.00 (bound)	−0.19 (bound)
0.27	−0.07 (virtual)	0.03–i0.09 (unbound)	−0.02 (bound)
0.26	−0.19 (virtual)	−0.00–i0.27 (unbound)	0.04–i0.03 (unbound)
0.25	−0.40 (virtual)	−0.08–i0.54 (unbound)	0.05–i0.11 (unbound)
0.24	−0.74 (virtual)	−0.21–i0.89 (unbound)	−0.02–i0.19 (unbound)
0.23	−1.26 (virtual)	−0.38–i1.32 (unbound)	−0.18–i0.28 (unbound)

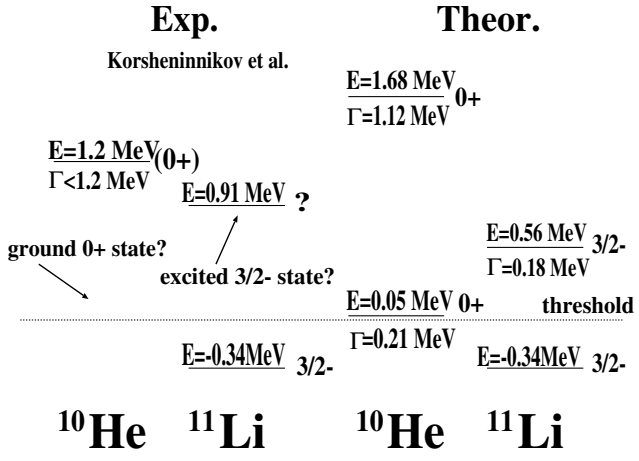

FIG. 2. Observed resonance in ^{10}He [3] (first column) and the observed states in ^{11}Li [12] (second column). The calculated 0^+ states in ^{10}He (third column) and the calculated $3/2^-$ states in ^{11}Li [11] (fourth column) are shown.

corresponding state in each nucleus is important in order to consistently understand the binding mechanism of the three-body S wave.

In this work, we investigated the ground state of ^{10}He by using the ACCC. The obtained energy of the unbound states is very near to the $^8\mathrm{He} + n + n$ threshold energy within a reasonable ^{8}He-n potential ($\delta = \sim 0.24$–0.27). Furthermore, by using any parameter of the potential strength, we could not obtain a ground-state solution which corresponds to the observed resonance energy around $E_r = 1.2$ MeV (the calculated maximum energy of the ground state is 0.05 MeV). On the contrary, the solution of the first excited state was obtained at $E_r = 1.68$ MeV, where the $^8\mathrm{He} + n$ subsystem was also reproduced at the same time. Then, the observed resonance in ^{10}He is considered to have the main component of $[p_{1/2}p_{1/2}]_{0+}$, which is the same conclusion as in a previous paper [8]. Therefore, we conclude that the ground state of ^{10}He has not yet been observed. Also, it should exist in the energy region of the $^8\mathrm{He} + n + n$ threshold. This conclusion is also supported by an analogy from the neighboring nucleus, ^{11}Li. Therefore, we think that experimental searching of the missing ground state of ^{10}He is important.

As far as the ACCC is concerned, we understand that it is also a very useful method to study the unbound state. In Ref. [19], Tanaka, Suzuki, Varga, and Lovas emphasize that the ACCC would be useful for the analysis of a three-body S wave. We predict a new state of the three-body S wave in ^{10}He, which is not treated using the conventional method. Since other interesting problems have been discussed for the three-body S wave, e.g., the Efimov state, the effective barrier, etc., it would be very meaningful to apply the ACCC to a wide region. In the future, we will apply the ACCC to ^{11}Li by directly comparing with ^{10}He.

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