Chaotic Dot-Superconductor Analog of the Hanbury Brown–Twiss Effect

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(Received 8 March 2002; published 2 July 2002)

As an electrical analog of the optical Hanbury Brown–Twiss effect, we study current cross correlations in a chaotic quantum dot-superconductor junction. One superconducting and two normal reservoirs are connected via point contacts to a chaotic quantum dot. For a wide range of contact widths and transparencies, we find large positive current correlations. The positive correlations are generally enhanced by normal backscattering in the contacts. Moreover, for normal backscattering in the contacts, the positive correlations survive when suppressing the proximity effect in the dot with a weak magnetic field.

In quantum theory, identical particles are indistinguishable. Under exchange of any pair of particles, the many-body wave function remains invariant up to a sign, positive for bosons and negative for fermions. This exchange symmetry was used in the pioneering interferometer experiment with photons, by Hanbury Brown and Twiss [1]. Several theoretical works [2–4] have suggested different analogs for this experiment with electrons in mesoscopic multiterminal conductors. It has been shown [3] that the fermionic statistics of electrons leads to negative correlations between currents flowing in different terminals. Such negative correlations were also recently observed experimentally [5].

When normal conductors are connected to a superconductor, correlations are introduced between electrons and holes due to Andreev reflections at the normal-superconductor interface, a phenomenon known as the proximity effect. The influence of the proximity effect on the current auto-correlations, i.e., the shot noise, in two-terminal diffusive normal-superconductor junctions was recently studied [6,7].

In multiterminal conductors, Andreev reflection can lead to positive cross correlations between currents flowing in the contacts to the normal reservoirs [8,9]. So far, positive correlations have been predicted only for single mode junctions [8–10]. Moreover, in multiterminal diffusive junctions it was found that cross correlations are negative in the absence of the proximity effect [11]. This raises two important questions: (i) Are the positive correlations in normal-superconducting junctions a large effect, of the order of the number of modes in multimode junctions, and if this is the case (ii) is the proximity effect necessary to obtain these positive correlations?

In this Letter, we give an answer to these two questions. The positive correlations are large, and, surprisingly, become enhanced by normal backscattering at the normal-superconducting interface. Moreover, positive correlations can exist even in the absence of the proximity effect, if the normal-superconductor interface is nonideal.

We study the current correlations in a system consisting of a chaotic quantum dot connected via point contacts to one superconducting and two normal reservoirs. Systems consisting of chaotic dots coupled to superconductors have recently [12] attracted a lot of interest. The generic properties of the model makes our result qualitatively relevant for multiterminal normal-superconducting structures with random scattering.

A schematic picture of the system is shown in Fig. 1. A quantum dot is connected to two normal reservoirs (N_1 and N_2) and one superconducting reservoir (S) via quantum point contacts. The contacts to the normal and superconducting reservoirs have mode independent transparency Γ_N and Γ_S, respectively, and support N and M transverse modes. The conductances of the point contacts are much larger than the conductance quanta $2e^2/h$, i.e., $N\Gamma_N, M\Gamma_S \gg 1$, so Coulomb blockade effects in the dot can be neglected. The two normal reservoirs are held at the same potential V and the potential of the superconducting reservoir is zero.

We consider the case where the classical motion in the dot is chaotic on time scales longer than the ergodic time τ_{erg}. The quasiparticle dwell time in the dot, $\hbar/E_{\mathrm{Th}}$, is assumed to be much larger than τ_{erg}, but much smaller than the inelastic scattering time. Here $E_{\mathrm{Th}} = (2N\Gamma_N + M\Gamma_S)\delta/\pi$, where δ is the mean level spacing in the dot. Under these conditions random matrix theory [13] can be used to describe the statistical properties of the scattering matrix S_d of the dot.
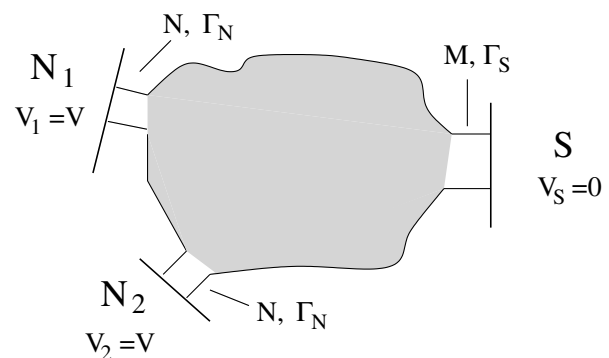

FIG. 1. A chaotic quantum dot, acting as a beam splitter, is connected to two normal (N_1 and N_2) and one superconducting (S) reservoirs via quantum point contacts.

Because of the random scattering in the dot, the current $I_i(t)$ in contact i fluctuates around its quantum statistical average $\bar{I}_i$. We study the zero-frequency spectral density of the current cross correlations $P_{12} = 2 \int dt \, \overline{\Delta I_1(t) \Delta I_2(0)}$, where $\Delta I_j(t) = I_i(t) - \bar{I}_i$. The correlation can be expressed in terms of the total scattering matrix S of the dot and superconductor. We consider the limit of zero temperature and a potential eV much lower than E_{Th} and the superconducting gap Δ, where the energy dependence of the scattering matrix S can be neglected. Moreover, at $|E| < \Delta$ no quasiparticles can escape into the superconductor, and S describes [13] only scattering between the normal reservoirs,

$$S = \begin{pmatrix} S^{ee} & S^{eh} \\ S^{he} & S^{hh} \end{pmatrix}, \qquad S^{\alpha\beta} = \begin{pmatrix} S_{11}^{\alpha\beta} & S_{12}^{\alpha\beta} \\ S_{21}^{\alpha\beta} & S_{22}^{\alpha\beta} \end{pmatrix}, \quad (1)$$

where $S_{ij}^{\alpha\beta}$ are matrix amplitudes ($N \times N$) for injected quasiparticles (e or h) of type β in lead j to be backreflected as quasiparticles of type α in lead i.

Noting that the current fluctuation is just the sum of the fluctuations of electron and hole currents, the noise power can be conveniently written [8] as

$$P_{12} = P_{12}^{ee} + P_{12}^{hh} + P_{12}^{eh} + P_{12}^{he}, \quad (2)$$

where the noise power $P_{12}^{\alpha\beta}$ for correlation between quasiparticle currents is given by

$$P_{12}^{\alpha\beta} = \sigma V \frac{4e^3}{h} \sum_{i,j=1,2} \mathrm{tr}[(S_{1i}^{\alpha e})^\dagger S_{1j}^{\alpha h} (S_{2j}^{\beta h})^\dagger S_{2i}^{\beta e}], \quad (3)$$

with $\sigma = +(-)$ for $\alpha = \beta$ ($\alpha \neq \beta$). The expression for the correlations in Eqs. (2)–(3) is an extension of the result [3] for a purely normal conductor, taking into account both electron and hole quasiparticles. However, unlike the normal cross correlations, which are manifestly negative, the cross correlation P_{12}, can be positive, because the correlations between different types of quasiparticles, $P_{12}^{eh} + P_{12}^{he}$, are positive.

The ensemble averaged correlations $\langle P_{12} \rangle$ are calculated using the statistical properties of the scattering matrix S_d of the dot. We first consider the case with ideal contacts $\Gamma_N, \Gamma_S = 1$ and no magnetic field in the dot, i.e., conserved time reversal symmetry. In this case it is useful to decompose S_d as (see, e.g., Ref. [13])

$$S_d = \begin{pmatrix} U & 0 \\ 0 & U' \end{pmatrix} \begin{pmatrix} r_{2N,2N} & t_{2N,M} \\ t_{M,2N} & r_{M,M} \end{pmatrix} \begin{pmatrix} U^T & 0 \\ 0 & U'^T \end{pmatrix}, \quad (4)$$

where U (U') is a unitary matrix of dimension $2N \times 2N$ ($M \times M$), uniformly distributed in the unitary group. The diagonal matrices $r_{2N,2N}$ and $r_{M,M}$ have $\min(2N, M)$ elements $\sqrt{1 - T_n}$ and the rest unity ($t_{2N,M}$ and $t_{M,2N}$ follow from the unitarity of S_d). Here T_n are the transmission eigenvalues, which have a density [14] $\rho(T) = (2N + M)/(2\pi)\sqrt{T - T_{\min}}/(T\sqrt{1 - T})$, where $T_{\min} = [(2N - M)/(2N + M)]^2$ is the smallest possible eigenvalue.

Inserting the decomposition into the expression for the total scattering matrix S we find [13] the quasiparticle

scattering amplitudes $S^{\alpha\beta}$ in Eq. (1) as, e.g., $S^{ee} = U\sqrt{1 - T}/(2 - T)U^*$ ($T = 1 - r_{2N,2N}^2$). By summing over injection contacts in the expression for the individual quasiparticle current correlators in Eq. (3) and inserting the scattering amplitudes, we obtain

$$P_{12}^{ee} = V \frac{4e^3}{h} \mathrm{tr}[U(1 - \mathcal{R})U^* C_1 U^T \mathcal{R} U^\dagger C_2] \quad (5)$$

and similar for the other terms $P_{12}^{\alpha\beta}$. Here $\mathcal{R} = T^2/(2 - T)^2$ are eigenvalues of the matrix product $(S^{eh})^\dagger S^{eh}$ and the matrix C_1 is diagonal with elements $(C_1)_n = 1$ for $n \leq N$ and 0 otherwise. The matrix $C_2 = 1 - C_1$. The ensemble average of Eq. (5) is carried out in two steps. $P_{12}^{\alpha\beta}$ is averaged over the unitary matrix U, using the diagrammatic technique in Ref. [15]. This gives, to leading order in N, M (i.e., neglecting weak localization corrections) for the total current corellations in Eq. (2),

$$\langle P_{12} \rangle_U = V \frac{4e^3}{h} \left(\mathrm{tr}[\mathcal{R}(1 - \mathcal{R})] - \frac{\mathrm{tr}(\mathcal{R})\mathrm{tr}(1 - \mathcal{R})}{4N} \right). \quad (6)$$

We then perform the average over transmission eigenvalues by integrating $\langle P_{12} \rangle_U$ weighted by the density $\rho(T)$. By using that to leading order in N, we have $\langle \mathrm{tr}(\mathcal{R})\mathrm{tr}(1 - \mathcal{R}) \rangle_T = \langle \mathrm{tr}(\mathcal{R}) \rangle_T \langle \mathrm{tr}(1 - \mathcal{R}) \rangle_T$, we obtain ($\langle \cdots \rangle = \langle \cdots \rangle_{U,T}$)

$$\frac{\langle P_{12} \rangle}{P_0} = \frac{(1 + \gamma)^2}{2\gamma^2} \left(\frac{1}{1 + \gamma} - 2\frac{\gamma}{f} - \frac{f^2 - 16\gamma^3}{f^{5/2}} \right), \quad (7)$$

where $f(\gamma) = 1 + 6\gamma + \gamma^2$ and $\gamma = 2N/M$, the ratio between the conductances of the point contacts connected to the normal and the superconducting reservoirs. The correlation is normalized with $P_0 = 4VNe^3/h$, i.e., it is large, of order N. The correlation $\langle P_{12} \rangle$, plotted in Fig. 2, is positive for a dominating coupling to the superconductor, but
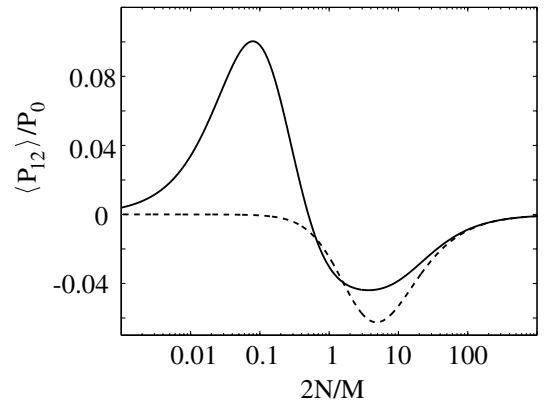

FIG. 2. The ensemble averaged current cross correlation $\langle P_{12} \rangle$ with (solid line) and without (dashed line) a proximity effect in the dot, as a function of $\gamma = 2N/M$. The correlations in the presence of the proximity effect are positive for a dominating coupling to the superconductor ($\gamma < 0.5$), but are negative for all γ when suppressing the proximity effect with a weak magnetic field in the dot.

crosses over (at $\gamma \approx 0.5$) to negative values upon increasing the coupling to the normal reservoirs.

In the general case, it is difficult to provide a simple explanation for the sign and magnitude of the correlations $\langle P_{12} \rangle$, since they result from a competition between $\langle P_{12}^{ee} \rangle + \langle P_{12}^{hh} \rangle$ and $\langle P_{12}^{eh} \rangle + \langle P_{12}^{he} \rangle$, which have opposite signs and are generally of the same magnitude. It is, however, possible to get a qualitative picture in certain limits by studying the different contributing scattering processes.

In the limit $\gamma \rightarrow 0$, the coupling to the normal reservoirs is negligible and a gap in the spectrum opens up around Fermi energy in the dot [12]. As a consequence, quasiparticles injected from one normal reservoir are Andreev reflected, effectively direct at the contact-dot interface [16], with unity probability back to the same reservoir. There is thus no partition of incoming quasiparticles and hence no noise, $P_{12} = 0$.

Increasing the coupling to the normal reservoir, the probability of normal reflection ($\sim \gamma$) as well as cross Andreev reflection ($\sim \gamma^2$), from one reservoir to the other, becomes finite. Since normal reflection is the dominant process, we can neglect the terms in P_{12} in Eq. (3) containing the cross Andreev reflection amplitude (e.g., S_{12}^{eh}). This gives $\langle P_{12} \rangle = 8V(e^3/h) \langle \mathrm{tr}[S_{12}^{ee}(S_{12}^{ee})^\dagger \times S_{11}^{eh}(S_{11}^{eh})^\dagger] \rangle = 2P_0\gamma$, positive since only terms in $\langle P_{12}^{eh} \rangle + \langle P_{12}^{he} \rangle$ contribute. This shows that the correlations can be explained as the partition noise of injected electrons, which have a probability $\lesssim 1$ to be Andreev reflected (effectively at the contact-dot interface) and a probability $\sim \gamma$ to be normally reflected.

In the opposite limit, $\gamma \gg 1$, the coupling to the superconductor is weak and the Andreev reflection probability is small, of order $1/\gamma$. The correlation can be written as $\langle P_{12} \rangle = -16V(e^3/h) \langle \mathrm{tr}[S_{11}^{ee}(S_{11}^{ee})^\dagger S_{21}^{eh}(S_{21}^{eh})^\dagger] \rangle = -P_0/\gamma$, which is negative since we find that in this limit only terms in $\langle P_{12}^{ee} \rangle + \langle P_{12}^{hh} \rangle$ contribute. This shows that the correlation can be explained as partition noise of electrons which have probability $\lesssim 1/2$ to be normally reflected (without reaching the dot-superconductor contact) and probability $\sim 1/\gamma$ to be Andreev reflected.

When the proximity effect is suppressed by a weak magnetic field in the dot [16], it is no longer possible to express the correlations directly in terms of the transmission eigenvalues T, as in Eq. (6). We instead use the fact that the scattering matrix S_d of the dot itself is uniformly distributed in the unitary group, without constraints on the symmetry of S_d. Since the scattering matrix amplitudes $S^{\alpha\beta}$ in Eq. (3) can be expressed [13] in terms of S_d, the correlations $P_{12}^{\alpha\beta}$ can be directly averaged over S_d using the diagrammatic technique in Ref. [15]. This gives, for the total correlations,

$$\frac{\langle P_{12} \rangle}{P_0} = -\frac{\gamma^2(1 + \gamma)}{(2 + \gamma)^4}, \tag{8}$$

plotted in Fig. 2. The gap in the spectrum is suppressed and the scattering processes responsible for the positive

correlations in the presence of the proximity effect are strongly modified, most notably in the limit $\gamma \ll 1$. As a consequence, the correlations are manifestly negative for all γ, i.e., there are no positive correlations in the absence of the proximity effect, similar to what was found for a metallic diffusive system [11]. It is important to note here that the weak magnetic field suppresses only the correlation between the electrons and holes inside the dot, i.e., the proximity effect; it does not affect the Andreev reflection process at the dot-superconductor contact.

Until now we have considered only ideal point contacts, with $\Gamma_N, \Gamma_S = 1$. In an experimental situation, it is often difficult to obtain an ideal contact between the dot and the superconductor. It is therefore of interest to study the situation with a nonideal dot-superconductor contact. We consider first the case with the proximity effect in the dot. To calculate the current correlations in this case we note [13] that a nonideal interface changes the density of transmission eigenvalues, $\rho(T)$, but not the distributions of the unitary matrix U in Eq. (5). The transmission eigenvalue density is calculated numerically [14,15] for different contact transparencies $\Gamma_S < 1$, and the integrals in Eq. (6) are then performed.

The resulting correlations are plotted in Fig. 3. Surprisingly, the main effect of normal backscattering at the dot-superconductor contact is to cause a crossover from negative to positive correlation for a strong coupling to the normal reservoirs. In this limit, $\gamma \gg 1$, injected quasiparticles undergo at the most one scattering event at the dot-superconductor contact before leaving the junction. The expression for the scattering matrices $S_{ij}^{\alpha\beta}$ in Eq. (3) simplifies considerably and we can derive an analytical expression for the correlations, giving [17]
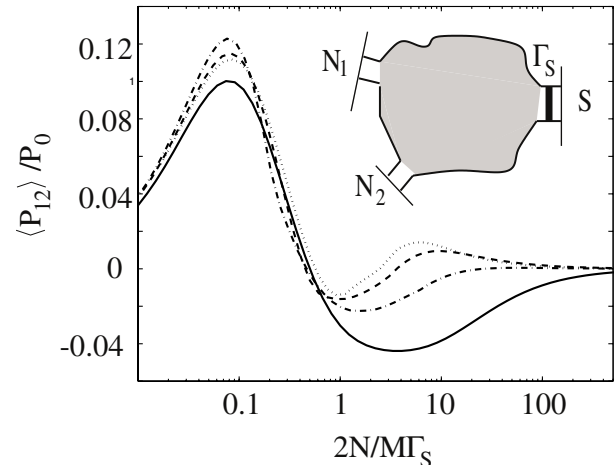

FIG. 3. Ensemble averaged current cross correlations P_{12} in the presence of normal backscattering at the normal-superconducting interface (see inset). The contact transparencies Γ_S are 1 (solid line), 0.8 (dash-dotted line) 0.6 (dashed line), and 0.4 (dotted line). The correlations are plotted as a function of $2N/M\Gamma_S$. Note that the correlations for a dominating coupling to the normal reservoirs cross over from negative to positive on *increasing* the normal backscattering.

$$\frac{\langle P_{12}\rangle}{P_0} = \frac{1}{\gamma} R_{eh}(1 - 2R_{eh}), \qquad (9)$$

where $R_{eh} = \Gamma_S^2/(2 - \Gamma_S)^2$ is the Andreev reflection probability of quasiparticles incident in the dot-superconductor contact. There is a crossover from negative to positive correlations already for $R_{eh} = 1/2$, i.e., $\Gamma_S = 2(\sqrt{2} - 1) \approx 0.83$, in agreement with the full numerics in Fig. 3.

Since $\Gamma_S < 1$, now R_{eh} is smaller than one. As a consequence, there is additional partition due to the possibility of normal reflection at the dot-superconductor contact. The partition noise discussed above, from electrons being either Andreev reflected or normally reflected (without reaching the dot-superconductor contact), is thus reduced to $-P_0 R_{eh}/\gamma$. However, the additional normal reflection at the dot-superconductor contact gives rise to a noise term $2P_0 R_{eh}(1 - R_{eh})/\gamma$, with opposite sign [together they give Eq. (9)]. For $R_{eh} < 1/2$, the second term is dominating, giving positive correlations.

Interestingly, in the absence of a proximity effect in the dot, we find in the same way that the correlation for $\gamma \gg 1$ is also given by Eq. (9). The proximity effect thus plays no role when the quasiparticles undergo at most one Andreev reflection. In this limit it is possible to explain the positive correlations by particle counting arguments. We note that an Andreev reflection leads to a simultaneous injection of a pair of particles (electrons or holes) into the dot. Each particle of the pair has an independent probability $1/2$ to go into each normal reservoir (no backreflection to the superconductor due to the weak coupling). Assuming uncorrelated attempts of the pairs to enter the dot gives the probability for Q_1 (Q_2) particles to end up in reservoir 1 (2) when Q_p pairs try to enter. From this probability one directly obtains $\langle P_{12}\rangle \propto R_{eh}(1 - 2R_{eh})$, as in Eq. (9). This shows that, for a weak coupling to the superconductor, nonperfect Andreev reflection at the normal-superconductor interface together with independent partitioning of particles inside the dot give rise to positive correlations.

Finally, we note that the effect of normal backscattering in the contacts between the dot and the normal reservoirs is to enhance the positive correlations for a dominating coupling to the superconducting reservoir. In the limit with tunnel barriers in all contacts, the transmission eigenvalue density is known analytically [14] and the correlation $\langle P_{12}\rangle$ follows from Eq. (6) [18]:

$$\frac{\langle P_{12}\rangle}{P_0 \Gamma_N} = \frac{\bar{\gamma}}{(1 + \bar{\gamma}^2)^{3/2}} \left(1 - 5\frac{\bar{\gamma}^2}{(1 + \bar{\gamma}^2)^2}\right), \qquad (10)$$

where $\bar{\gamma} = 2N\Gamma_N/(M\Gamma_S)$. The correlations are thus positive for $\bar{\gamma} < (\sqrt{5} - 1)/2$ and $\bar{\gamma} > (\sqrt{5} + 1)/2$, and negative for intermediate values.

In conclusion, we have studied the current cross correlations in a three-terminal superconducting-chaotic dot

analog of the Hanbury Brown–Twiss interferometer. We find that the correlations are positive for a wide range of junction parameters, and survive even in the absence of a proximity effect in the dot. The magnitude of the positive correlations is large, proportional to the number of transport modes in the contacts to the dot, which should simplify an experimental observation.

Note added. — In the final stages of this work we became aware of the work of Boerlin *et al.* [19] who investigate a three-terminal junction in the tunnel limit. In this limit, our results coincide.

[1] R. Hanbury Brown and R. Q. Twiss, Nature (London) **177**, 27 (1956).
[2] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992).
[3] M. Büttiker, Phys. Rev. B **46**, 12 485 (1992).
[4] For a review, see Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[5] M. Henny *et al.,* Science **284**, 296 (1999); W. D. Oliver *et al.,* Science **284**, 299 (1999); S. Oberholzer *et al.,* Physica (Amsterdam) **6E**, 314 (2000).
[6] X. Jehl *et al.,* Nature (London) **405**, 50 (2000); A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober, Phys. Rev. Lett. **84**, 3398 (2000).
[7] W. Belzig and Y. Nazarov, Phys. Rev. Lett. **87**, 067006 (2001).
[8] M. P. Anantram and S. Datta, Phys. Rev. B **53**, 16 390 (1996).
[9] J. Torrés and Th. Martin, Eur. Phys. J. B **12**, 319 (1999).
[10] T. Gramespacher and M. Büttiker, Phys. Rev. B **61**, 8125 (2000).
[11] K. E. Nagaev and M. Büttiker, Phys. Rev. B **63**, 081301 (2001).
[12] J. Melsen *et al.,* Europhys. Lett. **35**, 7 (1996); D. Taras-Semchuk and A. Altland, Phys. Rev. B **64**, 014512 (2001).
[13] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[14] Yu. V. Nazarov in *Quantum Dynamics of Submicron Structures,* edited by H. A. Cerdeira *et al.,* NATO ASI Ser. E, Vol. 291 (Kluwer, Dordrecht, 1995), p. 687.
[15] P. W. Brouwer and C. W. J. Beenakker, J. Math. Phys. **37**, 4904 (1997).
[16] A. A. Clerk, P. W. Brouwer, and V. Ambegaokar, Phys. Rev. B **62**, 10 226 (2000).
[17] We note that this result coincides with what is found for the single mode model of Ref. [9] substituting $\Gamma_S = 2\epsilon$.
[18] The same result was obtained in Ref. [19], see comment at the end of the paper.
[19] J. Böerlin, W. Belzig, and C. Bruder, Phys. Rev. Lett. **88**, 197001 (2002).