New U(1) Gauge Extension of the Supersymmetric Standard Model

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In extending the minimal standard model of quarks and leptons to include supersymmetry, the conservation of baryon and lepton numbers is no longer automatic. I show how the latter may be achieved with a new U(1) gauge symmetry and new supermultiplets at the TeV scale. Neutrino masses and a solution of the μ problem are essential features of this proposed extension.

It is well known that the minimal standard model of quarks and leptons conserves both baryon number B and lepton number L automatically [as the consequence of the assumed $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and its representation content]. It is also well known that this is not true any more once it is extended to include supersymmetry. Thus any such extension must be supplemented by a new symmetry which forbids the violation of B or L or both. There are many ways to do this; the most direct is to impose the conservation of an odd-even discrete symmetry, i.e., $R \equiv (-1)^{2j+3B+L}$, which is of course the defining hypothesis of the minimal supersymmetric standard model (MSSM).

There are two additional features of the MSSM which are often called into question. One is the absence of neutrino masses. This is, however, easily remedied by the addition of three neutral singlet lepton superfields (analogs of the three right-handed singlet neutrinos of the nonsupersymmetric standard model). The other is the presence of the so-called μ term in the MSSM superpotential, i.e., $\mu \hat{\phi}_1 \hat{\phi}_2$, where $\hat{\phi}_{1,2}$ are the two Higgs superfields which spontaneously break the electroweak gauge symmetry. Since this term is allowed by the gauge symmetry and the supersymmetry, there is no understanding of why μ should be of the order of the electroweak breaking scale, rather than some very large unification scale.

Whereas there are piecemeal solutions of all the above three problems of the MSSM, it is clearly desirable to have a single principle which works for all three at the same time. In this paper I show how a new simple U(1) gauge extension of the MSSM may be used exactly for this purpose [1].

Consider the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. The usual quark and lepton (left-handed) chiral superfields transform as follows:

$$(\hat{u}, \hat{d}) \sim (3, 2, 1/6; n_1), \qquad \hat{u}^c \sim (3^*, 1, -2/3; n_2),$$
$$\hat{d}^c \sim (3^*, 1, 1/3; n_3), \tag{1}$$

$$(\hat{\nu}, \hat{e}) \sim (1, 2, -1/2; n_4), \qquad \hat{e}^c \sim (1, 1, 1; n_5),$$
$$\hat{N}^c \sim (1, 1, 0; n_6). \tag{2}$$

They are supplemented by the two Higgs doublet superfields

$$\hat{\phi}_1 \sim (1, 2, -1/2; -n_1 - n_3),$$
$$\hat{\phi}_2 \sim (1, 2, 1/2; -n_1 - n_2), \tag{3}$$

with

$$n_1 + n_3 = n_4 + n_5, \qquad n_1 + n_2 = n_4 + n_6, \tag{4}$$

as in the MSSM. However, the μ term is replaced by the trilinear interaction $\hat{\chi} \hat{\phi}_1 \hat{\phi}_2$, where $\hat{\chi}$ is a Higgs singlet superfield transforming as

$$\hat{\chi} \sim (1, 1, 0; 2n_1 + n_2 + n_3). \tag{5}$$

Thus

$$2n_1 + n_2 + n_3 \neq 0 \tag{6}$$

is required so that the effective μ parameter of this model is determined by the $U(1)_X$ breaking scale, i.e., $\langle \hat{\chi} \rangle$.

To complete this model, I add two copies of the singlet quark superfields

$$\hat{U} \sim (3, 1, 2/3; n_7), \qquad \hat{U}^c \sim (3^*, 1, -2/3; n_8), \tag{7}$$

and one copy of

$$\hat{D} \sim (3, 1, -1/3; n_7), \qquad \hat{D}^c \sim (3^*, 1, 1/3; n_8), \tag{8}$$

with

$$n_7 + n_8 = -2n_1 - n_2 - n_3, \tag{9}$$

so that their masses are also determined by the $U(1)_X$ breaking scale.

To ensure the absence of the axial-vector anomaly [2], the following conditions are considered [3]:

$$[SU(3)]^2 U(1)_X : 2n_1 + n_2 + n_3 + n_7 + n_8 = 0, \tag{10}$$

$$[SU(2)]^2 U(1)_X : 3(3n_1 + n_4) + (-n_1 - n_3)$$
$$+ (-n_1 - n_2) = 0, \tag{11}$$

$$[U(1)_Y]^2 U(1)_X : 3\left[6\left(\frac{1}{6}\right)^2 n_1 + 3\left(-\frac{2}{3}\right)^2 n_2 + 3\left(\frac{1}{3}\right)^2 n_3 + 2\left(-\frac{1}{2}\right)^2 n_4 + n_5\right] + 2\left[3\left(\frac{2}{3}\right)^2 n_7 + 3\left(-\frac{2}{3}\right)^2 n_8\right]$$

$$+ 3\left(-\frac{1}{3}\right)^2 n_7 + 3\left(\frac{1}{3}\right)^2 n_8 + 2\left(-\frac{1}{2}\right)^2 (-n_1 - n_3) + 2\left(\frac{1}{2}\right)^2 (-n_1 - n_2) = 0, \tag{12}$$

$$U(1)_Y [U(1)_X]^2 : 3\left[6\left(\frac{1}{6}\right) n_1^2 + 3\left(-\frac{2}{3}\right) n_2^2 + 3\left(\frac{1}{3}\right) n_3^2 + 2\left(-\frac{1}{2}\right) n_4^2 + n_5^2\right] + 2\left[3\left(\frac{2}{3}\right) n_7^2 + 3\left(-\frac{2}{3}\right) n_8^2\right]$$

$$+ 3\left(-\frac{1}{3}\right) n_7^2 + 3\left(\frac{1}{3}\right) n_8^2 + 2\left(-\frac{1}{2}\right) (-n_1 - n_3)^2 + 2\left(\frac{1}{2}\right) (-n_1 - n_2)^2 = 0, \tag{13}$$

$$[U(1)_X]^3 : 3[6n_1^3 + 3n_2^3 + 3n_3^3 + 2n_4^3 + n_5^3 + n_6^3] + 3(3n_7^3 + 3n_8^3) + 2(-n_1 - n_3)^3 + 2(-n_1 - n_2)^3$$
$$+ (2n_1 + n_2 + n_3)^3 = 0. \tag{14}$$

Using Eq. (9), it is clear that Eq. (10) is automatically satisfied. Using Eqs. (4) and (9), it is easily shown that both Eqs. (11) and (12) are satisfied by the single condition

$$n_2 + n_3 = 7n_1 + 3n_4. \tag{15}$$

Using Eqs. (4), (9), and (15), it is then simple to show that Eq. (13) becomes

$$6(3n_1 + n_4)(2n_1 - 4n_2 - 3n_7) = 0. \tag{16}$$

Using Eq. (15), it is clear that $3n_1 + n_4 = 0$ contradicts Eq. (6). Hence only the condition

$$2n_1 - 4n_2 - 3n_7 = 0, \tag{17}$$

will be considered from here on.

At this point, the eight parameters (n_1 to n_8) are constrained by the five conditions given by Eqs. (4), (9), (15), and (17). Consider n_1, n_4, and n_6 as the independent parameters. The others are then given by

$$n_2 = -n_1 + n_4 + n_6, \tag{18}$$

$$n_3 = 8n_1 + 2n_4 - n_6, \tag{19}$$

$$n_5 = 9n_1 + n_4 - n_6, \tag{20}$$

$$n_7 = 2n_1 - \frac{4}{3} n_4 - \frac{4}{3} n_6, \tag{21}$$

$$n_8 = -11n_1 - \frac{5}{3} n_4 + \frac{4}{3} n_6. \tag{22}$$

It is now straightforward to simplify Eq. (14) to read

$$-36(3n_1 + n_4)(9n_1 + n_4 - 2n_6)$$
$$\times (6n_1 - n_4 - n_6) = 0. \tag{23}$$

Whereas one factor, i.e., $3n_1 + n_4$, must be nonzero, there remain two possible solutions, i.e.,

$$(A) \quad n_6 = \frac{1}{2} (9n_1 + n_4), \tag{24}$$

$$(B) \quad n_6 = 6n_1 - n_4. \tag{25}$$

which render $U(1)_X$ free of the axial-vector anomaly. This exact factoring of the sum of eleven cubic terms is certainly not a trivial result [4].

Solution (A) is thus given by

$$n_2 = n_3 = \frac{1}{2} (7n_1 + 3n_4),$$

$$n_5 = n_6 = \frac{1}{2} (9n_1 + n_4), \tag{26}$$

$$n_7 = -4n_1 - 2n_4, \qquad n_8 = -5n_1 - n_4. \tag{27}$$

In the MSSM, $\hat{L}$ and $\hat{\phi}_1$ transform identically under $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here $\hat{L}$ and $\hat{\phi}_1$ are distinguished by $U(1)_X$ if

$$9n_1 + 5n_4 \neq 0. \tag{28}$$

Hence the lepton number L may be automatically conserved as in the nonsupersymmetric standard model.

In the MSSM, the term $\hat{u}^c \hat{d}^c \hat{d}^c$ is allowed in the superpotential. Here it is forbidden if

$$7n_1 + 3n_4 \neq 0. \tag{29}$$

Hence the baryon number B may be automatically conserved as well.

Solution (B) has

$$n_2 = 5n_1, \quad n_3 = 2n_1 + 3n_4, \quad n_5 = 3n_1 + 2n_4, \tag{30}$$

$$n_6 = 6n_1 - n_4, \quad n_7 = -6n_1, \quad n_8 = -3n_1 - 3n_4. \tag{31}$$

Hence L is automatically conserved if

$$3n_1 + 4n_4 \neq 0, \tag{32}$$

and B is automatically conserved if

$$3n_1 + 2n_4 \neq 0. \tag{33}$$

Note that solutions (A) and (B) are identical if $n_4 = n_1$. This turns out to be also the condition [5] for $U(1)_X$ to be orthogonal to $U(1)_Y$, i.e.,

$$3\left[6\left(\frac{1}{6}\right)n_1 + 3\left(-\frac{2}{3}\right)n_2 + 3\left(\frac{1}{3}\right)n_3 + 2\left(-\frac{1}{2}\right)n_4 + n_5\right] + 2\left[3\left(\frac{2}{3}\right)n_7 + 3\left(-\frac{2}{3}\right)n_8\right] + 3\left(-\frac{1}{3}\right)n_7 + 3\left(\frac{1}{3}\right)n_8$$

$$+ 2\left(-\frac{1}{2}\right)(-n_1 - n_3) + 2\left(\frac{1}{2}\right)(-n_1 - n_2) = 0. \tag{34}$$

There are two more anomalies to consider. The global SU(2) chiral gauge anomaly [6] is absent because the number of $SU(2)_L$ doublets is even. The mixed gravitational-gauge anomaly [7] is proportional to the sum of $U(1)_X$ charges, i.e.,

$$3(6n_1 + 3n_2 + 3n_3 + 2n_4 + n_5 + n_6) + 3(3n_7 + 3n_8) + 2(-n_1 - n_3) + 2(-n_1 - n_2)$$

$$+ (2n_1 + n_2 + n_3) = 6(3n_1 + n_4), \tag{35}$$

which is not zero. This anomaly may be tolerated if gravity is neglected. On the other hand, it may be rendered zero by adding $U(1)_X$ supermultiplets as follows: one with charge $3(3n_1 + n_4)$, three with charge $-2(3n_1 + n_4)$, and three with charge $-(3n_1 + n_4)$. Hence they contribute $3 + 3(-2 - 1) = -6$ (in units of $3n_1 + n_4$) to Eq. (35), but $27 + 3(-8 - 1) = 0$ to Eq. (14).

The allowed terms in the superpotential of either solution (A) or (B) consist of the usual allowed terms of the MSSM with $\mu\hat{\phi}_1\hat{\phi}_2$ replaced by $\hat{\chi}\hat{\phi}_1\hat{\phi}_2$. In (A), the usual R-parity violating terms are forbidden by Eqs. (28) and (29). As for the interactions of the exotic quark singlets of Eqs. (7) and (8), $n_1 + n_4 \neq 0$ forbids $\hat{U}^c\hat{d}^c\hat{d}^c$, $\hat{u}^c\hat{d}^c\hat{D}^c$, and $(\hat{u}_i\hat{d}_j - \hat{d}_i\hat{u}_j)\hat{D}$; $13n_1 + n_4 \neq 0$ forbids $\hat{U}^c\hat{d}^c\hat{D}^c$; and $n_1 \neq 0$ forbids $\hat{e}^c\hat{u}^c\hat{D}$, $(\hat{\nu}\hat{d} - \hat{e}\hat{u})\hat{D}^c$, $\hat{N}^c\hat{u}^c\hat{U}$, and $\hat{N}^c\hat{d}^c\hat{D}$. This means that if $n_4 = -n_1$, then $\hat{U}^c$ and $\hat{D}^c$ are diquark superfields, and, if $n_1 = 0$, then $\hat{U}$ and $\hat{D}$ are leptoquark superfields.

In solution (B), the usual R-parity violating terms are forbidden by Eqs. (32) and (33). Furthermore, $n_1 + 3n_4 \neq 0$ forbids $\hat{U}^c\hat{d}^c\hat{d}^c$; $n_1 \neq 0$ forbids $\hat{u}^c\hat{d}^c\hat{D}^c$ and $(\hat{u}_i\hat{d}_j - \hat{d}_i\hat{u}_j)\hat{D}$; $4n_1 + 3n_4 \neq 0$ forbids $\hat{U}^c\hat{d}^c\hat{D}^c$; $n_1 + n_4 \neq 0$ forbids $\hat{e}^c\hat{u}^c\hat{D}$, $(\hat{\nu}\hat{d} - \hat{e}\hat{u})\hat{D}^c$, and $\hat{N}^c\hat{d}^c\hat{D}$; and $5n_1 - n_4 \neq 0$ forbids $\hat{N}^c\hat{u}^c\hat{U}$. This means that $\hat{U}^c$ is a diquark if $n_4 = -n_1/3$, and $\hat{U}$ is a leptoquark if $n_4 = 5n_1$; whereas $\hat{D}^c$ is a diquark if $n_1 = 0$, and $\hat{D}$ is a leptoquark if $n_4 = -n_1$.

Even with the imposition of R parity, there are higher-dimensional operators in the MSSM which may induce proton decay, i.e., $\hat{q}\hat{q}\hat{q}\hat{l}$ and $\hat{u}^c\hat{u}^c\hat{d}^c\hat{e}^c$. In the nonsupersymmetric standard model, since quarks and leptons are fermions, these operators have dimension six, but in the MSSM they have dimension-five pieces. Hence proton decay may not be sufficiently suppressed, which is a well-known problem of the MSSM. In this model, these terms are forbidden [in both solutions (A) and (B)] by $3n_1 + n_4 \neq 0$, i.e., the same condition that forbids the μ term.

As it stands, this model pairs ν with N^c to form a Dirac neutrino with mass proportional to $\langle\phi_2\rangle$. This would require extremely small Yukawa couplings and is generally considered to be very unnatural. On the other hand, if $n_6 = 3n_1 + n_4$ is assumed [i.e., $n_4 = 3n_1$ in solution (A) or $n_4 = 3n_1/2$ in solution (B)], then the extra singlets used to cancel the mixed gravitational-gauge anomaly of Eq. (35) are exactly the right number and structure to allow

neutrinos to acquire naturally small seesaw Dirac masses, as shown below.

In addition to the three singlets N^c of $U(1)_X$ charge n_6, there are now also three singlets N of charge $-n_6$ and three singlets S^c of charge $-2n_6$. The 12×12 mass matrix spanning (ν, S^c, N, N^c) is then of the form

$$\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & m_2 & 0 \\ 0 & m_2 & 0 & M \\ m_1 & 0 & M & 0 \end{bmatrix}, \tag{36}$$

where m_1 comes from $\nu N^c\phi_2^0$ with $\langle\phi_2^0\rangle \neq 0$, m_2 comes from $NS^c\chi$ with $\langle\chi\rangle \neq 0$, and M is an allowed invariant mass. Thus $m_1 \sim 10^2$ GeV, $m_2 \sim 10^3$ GeV, and $M \sim 10^{16}$ GeV are expected. In the reduced (ν, S^c) sector, the effective 6×6 mass matrix is still exactly of the Dirac form, i.e.,

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -m_1m_2/M \\ -m_1m_2/M & 0 \end{bmatrix}, \tag{37}$$

and $m_1m_2/M \sim 10^{-2}$ eV is the right order of magnitude for realistic neutrino masses.

In conclusion, a remarkable new U(1) gauge symmetry has been identified in a simple extension of the supersymmetric standard model which is capable of enforcing B or L conservation or both, as well as the absence of the μ term and the presence of neutrino masses. Two solutions have been obtained [from the exact factoring of Eq. (14)

TABLE I. Solutions (A) and (B), where $n_i = an_1 + bn_4$.

| | (A) | | (B) | |
	a	b	a	b
n_2	7/2	3/2	5	0
n_3	7/2	3/2	2	3
n_5	9/2	1/2	3	2
n_6	9/2	1/2	6	−1
n_7	−4	−2	−6	0
n_8	−5	−1	−3	−3
$-n_1 - n_3$	−9/2	−3/2	−3	−3
$-n_1 - n_2$	−9/2	−3/2	−6	0
$2n_1 + n_2 + n_3$	9	3	9	3

TABLE II.   Conditions on n_1 and n_4 in (A) and (B).

| (A) | | (B) | | |
c	d	c	d	$cn_1 + dn_4 \neq 0$ forbids
3	1	3	1	μ term
9	5	3	4	L violation
7	3	3	2	B violation
1	1	1	3	U^c as diquark
1	1	1	0	D^c as diquark
1	0	5	−1	U as leptoquark
1	0	1	1	D as leptoquark
13	1	4	3	U^c, D^c as semiquarks

to become Eq. (23)] with many possible variations regarding new interactions beyond the MSSM, as summarized in Tables I and II. The origin of this new U(1) gauge symmetry is unknown at present; it has no obvious fit into any simple model of grand unification or string theory.

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[4] Consider, for example, $n_1 = 2/3$, $n_4 = -1$ in solution (A), then Eq. (14) is realized as $18(2/3)^3 + 18(5/6)^3 + 6(-1)^3 + 6(5/2)^3 + 9(-2/3)^3 + 9(-7/3)^3 + 4(-3/2)^3 + (3)^3 = 0$.

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