Conditional-Phase Switch at the Single-Photon Level

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We present an experimental realization of a two-photon conditional phase switch, related to the "c-ϕ" gate of quantum computation. This gate relies on quantum interference between photon pairs and generates entanglement between two optical modes through the process of spontaneous parametric down-conversion (SPDC). The interference effect serves to enhance the effective nonlinearity by many orders of magnitude, so it is significant at the quantum (single-photon) level. By adjusting the relative optical phase between the classical pump for SPDC and the pair of input modes, one can impress a large phase shift on one beam which depends on the presence or absence of a single photon in a control mode.

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A great deal of effort has gone into the search for a practical architecture for quantum computation. As was recognized early on, single-photon optics provides a nearly ideal arena for many quantum-information applications [1]; unfortunately, the absence of significant nonlinear effects at the quantum level (photon-photon interactions) appeared to limit the usefulness of quantum optics to applications in communications as opposed to computation. (Nevertheless, two recent proposals [2,3] have resurrected the possibility of quantum computation using purely linear optics.) Therefore, work has focused on NMR [4], solid-state [5], and atomic [6–9] proposals for quantum logic gates, but so far none of these systems has demonstrated all of the desired features such as strong coherent interactions, low decoherence, and straightforward scalability. Typical optical nonlinearities are so small that the dimensionless efficiency of photon-photon interactions rarely exceeds the order of 10^{-10}. We have recently used quantum interference to enhance these nonlinearities by as much as 10 orders of magnitude, leading to near-unit-efficiency sum-frequency generation of individual photon pairs [10]. In this Letter, we demonstrate that a similar experimental geometry can be used to make a conditional phase switch. Our switch is very similar to an enhanced Kerr or cross-phase-modulation effect, in which the presence or absence of a single photon in one mode may lead to a significant phase shift of the other mode. This is also similar to experiments performed in cavity QED [6] (and to theoretical proposals for atomic vapors, in systems relying on atomic coherence effects [11] or photon exchange interactions [12]), but occurs in a relatively simple and robust system relying only on beams interacting in a nonresonant nonlinear crystal.

The controlled-phase or c-ϕ gate performs the mapping $|m\rangle_1|n\rangle_2 \rightarrow \exp(imn\phi)|m\rangle_1|n\rangle_2$, where the subscripts 1 and 2 indicate the two qubits, stored in two distinct optical modes, and m and n can take the values 0 and 1 representing zero- and one-photon states [13]. This shifts the phase of $|1\rangle_1|1\rangle_2$ by ϕ, leaving the other three basis states unchanged. Although in quantum mechanics an overall phase factor is meaningless, this unitary transformation is nontrivial when we consider what happens to superpositions of photon number. The operation induces a relative phase of ϕ *between* the $|0\rangle$ and $|1\rangle$ states of qubit 2, if and only if qubit 1 is in state $|1\rangle$. (It is this *relative* phase which is referred to as the "optical phase" of mode 2 [14].)

Since our experiment relies on interference, its operation is sensitive to the phase and amplitude of the initial state, and we must limit ourselves to a specific set of inputs. In particular, we illuminate our switch with two classical fields in weak coherent states, $|\Psi\rangle = |\alpha\rangle \otimes |\beta\rangle \approx [|0\rangle_1 + \alpha|1\rangle_1] \otimes [|0\rangle_2 + \beta|1\rangle_2]$, for $|\alpha|, |\beta| \ll 1$. This state includes contributions of all four two-qubit computational-basis states. As we show theoretically and experimentally, the lowest-order action of the gate is to shift the phase of only the $|1\rangle_1|1\rangle_2$ state, as desired for c-ϕ operation.

This gate differs from the canonical c-ϕ concept in several regards. Principally, the input cannot be in a pure Fock state (e.g., $|1\rangle_1|1\rangle_2$), or an arbitrary superposition of the computational-basis states, because the appropriate relative phase of $|0\rangle_1|0\rangle_2$ and $|1\rangle_1|1\rangle_2$ must be chosen at the outset. Nevertheless, the gate produces significant entanglement at the output and may be useful in nondeterministic operation [2]; in other words, it may be possible to postselect the desired value of a given qubit rather than supplying it at the input. Alternatively, such a gate might be used in the polarization rather than the photon-number basis. The interaction can be controlled through phase-matching conditions such that the phase shift is impressed only if both photons have, for example, vertical polarization. Thus, two-photon entangled states as typically produced in down-conversion systems, which are more properly described as $|\Psi\rangle = |0\rangle|0\rangle + \varepsilon\{a|H\rangle|H\rangle + b|H\rangle|V\rangle + c|V\rangle|H\rangle + d|V\rangle|V\rangle\}$, could store the amplitudes of the four computational-basis states in the amplitudes a, b, c, and d, with the (small) coefficient ε ensuring that εd exhibits the appropriate phase relationship with the vacuum. Although the vacuum term would dominate, as in most down-conversion experiments, the computation would have the desired effect contingent simply on the eventual detection of a photon pair. Potential contamination due to states outside the computational basis (e.g., states in which two photons are present in the same mode) can be avoided by operating in the low-photon-number regime. Finally, the question as to whether the entanglement produced by these interactions might be useful as a generalized quantum gate in some larger Hilbert space (e.g., higher photon-number states) remains open.

Our experiment can be described as a modified Mach-Zehnder interferometer (MZI) (Fig. 1). The input beam is a weak laser pulse of frequency ω (containing much less than one photon per pulse on average) which enters the interferometer and is split into the signal (mode 1) and phase reference (mode 3). Modes 1 and 3 are recombined at a beam splitter after mode 1 passes through a $\chi^{(2)}$ nonlinear crystal which is simultaneously illuminated by a pump beam at frequency 2ω. The output fringes from the MZI serve to measure the relative phase introduced between the two arms by the action of the crystal. Our control beam (mode 2) is another very weak coherent state at ω that crosses mode 1 inside the nonlinear crystal. Photon-counting detectors monitor one output of the interferometer and mode 2. In order to demonstrate the conditional phase operation of the device, we measure the phase of the fringes at det. 1 and compare the cases in which the control detector (det. 2) does or does not fire. This "conditional homodyne" measurement [15] is similar to recent studies of "wave-particle correlations" in cavity QED [16].

A more detailed schematic of the experiment is shown in Fig. 2. The beam from a Ti:sapphire oscillator (center wavelength 810 nm, rep rate 80 MHz, and pulse duration 50 fs) is used to create the four beams used in the experiment. The phase reference, signal, and control beams are created by separating a small amount of the fundamental beam with beam splitters (BS) 3 and 1—all beam splitters are 90/10 (T/R). The signal and control beams are made by rotating the polarization after BS1 and treating the horizontal and vertical components independently. All three of these beams are subsequently attenuated using neutral density filters. The majority of the pump undergoes second-harmonic generation (SHG) in a type-I β-barium borate (BBO) crystal. With the fundamental removed, this 405-nm pulse serves as the pump laser for parametric down-conversion. The signal and control beams are recombined with the pump laser at BS 4 and all three beams are focused onto a second 0.5-mm BBO crystal phase matched for type-II down-conversion and, therefore, type-II SHG. The spot created on the down-conversion crystal is imaged through a spatial filter to select a single spatial mode [10]. The output from the spatial filter is separated by a polarizing beam splitter (PBS) such that the vertically polarized control beam is sent to detector 2 for direct photodetection, while the horizontally polarized signal beam interferes with the phase reference at BS 2. Detector 1 measures the output from one port of BS 2. Both detectors are silicon avalanche photodiodes. Interference filters, with center wavelengths of 810 nm and bandwidths of 10 nm, are placed in front of each detector.

In previous work [10], we demonstrated that quantum interference leads to a phase-sensitive photon-pair production rate in a similar geometry. The interference can be understood as follows. Initially, modes 1 and 2 contain weak coherent states and mode p contains an intense (classical) pump laser: $|\Psi\rangle = |\gamma\rangle_p \otimes [|00\rangle + \alpha|10\rangle + \beta|10\rangle + \alpha\beta|11\rangle]$. Under the interaction Hamiltonian, $\mathcal{H}_{\rm int} = g a_1^\dagger a_2^\dagger a_p + g^* a_1 a_2 a_p^\dagger$, the lowest order action of the pump laser is simply to add an amplitude for a photon pair through parametric down-conversion. The final state becomes $|\Psi\rangle = |\gamma\rangle_p \otimes [|00\rangle + \alpha|10\rangle + \beta|01\rangle + (\alpha\beta + A_{\rm DC})|11\rangle]$, where $A_{\rm DC} \propto \gamma g$ is the amplitude for down-conversion. In [10], we observed the modulation in the

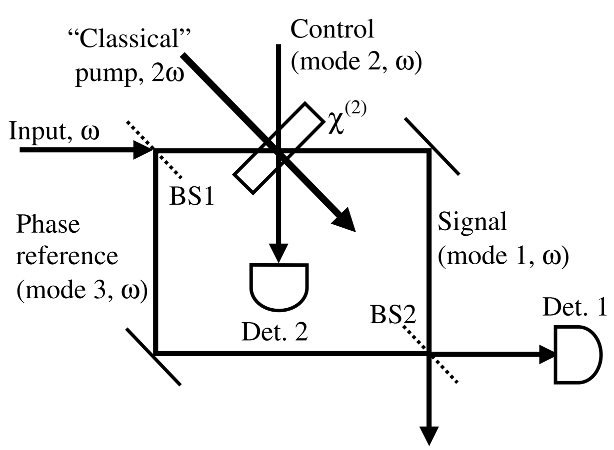

FIG. 1. A cartoon of the experiment. The signal beam, a weak ($|\alpha| \ll 1$) coherent state, is passed through a Mach-Zehnder interferometer in order to measure the phase shift. This shift is imprinted by a $\chi^{(2)}$ crystal pumped with a strong classical pump (p), only when the control beam (also a weak coherent state with mean photon number $|\beta|^2 \ll 1$) contains a photon. This conditional phase operation is verified by correlating the MZ output fringes at det. 1 with detection of a control photon at det. 2.

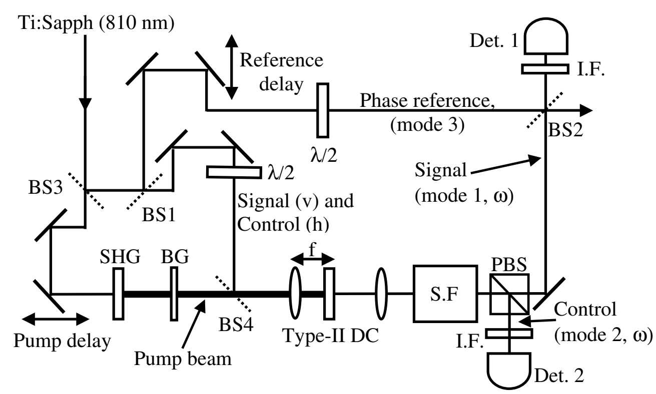

FIG. 2. Schematic of the experiment. BS 1–4 are 90/10 (T/R) beam splitters; SHG consists of two lenses and a 0.1-mm BBO crystal for type-I second harmonic generation; $\lambda/2$ are half-wave plates; S.F. is a spatial filter; I.F. are interference filters; BG is a blue filter; PBS is a polarizing beam splitter; det. 1 and 2 are photon counters. The pump laser at 405 nm is separated from the 810 nm light by using a fused-silica prism, not shown for clarity.

photon pair production rate by performing direct photon coincidence counting on modes 1 and 2. We changed the phase of the amplitude A_{DC} by changing the delay of the pump laser and, in so doing, changed the value of $|\alpha\beta + A_{DC}|^2$—the probability of producing a photon pair. However, this process also affects the *phase* of that amplitude, i.e., $\arg(\alpha\beta + A_{DC})$. This is the "cross-phase modulation" we study. The absolute phase of a state is never experimentally observable; we therefore study the relative phase between $|11\rangle$ and $|01\rangle$, contrasting it with the case of no control photon: $|10\rangle$ vs $|00\rangle$. This relative phase is precisely the optical phase measured by our Mach-Zehnder interferometer. The final state of modes 1 and 2 can be rewritten as follows:

$$|\Psi\rangle = (|0\rangle_1 + \alpha|1\rangle_1)|0\rangle_2 + \beta\left[|0\rangle_1 + \left(\alpha + \frac{A_{DC}}{\beta}\right)|1\rangle_1\right]|1\rangle_2 . \quad (1)$$

In this form, it is evident that entanglement is generated between the photon number in mode 2 and the optical phase in mode 1; the conditions that $|\alpha|$, $|\beta| \ll 1$ limit the state to one of nonmaximal entanglement. Nonetheless, maximal entanglement can be produced in polarization within the coincidence subspace [17]. When $|A_{DC}| \ll |\alpha\beta|$, (i.e., the down-conversion rate is much less than the "accidental" coincidence rate from the signal and control beams) there is a small phase shift but, to first order, no change in rate. In the opposite limit, when $|A_{DC}| > |\alpha\beta|$, the maximum phase shift is 180° and occurs at the point of maximum destructive interference.

To explore the small phase-shift regime, we adjusted our signal and control beam intensities to obtain, in the absence of interference, a coincidence rate of $(256 \pm 3)\ \mathrm{s}^{-1}$ between det. 1 and det. 2. Our coincidence rate from down-conversion alone was $(4.7 \pm 0.2)\ \mathrm{s}^{-1}$. The singles rates at det. 1 (again in the absence of interference) were $88 \times 10^3\ \mathrm{s}^{-1}$ from the signal beam alone and $79 \times 10^3\ \mathrm{s}^{-1}$ from the phase reference; det. 2 received a singles rate of $282 \times 10^3\ \mathrm{s}^{-1}$ from the control beam. This corresponds to several photons per thousand laser pulses. The singles rates due to down-conversion were $400\ \mathrm{s}^{-1}$ at det. 1 and $300\ \mathrm{s}^{-1}$ at det. 2. To perform the experiment, the phase reference was blocked and pump delay moved in sub-wavelength steps to observe fringes in the photon pair production rate (described in [10]). The pump delay was then stopped at a fixed phase relative to the maximum of the pair-production fringes. We then scanned over a few Mach-Zehnder interference fringes by stepping the reference delay in 0.04-μm steps and recorded the singles rates at the two detectors and their coincidence rate. Because of the low probability of having a photon in any given control pulse, the interference fringes in det. 1's singles rate are dominated by the case where zero photons are present in the control mode; the coincidence rate shows the phase-shifted fringes when a control photon is detected.

A sample data set is shown in Fig. 3 for a pump delay of -1.6 fs (about $-455°$). For clarity, the fringes shown are taken in the large phase-shift regime, with $|A_{DC}| > |\alpha\beta|$. To achieve this regime, we reduced our coincidence rate from the signal and control beams to $(1.1 \pm 0.1)\ \mathrm{s}^{-1}$ in the absence of interference; our down-conversion coincidence rate was $(5.2 \pm 0.2)\ \mathrm{s}^{-1}$. Det. 1 received about a $700\ \mathrm{s}^{-1}$ singles rate from the signal and $8600\ \mathrm{s}^{-1}$ from the phase reference; det. 2 had a singles rate of $129 \times 10^3\ \mathrm{s}^{-1}$ from the control beam. The coincidence counts have been averaged over 40-sec intervals due to the considerable shot noise. The fringes were fitted to cosine curves where the period of the coincidence fringes was constrained to equal that of the singles fringes. The phase difference was then extracted modulo 360°.

Relative phases were measured in this way for many different pump phase delays; those values are summarized in Fig. 4. The phase shifts measured for the low phase-shift regime are the open circles (right-hand scale). The dashed line is the theoretical prediction based on the experimentally observed ratio of coincidence rates, with no adjustable parameters. In this regime, the phase shift is limited to approximately $|A_{DC}|/|\alpha\beta|$—about 8° for the experimental ratio of coincidence rates. The phase shift is approximately sinusoidal in the pump phase for this ratio. The shifts in the large phase-shift regime are shown in Fig. 4 as solid circles (left-hand scale). Theory is shown as a solid line and, again, involves no free parameters. It is clear that in this regime we are able to access any phase shift. In this regime, the phase shift does not follow a sinusoidal modulation but rather increases monotonically with the pump phase, modulo 360°. There is strong agreement

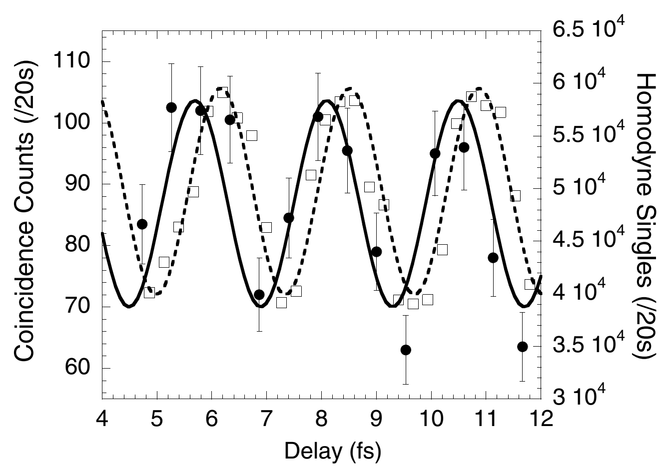

FIG. 3. Phase-shifted fringes in the large phase-shift regime. The det. 1 singles rate (open squares, dashed line) and coincidence rate between det. 1 and det. 2 (closed circles, solid line) are shown as a function of the reference delay. The coincidence fringes display the phase of the signal for cases in which a control photon was present; the singles are dominated by cases in which no photon was present. For this particular pump phase, the coincidence counting rate *lags* the singles rate by $(65 \pm 8)°$.

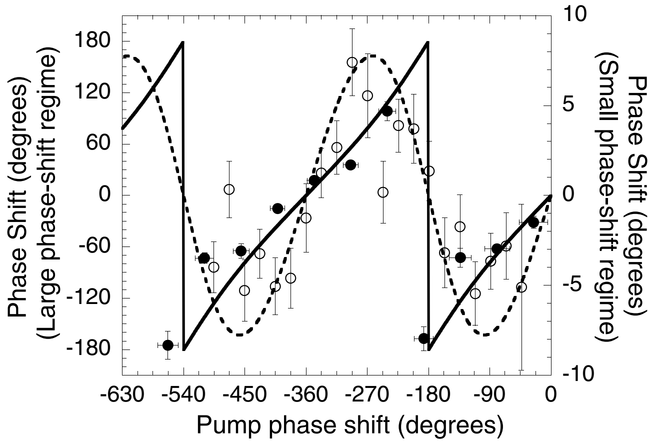

FIG. 4. Phase shift versus pump phase delay. The phase of the pump laser was changed via the pump delay and was estimated using the accompanying modulation in the mean coincidence rate [10]. The phase shift between the coincidence and singles fringes is plotted against the pump phase delay for both the large phase-shift regime (solid circles) and the small phase-shift regime (open circles). The solid and dashed lines show the theoretical predictions for these two cases, respectively, based only on the measured ratio of the individual-path rates, and with no adjustable parameters.

between theory and experiment, with slightly reduced phase shifts in the low phase-shift regime possibly attributable to background.

We have demonstrated the correlation between the photon number in one mode and the optical phase in another in a coherent conditional phase switch. Our theoretical description of the device shows that entanglement between the two modes is generated, but explicit demonstration requires additional measurements. This is a new type of asymmetric entanglement [15], of the sort required for the quantum c-ϕ gate. However, our switch differs from the c-ϕ, since the switch's reliance on quantum interference makes it intrinsically dependent on the optical phase of the input beams. While this phase dependence will not allow the gate to operate on Fock states, the gate does act exactly as a c-ϕ in the coincidence basis in some interesting situations [17]. Methods such as the one described in this Letter of creating and controlling entanglement at the single-photon level are very exciting for the field of nonlinear quantum optics and are promising steps towards all-optical quantum computing.

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