Solitonlike Beam Propagation along Light-Induced Singularity of Space Charge in Fast Photorefractive Media

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We investigate light beam propagation in a fast photorefractive medium placed in an alternating electric ac field to enhance the nonlinear response. It is shown that the joint action of the optical and material nonlinearities leads to formation of a narrow singularity of the light-induced space charge at the intensity maximum and to self-trapping of the light energy near the corresponding discontinuity of the index profile. Owing to the strong saturation of the material nonlinearity, the trapped beam propagates over long distances with only a weak loss of its power.

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It has become clear in recent years that the photorefractive (PR) nonlinearity manifests itself in a wealth of exciting optical phenomena. Among them are propagation of solitons [1] and surface waves [2], pattern formation [3], parametric scattering [4], subharmonic generation [5,6], and critical enhancement [7]. Most of the mentioned PR effects are generic; they are easily accessible at low light intensities and distinguished by a wide variety of forms.

The optical phenomena specific of PR media are remarkable in the sense that they result from a joint action of the optical and material nonlinearities. As shown recently, hybridization of these nonlinearities is able to provide an almost infinite growth (singularity) of the rate of spatial amplification when approaching the threshold of the subharmonic generation [7].

Another surprising effect of the same category, closely related to the soliton propagation, has come onto the scene in 2000 [8]. It was found that under certain conditions a localized light beam creates a narrow singularity of space charge at the intensity maximum, which leads to a discontinuity (shock wave) of the refractive index owing to the electro-optic effect. This discontinuity separates regions of strong self-focusing/defocusing in the medium. The necessary conditions for the formation of the charge singularity can be easily achieved in fast PR crystals (sillenites and semiconductors) placed in a quickly oscillating electric ac field.

The singular behavior of nonlinear systems has always attracted considerable interest. As relevant examples we mention the shock waves described by the Burgers equation [9], and the 2D and 3D charge singularities (Langmuir collapse) in plasma physics [10]. In our case, the effect of charge singularity on beam propagation may exhibit two opposing tendencies: On the one hand, the material nonlinearity works to support the shock wave. On the other hand, diffraction by the abrupt index profile tends to wash out the singularity. What is the scenario of beam propagation in this case? Is it beam broadening accompanied by disappearance of the charge singularity, or the soliton propagation coupled with the refractive index discontinuity, or something else? Solution of this new and challenging problem seems to be of general interest. In this Letter we intend to give such a solution.

Prior to coming to the matter, we mention a few aspects related to the actual problem. The PR solitons are usually associated with a local nonlinear response [11] (a spatially symmetric beam produces a symmetric index profile). The results known for the nonlocal nonlinearity refer to the gradient response (the index change is proportional to the intensity gradient). They include two exact solutions that describe propagation of a specially shaped planar light beam with stationary transverse profile [12] and propagation of 1D Gaussian input beams [13]. Since these Gaussian beams exhibit diffraction, soliton propagation is not expected for the gradient response.

Now let a planar light beam propagate along the z axis and x be the transverse coordinate. The nonlinear index change δn is expressed as $\delta n = -n_0^3 rE/2$, where $E \simeq E_x = E(x,z)$ is the light-induced space-charge field, n_0 the nonperturbed refractive index, and r the electro-optic coefficient. Since $\delta n \ll n_0$, the dependence $\delta n(x)$ can be found from the 1D nonlinear equation derived in [8] for fast PR crystals on the basis of the conventional one-species charge transport model [5]. This material equation written for the normalized field $e = E/E_0$ (E_0 is the applied ac-field amplitude) reads

$$\left[\frac{(1+I)(1-e^2)}{1+l_s e_x}\right]_x + \frac{1}{l_0}\,e(1+I) = 0. \qquad (1)$$

The subscript x stands for the transverse differentiation, $l_0 = \mu\tau E_0$ and $l_s = \varepsilon\varepsilon_0 E_0/qN_t$ are the drift and saturation lengths, $\mu\tau$ is the mobility-lifetime product for photoelectrons, $\varepsilon\varepsilon_0$ the dielectric constant, q the elementary charge, and N_t the trap concentration. The beam intensity I is normalized to the effective background intensity characterizing the dark conductivity [1].

The material Eq. (1) is derived by averaging over the rapid ac oscillations [8]. Its structure reflects that the progressive importance of the higher spatial harmonics starts from very low values of the light contrast [5,6]. The term e^2 in the numerator originates from the drift nonlinearity. The term $l_s e_x$ in the denominator, including the smallest characteristic length and defining the (second) order of the differential equation, describes trap saturation. It is important only in the vicinity of the discontinuity. With this term omitted it is impossible to obtain a localized solution for $e(x)$. This resembles indeed the physics described by the Burgers equation. The main conditions for validity of Eq. (1) are not very restrictive [8]. They are as follows: smallness of the lifetime and the ac-field period as compared to the dielectric relaxation time and the inequality $E_0 \gg (k_B T N_t/\varepsilon\varepsilon_0)^{1/2}$. The feature of fast PR crystals crucial for the discontinuity formation is the inequality $l_s/l_0 \ll 1$. Typically, in the sillenites (BSO, BTO, BGO) and semiconductors (GaAs, CdTe, etc.) we have $l_s/l_0 \lesssim 10^{-2}$.

Equation (1) with the boundary conditions $e(\pm\infty) = 0$ can be solved numerically by the relaxation method [14] for an arbitrary beam profile. In our calculations we use the following parameters representative for experiments with the sillenites: $\varepsilon = 56$, $\mu\tau = 2.4 \times 10^{-7}$ cm^2/V, $N_t = 2 \times 10^{16}$ cm^{-3}, and $E_0 = 25$ kV/cm. They correspond to the drift length $l_0 = 60$ μm and the saturation length $l_s \simeq 0.4$ μm.

Figure 1 illustrates the light-induced field profile for an input Gaussian beam, $I(x,0) = I_0 \exp(-4x^2/d^2)$, of width $d = 36$ μm for four values of the peak intensity I_0. Obviously, we are dealing with a kind of nonlocal response. For $I_0 \gtrsim 1$ (dominating photoconductivity) the field profile is characterized by a highly pronounced discontinuity at the beam center. The value $|e(x)|_{\max}$ is below 1 but it approaches quickly unity with increasing I_0. This strong saturation of the field amplitude implies that the concentration of ionized donors is comparable to that of acceptors. Then, from the Poisson equation one concludes that the characteristic width of the discontinuity is of the order of the saturation length $l_s \ll d, l_0$. When the beam width d increases above $(5-6)l_0$ the discontinuity abruptly disappears. The gradient response ($e = -l_0 I_x$) corresponds to extremely small values of the beam amplitude, $I_0 \ll 0.1$.

To describe the nonlinear beam propagation, we supplement Eq. (1) by an optical equation for the beam envelope Ψ. The later has the form of a $(1+1)D$ nonlinear Schrödinger equation (see, e.g., [9]):

$$2ikn_0\Psi_z + \Psi_{xx} = k^2 n_0^4 r E_0 e \Psi\,, \qquad (2)$$

where k is the vacuum wave vector and $I = |\Psi|^2$. In the case of nonlocal dependence $e(I)$, Eq. (2) does not possess the Hamiltonian structure, which creates serious problems for analytical treatment.

The modified beam propagation method (BPM) [15] was used to solve numerically Eq. (2) in conjunction with Eq. (1). For n_0 and r we used the values 2.5 and 5 pm/V representative for the sillenites. The results of our calculations for the distributions $I(x,z)$ and $\delta n(x,z)$ are shown in Figs. 2a and 2b. The chosen incident peak intensity and beam width are $I_0 = 10$ and $d = 36$ μm, the propagation distance $z_0 = 6$ mm. This distance exceeds considerably the characteristic nonlinear length for the intensity changes, $(kn_0^3 r E_0)^{-1} \approx 0.5$ mm, thus the depicted spatial evolution is strongly nonlinear. The solid lines in Fig. 3 are the snapshots of the transverse intensity distribution for several values of the propagation coordinate z. The dotted lines show the corresponding profiles for the gradient nonlinear response $[\delta n \propto I_x/(1+I)]$ ensuring the same magnitude $|\delta n(x)|_{\max}$ at $z = 0$.

As seen from Figs. 2 and 3, already at a distance $z = 1$ mm, where the index profile is not yet substantially different from the initial one (the line 4 in Fig. 1), the beam experiences remarkable changes. It splits into a main central beam (it is slightly shifted to the right, larger in the

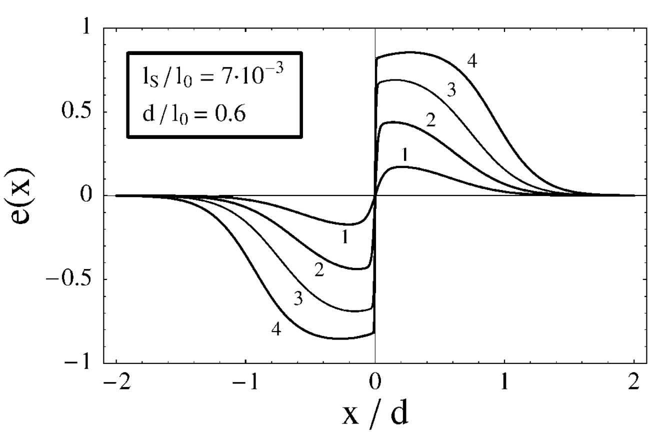

FIG. 1. Normalized space-charge field versus the transverse coordinate; the curves 1, 2, 3, and 4 correspond to the normalized peak intensity $I_0 = 0.1$, 0.5, 2, and 10, respectively.

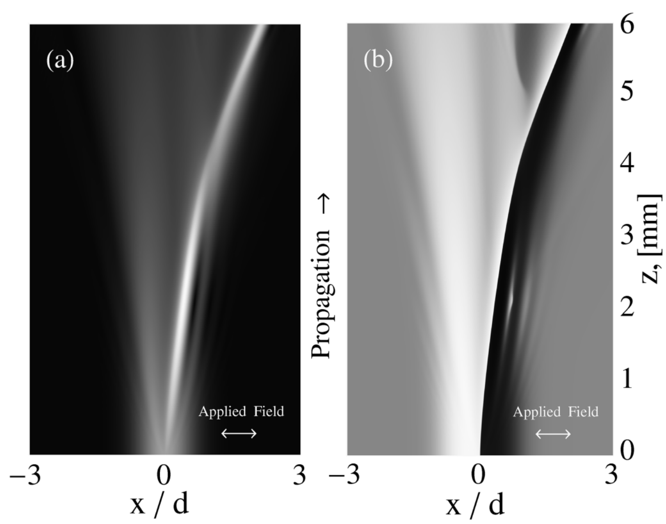

FIG. 2. Spatial evolution of (a) the light intensity $I(x,z)$ and (b) the nonlinear refractive index $\delta n(x,z)$ for an input Gaussian beam. The bright areas correspond to high values of the variables.

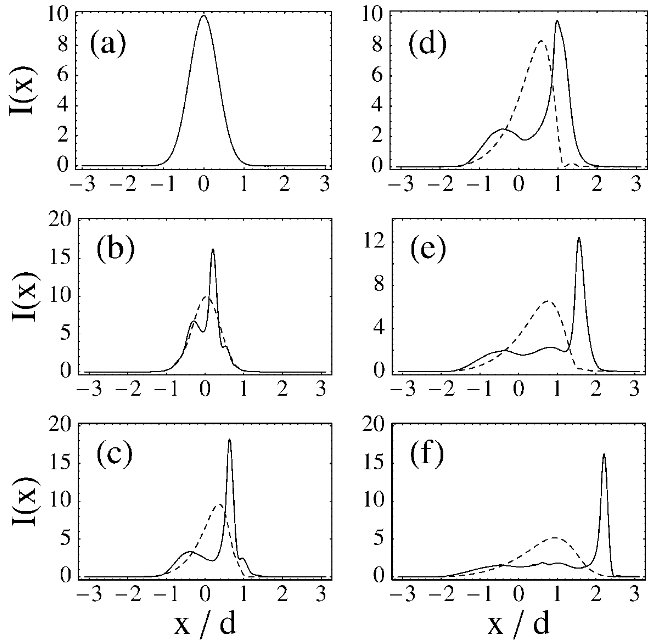

FIG. 3. Beam profiles (a)–(f) corresponding to propagation distances $z = 0$, 1, 3, 4, 5, and 6 mm, respectively.

peak value, and narrower in size as compared to the input beam) and two relatively weak side filaments. These changes are mainly caused by diffraction at the discontinuity and focusing/defocusing on the negative/positive regions of the initial index profile. This was proven by a direct simulation of the corresponding linear propagation problem for a z-independent index.

The subsequent z evolution is strongly nonlinear. It is characterized by the following basic features: The main beam (the core) remains of essentially the same width after the initial compression. Its amplitude pulsates modestly with z without a significant decrease. The shock wave of the refractive index does not disappear and does not branch into multiple shock waves related to the separate filaments; see Fig. 2b. It runs towards the right with a roughly quadratic displacement in z. The main beam is attached to the discontinuity and experiences a permanent bending. The self-trapping can be considered as the propagation of a nonlinear "surface" wave [2], with the index discontinuity acting as a light-guiding surface. Note that the effect of self-bending is known for beam propagation in PR media with a gradient nonlocal response [12,13]. However, the nonlinear behavior is essentially different in the gradient case; see the dotted lines in Fig. 3. The beam does not experience here any trapping and disperses quickly because of diffraction.

A number of other features of the nonlinear beam propagation are also worthy of attention. The side filaments evolve in a manner which is strongly different from that of the main trapped beam. The right filament disappears soon after its formation owing to the progressive bending of the main beam. The confluence of this filament with the trapped beam, which takes place at $z \approx 4$ mm (see Fig. 2a), is accompanied by a "transient" broadening of the compound peak, a decreasing amplitude, and increasing bending. The left filament does not experience any bending. It moves apart from the trapped beam and gradually disperses because of diffraction. The displacement of the intensity maximum is remarkably larger than that in the gradient case.

Basically, the described behavior resembles the formation of a spatial soliton. The main (trapped) beam does not show any noticeable decay. The separation of the side beams is similar to the "emission" of a nonsoliton part from an initial wave distribution as in the cases of the famous Korteweg–de Vries and cubic nonlinear Schrödinger equations [16]. This separation may occur because the chosen input beam width, $d = 36$ μm, exceeds the optimum one for self-trapping. One can expect on the basis of the data of Fig. 3 that a reduction of d by a factor of ≈ 3 facilitates the establishing of the balance between self-focusing on the positive segment of the index profile and diffraction mediated divergence, i.e., reduces the nonsoliton part of the input intensity distribution.

However, identification of the trapped beam with a soliton would be incorrect. The point is that such a soliton (if it does exists) must possess not only a narrow core but also a relatively long tail caused by the presence of the discontinuity. Stability of this soliton implies obviously a dynamic balance between the tail and core. It is not excluded that the trapped beam loses gradually its energy to maintain the tail and slowly disappears.

To clarify the question of the core stability, we have performed an additional numerical experiment. Instead of a fairly wide ($d = 36$ μm) Gaussian input beam, we used a $\simeq 12$ μm-wide asymmetric beam possessing a small and wide left hump. This choice has allowed us to reduce the filamentation process and energy losses; see Fig. 4a. The solid lines 1 and 2 in Fig. 4b show the change of the initial intensity profile after a 4-mm nonlinear propagation. The

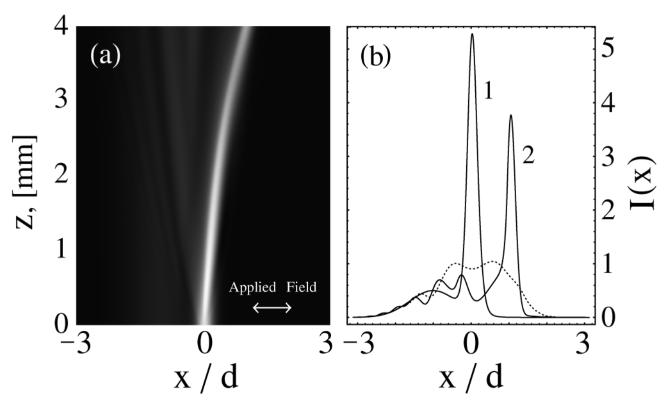

FIG. 4. (a) Spatial evolution of an asymmetric input light beam; (b) beam profiles. The solid lines 1 and 2 correspond to $z = 0$ and 4 mm, respectively. The dotted line is plotted for the gradient response at $z = 4$ mm.

dotted line displays the output profile for the case of the equally strong gradient nonlinear response. One sees that most of the light energy travels with no divergence and the trapped power is larger than that for the previous case, compare with Fig. 3. Furthermore, the difference with the gradient case, where the light beam quickly disperses, is even more pronounced than earlier. At the same time, a slow decrease of the trapped beam amplitude and a slow accumulation of the energy in the tail is obvious.

In this way propagation of the trapped beam along the charge singularity has to be associated not with a stationary but with a metastable long-living state. Correspondingly, we can speak of a solitonlike beam propagation. Since the propagation distance of the trapped beam is rather long, the difference between soliton and solitonlike propagation can be of minor importance for practical purposes. The strong distinction of our case from the case of the gradient response stems from the strong saturation of the function $|\delta n(I)|_{\max}$ and the presence of the discontinuity of the refractive index. These features greatly facilitate beam trapping.

Let us comment now on the results of a recent study of beam propagation in ac-biased BSO crystals [17]. The theoretical part was based on the known relations for the nonlinear response that are valid only in the low-light-contrast limit. Within this approach, the formation of the discontinuity and the effect of strong saturation were missed. The filamentation of a Gaussian input beam observed experimentally is in line with our results.

It is important to emphasize that the current studies on PR solitons deal mostly with slow ferroelectric materials [1]. Changeover to fast materials is vitally important for numerous real-time applications. The use of ac fields makes, as known, the nonlinear response of the sillenites and semiconductors fairly strong [5]. This ac response is, however, essentially different from the response of slow ferroelectrics. A fundamentally different possibility for soliton propagation in fast PR crystals (InP) has been demonstrated recently in [18].

In conclusion, we have investigated theoretically beam propagation in a fast PR crystal placed in an ac field to enhance the nonlinear response. It has been shown that the main part of the incident power is trapped near the singularity of the light-induced space charge and can propagate over fairly long distances with only a weak decay. This scenario strongly differs from that known for other types of PR nonlinear responses.

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[1] G. I. Stegeman and M. Segev, Science **286**, 1518 (1999), and references therein.

[2] G. S. García-Quirino, J. J. Sánchez-Mondragón, and S. I. Stepanov, Phys. Rev. A **51**, 1571 (1995); A. A. Kamshilin, E. Raita, and A. V. Khomenko, J. Opt. Soc. Am. B **13**, 2536 (1996).

[3] T. Honda, Opt. Lett. **18**, 598 (1993); A. V. Mamaev and M. Saffman, Phys. Rev. Lett. **80**, 3499 (1998); S. G. Odoulov, M. Yu. Goulkov, and O. A. Shinkarenko, Phys. Rev. Lett. **83**, 3637 (1999).

[4] B. I. Sturman, S. G. Odoulov, and M. Yu. Goul'kov, Phys. Rep. **275**, 197 (1996); M. Yu. Goul'kov *et al.,* Phys. Rev. Lett. **86**, 4021 (2001).

[5] L. Solymar, D. J. Webb, and A. Grunnet-Jepsen, *The Physics and Applications of Photorefractive Materials* (Claredon, Oxford, 1996).

[6] T. E. McClelland *et al.,* Phys. Rev. Lett. **73**, 3082 (1994); H. C. Pedersen and P. M. Johansen, Phys. Rev. Lett. **77**, 3106 (1996).

[7] E. Podivilov *et al.,* Phys. Rev. Lett. **85**, 1867 (2000).

[8] G. F. Calvo *et al.,* Phys. Rev. Lett. **84**, 3839 (2000).

[9] G. B. Whitham, *Linear and Nonlinear Waves* (Wiley-Interscience, New York, 1974).

[10] V. E. Zakharov, in *Handbook of Plasma Physics* (North-Holland, Amsterdam, 1984), Vol. 2, p. 81.

[11] M. Segev *et al.,* Phys. Rev. Lett. **73**, 3211 (1994).

[12] D. N. Christodoulides and T. H. Coskun, Opt. Lett. **21**, 1460 (1996).

[13] D. N. Christodoulides and M. I. Carvalho, Opt. Lett. **19**, 1714 (1994).

[14] W. H. Press *et al., Numerical Recipes in C* (Cambridge University Press, Cambridge, United Kingdom, 1994).

[15] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, New York, 2001).

[16] G. L. Lamb, Jr., *Elements of Soliton Theory* (Wiley, New York, 1980).

[17] C. A. Fuentes-Hernández and A. V. Khomenko, Phys. Rev. Lett. **83**, 1143 (1999).

[18] M. Chauvet *et al.,* Opt. Lett. **21**, 1333 (1996); R. Uzdin, M. Segev, and G. J. Salamo, Opt. Lett. **26**, 1547 (2001).