Repulsive Casimir Forces

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We discuss repulsive Casimir forces between dielectric materials with nontrivial magnetic susceptibility. It is shown that considerations based on the naive pairwise summation of van der Waals and Casimir-Polder forces may not only give an incorrect estimate of the magnitude of the total Casimir force but even the wrong sign of the force when materials with high dielectric and magnetic responses are involved. Indeed repulsive Casimir forces may be found in a large range of parameters, and we suggest that the effect may be realized in known materials. The phenomenon of repulsive Casimir forces may be of importance both for experimental study and for nanomachinery applications.

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It is well known that the fluctuations of electromagnetic fields in vacuum or in material media depend on the boundary conditions imposed on the fields. This dependence gives rise to forces which are known as Casimir forces, acting on the boundaries. The best known example for such forces is the attractive force experienced by parallel conducting plates in vacuum [1]. Casimir forces between similar, disjoint objects such as two conducting or dielectric bodies are known in most cases to be attractive [2] and are sometimes viewed as the macroscopic consequence of van der Waals and Casimir-Polder attraction between molecules.

In view of the dominance of the Casimir forces at the nanometer scale, where the attractive force could lead to restrictive limits on nanodevices [3,4], the study of repulsive Casimir forces is of increasing interest.

Repulsive van der Waals forces are known to be possible if the properties of the intermediate medium are intermediate between the properties of two polarizable molecules [5]. In such cases the Hamaker constant becomes negative, a property which was successfully employed to explain the wetting properties of liquid helium [6]. How can one get a repulsive behavior when the intermediate substance is vacuum? A partial answer can be obtained from the observation that a purely magnetically polarizable particle repels a purely electrically polarizable particle [7]. Motivated by this result, Boyer, following Casimir's suggestion, studied interplane Casimir force with one plate a perfect conductor while the other is infinitely permeable. He showed that in this case the plates repel [7]. This problem was reconsidered since in [8,9]. However, one must note that for most molecules the magnetic polarizability is negligible compared to the electric polarizability. Indeed, in many of the treatments of the subject it is assumed that the van der Waals interaction is dominated by the dielectric behavior of the materials. If one then considers the pairwise summation of van der Waals and Casimir-Polder forces as an approximation to the force between materials, the result is generally attractive.

Calculations of the interaction between macroscopic bodies by summation of pair interactions are based on the assumption of additivity of the interatomic interaction energies, which is justified only within second order perturbation theory [10]. It was pointed out by Axilrod and Teller [11] that many-particle interactions may lead to substantial corrections to the so-called "additive" result. Sparnaay [12] estimated the corrections for some simple many-body systems to be as large as 30%. These corrections are usually taken to affect the *magnitude* of the force but not its sign.

In this Letter we emphasize that for materials with high magnetic susceptibility pairwise summation is no longer a good approximation for the macroscopic Casimir force, due to the collective response of the material. Indeed, we show that these considerations may not only give an incorrect estimate of the magnitude of the force but in some cases even of its sign. Thus restrictions imposed on the sign of the force from pairwise consideration can be misleading, and repulsive forces can be expected in a wider range of dielectric-magnetic materials. We study the Casimir force between materials with general permittivity and permeability and show that, for large permeability and permittivity, the transition between attractive and repulsive behavior depends only on the impedance $Z = \sqrt{\frac{\mu}{\epsilon}}$. In addition we show that at high temperatures there is always attraction, and thus in some cases the force changes sign as the temperature is increased.

We start by examining the pair interaction. The Casimir-Polder potential between two polarizable particles A and B is given by [13,14]

$$U(r) = -\frac{\hbar c}{4\pi r^7}[23(\alpha_E^A\alpha_E^B + \alpha_M^A\alpha_M^B) - 7(\alpha_E^A\alpha_M^B + \alpha_M^A\alpha_E^B)], \qquad (1)$$

where α_E, α_M are the electric and magnetic polarizability of the particles. From this equation it is immediately apparent that repulsion between particles can be obtained. For example, a purely electrically polarizable particle will repel a purely magnetically polarizable particle. What does this tell us about materials described by a given permittivity and permeability?

As an illustration of the subtle character pairwise summations may have we consider two materials with permeability and permittivity, ϵ_i, μ_i $(i = 1, 2)$. When one considers two polarizable balls in vacuum [15] the coefficient of the force can be read from (1) to be of the form

$$F_C \propto -23 \frac{\epsilon_1 - 1}{\epsilon_1 + 2} \frac{\epsilon_2 - 1}{\epsilon_2 + 2} - 23 \frac{\mu_1 - 1}{\mu_1 + 2} \frac{\mu_2 - 1}{\mu_2 + 2} + 7 \frac{\epsilon_1 - 1}{\epsilon_1 + 2} \frac{\mu_2 - 1}{\mu_2 + 2} + 7 \frac{\epsilon_2 - 1}{\epsilon_2 + 2} \frac{\mu_1 - 1}{\mu_1 + 2}. \quad (2)$$

To simplify the last expression we set $\mu_1 = 1$. In this case

$$F_C \propto -\frac{\epsilon_1 - 1}{\epsilon_1 + 2} \left(23 \frac{\epsilon_2 - 1}{\epsilon_2 + 2} - 7 \frac{\mu_2 - 1}{\mu_2 + 2} \right). \quad (3)$$

It can easily be shown that for $\epsilon_2 > \frac{37}{16}$ the force (3) is negative for any μ_2, and thus two such balls attract. Thus if one regards the two materials to be made of such "balls," and use as an approximation to the Casimir force summation of pairs of these, one comes to the conclusion that there is attraction whenever $\epsilon_2 > \frac{37}{16}$ and $\mu_1 = 1$. Next, however, looking at the whole Casimir energy we demonstrate that this last statement is wrong.

The Casimir interaction between two polarizable materials can be conveniently expressed in terms of the reflection coefficients at the boundaries ([3,16]). A general expression for the Casimir energy in planar geometry was obtained in Ref. [16]. Following [16] it is convenient to parametrize the field modes using $\mathbf{k} = (k_x, k_y, k_t)$, where k_t is defined by a Wick rotation $k_t \leftrightarrow i\omega$ [17]. In terms of $\mathbf{k}$ the Casimir energy per unit area of two infinite slabs separated by a distance a is given by

$$E_C = \frac{1}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \{\ln[1 - r(\epsilon_1, \mu_1) r(\epsilon_2, \mu_2) e^{-2a|\mathbf{k}|}] + \ln[1 - r(\mu_1, \epsilon_1) r(\mu_2, \epsilon_2) e^{-2a|\mathbf{k}|}]\}, \quad (4)$$

where the two terms on the right correspond to TE and TM modes, respectively. Here $r(\epsilon, \mu)$ for the TE mode is given by

$$r(\epsilon, \mu, \theta) = \frac{\sqrt{\epsilon\mu \cos^2(\theta) + \sin^2(\theta)} - \mu}{\sqrt{\epsilon\mu \cos^2(\theta) + \sin^2(\theta)} + \mu}, \quad (5)$$

and a similar expression with $\epsilon \leftrightarrow \mu$ holds for the TM mode [i.e., $r_{\mathrm{TM}}(\epsilon, \mu) = r_{\mathrm{TE}}(\mu, \epsilon)$] $r(\epsilon, \mu, \theta)$ is related to the usual reflection coefficient at the interface between vacuum and a medium with given ϵ and μ (see, for example, Eq. 7.39 in [18]) by adapting it to our notations (in particular our θ is not the usual angle of incidence, although it can be related to it). Hence the energy is given by

$$E_C = \frac{1}{8\pi^2} \int_0^\pi d\theta \sin\theta \int_0^\infty dk\, k^2 \times \{\ln[1 - r(\epsilon_1, \mu_1) r(\epsilon_2, \mu_2) e^{-2ak}] + \epsilon \leftrightarrow \mu\}. \quad (6)$$

The k integration can be done by expanding the logarithm and integrating term by term to obtain

$$E_C = -\frac{1}{32\pi^2 a^3} \int_0^\pi \mathrm{Li}_4[r(\epsilon_1, \mu_1, \theta) r(\epsilon_2, \mu_2, \theta)] \times \sin\theta\, d\theta + \epsilon \leftrightarrow \mu, \quad (7)$$

where $\mathrm{Li}_4(x) = \sum_{n=1}^\infty \frac{x^n}{n^4}$ is a polylogarithmic function.

In Fig. 1 we show positive and negative domains of the Casimir energy (7). Note that the condition $\epsilon_2 < \frac{37}{16}$ is not fulfilled in the repulsive regime, contradicting the argument given above based on the pairwise attraction picture. Thus determination of the sign of the Casimir force in the general case must involve the full expression (7), which predicts large repulsive regimes.

Although the full expression for the energy (7) is quite complicated, there is a simple statement to be made regarding the direction of the force: The border line between repulsive and attracting regimes for large values in the ϵ_1, μ_1 plane (as seen in the figures) is always linear, i.e., of the form $\frac{\epsilon_1}{\mu_1} \to \mathrm{const}$ when $(\epsilon_1, \mu_1) \to \infty$. This can be seen from the following argument: for large μ and ϵ we have

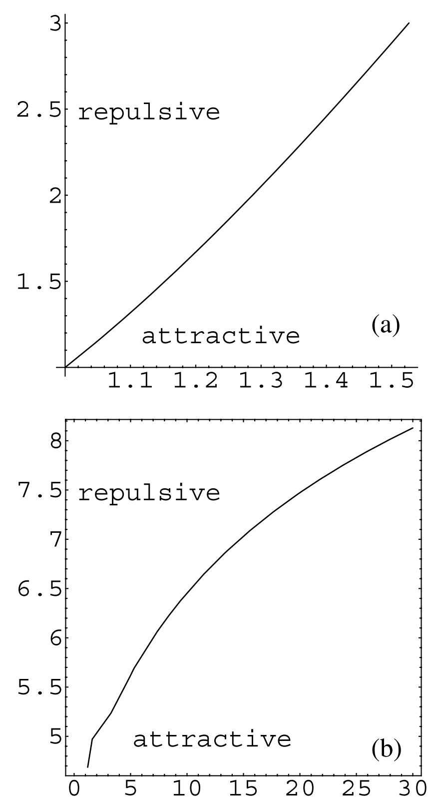

FIG. 1. Repulsive and attracting regions: (a) in the ϵ_2, μ_2 plane for $\epsilon_1 = 2$ and $\mu_1 = 1$ and (b) in the ϵ_1, ϵ_2 plane $\mu_1 = 1$ and $\mu_2 = 20$.

$$r(\epsilon,\mu,\theta) \sim \frac{\sqrt{\frac{\epsilon}{\mu}}|\cos(\theta)| - 1}{\sqrt{\frac{\epsilon}{\mu}}|\cos(\theta)| + 1} \tag{8}$$

(and a similar expression holds for $\epsilon \leftrightarrow \mu$). Thus in this limit the Casimir energy (7) can be written as a function of $\frac{\epsilon}{\mu}$ and vanishes for a certain value of this ratio. Note that one doesn't have to use very high values of the permittivity and permeability for the approximation (8) to be valid [see, for example, Fig. 1(a)], since the terms we omit are reduced by factors of $\frac{1}{\mu}$ or $\frac{1}{\epsilon}$.

We now point out several particular cases:

(i) When both of the materials have high permeability and permittivity, one can use the approximation (8) for both materials. To get an idea about the sign, we approximate the Casimir energy (7) by calculating the integral using just the first term in the polylogarithmic function and obtain

$$E_C \sim -\frac{1}{16\pi^2 a^3}\left[2 + I(Z_1, Z_2) + I\left(\frac{1}{Z_1}, \frac{1}{Z_2}\right)\right], \tag{9}$$

where $Z_i = \sqrt{\frac{\mu_i}{\epsilon_i}}$ are the impedances of the materials and $I(Z_1, Z_2) = 2\frac{Z_2+Z_1}{Z_2-Z_1}[Z_1 \ln(\frac{1}{Z_1} + 1) - Z_2 \ln(\frac{1}{Z_2} + 1)]$. The border curve defined by $E_C(Z_1, Z_2) = 0$ is shown in Fig. 2.

(ii) If one of the bodies is a perfect conductor then for large μ and ϵ the Casimir energy is given by

$$\begin{aligned} E_C &= \frac{-1}{32\pi^2 a^3}\int_0^\pi \mathrm{Li}_4[r(\epsilon_1,\mu_1,\theta)] + \mathrm{Li}_4[-r(\mu_1,\epsilon_1,\theta)]\sin\theta\, d\theta \\ &\sim \frac{-1}{16\pi^2 a^3}\left\{\frac{14}{8Z_1}\ln(Z_1 + 1) + \frac{3}{8}\left[1 - 6Z_1 \ln\left(1 + \frac{1}{Z_1}\right)\right]\right\}, \end{aligned} \tag{10}$$

where the first two terms in the Li_4 series where used. In this case for high permeability and permittivity the transition from an attractive to a repulsive regime takes place at $Z_1 = 1.037$, i.e., $\mu \sim 1.08\epsilon$.

(iii) The Casimir energy (7) can be most easily analyzed in the uniform velocity of light (UVL) case [19]. In this case the reflection coefficients (5) are independent of the angle, namely, $r(\epsilon,\mu) = \frac{1-\mu}{1+\mu}$. This makes the θ integral in (7) trivial, with the result

$$E_C = \frac{1}{8\pi^2 a^3}\mathrm{Li}_4\left[\left(\frac{1-\mu_1}{1+\mu_1}\right)\left(\frac{1-\mu_2}{1+\mu_2}\right)\right]. \tag{11}$$

This result agrees with the result obtained in [22] for a dilute medium (i.e., $|\mu_i - 1| \ll 1$ for $i = 1, 2$). In this case the force becomes repulsive if $\mu_1 > 1$ and $\mu_2 < 1$ or vice versa. However, the condition $\epsilon = \frac{1}{\mu}$ then implies that one of the materials will have $\epsilon(\omega) < 1$ on the imaginary axis which can be shown to be inconsistent with general properties of the dielectric function of a realistic material [23].

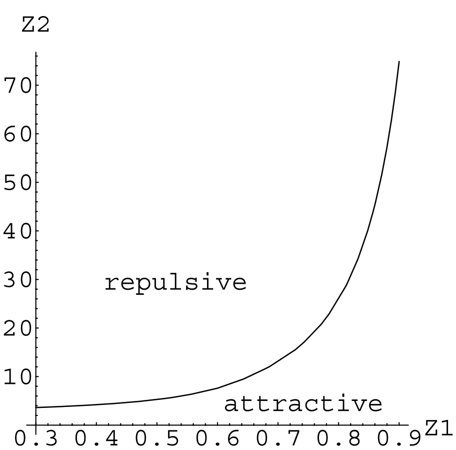

FIG. 2. Repulsive and attracting regions in the Z_1, Z_2 plane.

The leading term in the high temperature expansion for the free energy is obtained in the usual procedure by retaining only the zero Matsubara frequency [i.e., replacing $\int \frac{d^3k}{(2\pi)^4}$ by $\kappa_B T \int \frac{dk_x dk_y}{(2\pi)^2}$ in (4)]. Note that, in our mode parametrization, zero frequency corresponds to $\theta = 90°$ in (5) which greatly simplifies the integration. We find

$$\begin{aligned} F_C = -\frac{\kappa_B T}{16\pi a^2}\Bigg[&\mathrm{Li}_3\left(\frac{1-\mu_1}{1+\mu_1}\frac{1-\mu_2}{1+\mu_2}\right) \\ &+ \mathrm{Li}_3\left(\frac{1-\epsilon_1}{1+\epsilon_1}\frac{1-\epsilon_2}{1+\epsilon_2}\right)\Bigg]. \end{aligned} \tag{12}$$

This expression leads to an attractive force for any values of the permeability and permittivity provided μ_i, $\epsilon_i > 1$ ($i = 1, 2$). Thus even if at low temperature we have repulsion the force will change sign as we heat the system. However, the sign change is at temperature scales of $\kappa_B T \sim \frac{\hbar c}{4\pi a}$ which at 100 nm is of the order of 1000° K. This phenomenon can be qualitatively explained as follows: The dominant contribution to the free energy at high temperatures is due to static configurations (zero modes), since contributions from other Matsubara frequencies are exponentially suppressed. In particular the magnetic-electric interactions are nonstatic by nature (there is no static interaction between a magnetic dipole and an electric dipole). As a result the magnetic-electric parts of the interaction which are responsible for repulsive behavior [see Eq. (1)] will vanish in the high temperature limit [24].

In view of the growing interest and possibilities of measuring the Casimir effect [3,4,25–27] we wish to point out some advantages that repulsive Casimir forces might have for actual measurements: There is no problem of stiction, i.e., even for very close separations, the materials don't collapse on each other, in contrast to the usual effect where it can sometimes be difficult to hold them apart, a property which might be crucial for construction of nanomachines. Moreover it might be an easier task to align two materials in parallel, since this will be their natural tendency (in places where the two materials get closer, the repulsion is stronger). We would also wish to note, that although it is true that for many materials the magnetic response is negligible, there are classes of materials with high permeability, such as ferrites and garnets (notably YIG) which may be suitable for constructing a demonstration of repulsive Casimir forces.

We conclude by briefly summarizing the main novel results of this Letter: Repulsive Casimir forces are predicted for pairs of materials depending on their permeabilities and permittivities. In some cases estimation of the Casimir interaction by considering only two-body forces may indicate attraction between materials while the total Casimir force is repulsive. A generalized expression is given for the Casimir energy for arbitrary values of the permeabilities and permittivities of two parallel materials. For large values of the permeability and permittivity, the force depends only on the surface impedance. (This should lead to a convenient characterization of materials suitable for demonstrating repulsive Casimir forces.) However, even in the repulsive case, in the high temperature limit there is always attraction, thus the force will change direction as the temperature is raised.

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[1] H. B. G. Casimir, Proc. Koninkl. Ned. Akad. Wet. **51**, 793 (1948).
[2] O. Kenneth and S. Nussinov, Phys. Rev. D **65**, 085014 (2002).
[3] M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001).
[4] E. Buks and M. L. Roukes, Phys. Rev. B **63**, 033402 (2001).
[5] Jacob N. Israelachvili, *Intermolecular and Surface Forces* (Academic Press, London, 1992).
[6] I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Adv. Phys. **10**, 165 (1961).
[7] T. H. Boyer, Phys. Rev. A **9**, 2078 (1974).
[8] D. T. Alves, C. Farina, and A. C. Tort, Phys. Rev. A **61**, 034102 (2000).
[9] J. C. da Silva, A. Matos neto, H. Q. Placido, M. Revzen, and A. E. Santana, Physica (Amsterdam) **292A**, 411 (2001).
[10] F. London, Z. Phys. Chemie. (B) **11**, 222 (1930).
[11] B. M. Axilrod and E. Teller, J. Chem. Phys. **11**, 299 (1943).
[12] M. J. Sparnaay, Physica **25**, 353 (1959).
[13] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics,* Course of Theoretical Physics Vol. IV (Pergamon, Oxford, 1982).
[14] G. Feinberg and J. Sucher, Phys. Rev. A **2**, 2395 (1970).
[15] V. M. Mostepanenko and N. N. Trunov, *The Casimir Effect and its Applications* (Oxford University Press, New York, 1997).
[16] O. Kenneth, hep-th/9912102.
[17] Here **k** will be treated as a Euclidean vector. In particular $|\mathbf{k}| = \sqrt{k_x^2 + k_y^2 + k_t^2}$ and $\cos\theta \equiv k_t/|\mathbf{k}|$.
[18] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., Chap. 16.
[19] The UVL condition implies $\epsilon\mu =$ const. It was introduced by Lee [20] in an effective description of gluon fields and used by Brevik and Kolbensvet [21] in the context of electromagnetic Casimir forces.
[20] T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, Chur, 1980).
[21] I. Brevik and H. Kolbenstvedt, Ann. Phys. (N.Y.) **143**, 179 (1982).
[22] I. Klich, A. Mann, and M. Revzen, Phys. Rev. D **65**, 045005 (2002).
[23] This is a simple result of the Kramers-Krönig relations. Note, however, that if the intermediate vacuum is to be filled with another substance the relevant properties are relative to the intermediate substance, and may yield repulsive behavior.
[24] J. A. Grifols and F. Ferrer, Phys. Lett. B **460**, 371 (1999); **511**, 319(E) (2001).
[25] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, Phys. Rev. Lett. **87**, 211801 (2001).
[26] F. Chen, U. Mohideen, G. L. Klimchitskaya, and V. M. Mostepanenko, quant-ph/0201087.
[27] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).