Why an ac Magnetic Field Shifts the Irreversibility Line in Type-II Superconductors

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We show that for a thin superconducting strip placed in a transverse dc magnetic field—the typical geometry of experiments with high-T_c superconductors—the application of a weak ac magnetic field perpendicular to the dc field generates a dc voltage in the strip. This voltage leads to the decay of the critical currents circulating in the strip, and eventually the equilibrium state of the superconductor is established. This relaxation is not due to thermally activated flux creep but to the "walking" motion of vortices in the *two-dimensional* critical state of the strip with in-plane ac field. Our theory explains the shaking effect that was used for detecting phase transitions of the vortex lattice in superconductors.

Experimental investigation of the equilibrium properties of type-II superconductors can be performed only in the reversible region of the magnetic field (H) versus temperature (T) plane. But often strong flux-line pinning prevents the measurement of equilibrium properties. In this context, Willemin *et al.* [1] recently made an interesting observation. Their experiments revealed that application of an additional small oscillating magnetic field *perpendicular* to the main dc field leads to a fast decay of the currents circulating in the critical state of various high-T_c superconductors. This effect dramatically extends the observable reversible domain in the H-T plane. The relaxation of the irreversible magnetization in these experiments was exponential in time, and thus was obviously different from thermally activated flux creep, which leads to a logarithmic time law. Using this vortex-shaking process, the melting transition of the vortex lattice in $YBa_2Cu_3O_{7-\delta}$ crystals was detected [2] at temperatures very close to the critical temperature T_c, where the melting could not be investigated before. With the same shaking method, it was discovered [3] that the order-disorder transition in the vortex lattice of $Bi_2Sr_2CaCu_2O_8$ is of the first order at low temperatures, where vortex pinning usually masks the corresponding jump in the equilibrium magnetization. Thus, the vortex-shaking process opens new possibilities in experimental investigation of the H-T phase diagrams of superconductors. Nevertheless, the nature of this important effect so far remained unclear.

In this paper we give a quantitative explanation of this vortex-shaking effect for thin strips. It is shown that the relaxation is caused by the generation of a *dc electric field* by the *ac magnetic field*. We obtain this result in the framework of a quasistatic approach based on the standard critical state concept. Therefore, this effect is quite general and should occur in all type-II superconductors, not only in high-T_c materials. In contrast to the generation of an electric field in the case [4] when the ac and the dc magnetic fields are *parallel,* the above electric field appears in superconductors *without* a transport current. We also point

out a new way of measuring the critical current density j_c in superconductors.

The dc electric field in general depends on the orientation of the ac magnetic field with respect to the currents circulating in the superconductor. In the following we shall analyze the simplest situation: A thin superconducting strip fills the space $|x| \leq w$, $|y| < \infty$, $|z| \leq d/2$ with $d \ll w$; the constant and homogeneous external magnetic field H_a is directed along z, while the ac magnetic field $h \cos\omega t$ is applied along x, i.e., perpendicular to H_a and to the currents in the sample (Fig. 1). We also make here the usual Bean assumption that the critical current density j_c does not depend on the local induction B. The field H_a is assumed to be sufficiently large to exceed both the field of full penetration for the strip [5], $H_p = (j_c d/\pi) \ln(2ew/d)$, and the lower critical field H_{c1}, and so we may put $B = \mu_0 H$.

A qualitative explanation of the shaking effect is as follows: The currents flowing in the critical state of the strip generate a nonuniform distribution of the magnetic induction $B_z(x)$. The ac field periodically tilts the vortices in this state. However, at each point x with a nonzero sheet current $J(x)$ (the current density integrated over the thickness d), the tilt is *not symmetric* relative to the central plane of the strip $z = 0$, and during each cycle of the ac field, the asymmetry leads to a shift of vortices towards the center $x = 0$ of the strip. This shift tends to equilibrate $B_z(x)$, and it also generates a dc electric field which decreases $J(x)$. When $J(x)$ reaches zero, the asymmetry disappears, and the process stops.

It will be shown below that the generated dc electric field is proportional to the thickness of the sample, d. Thus, to describe the effect, the strip cannot be considered as infinitely thin, and the appropriate critical state problem is two dimensional. However, according to the approach of Ref. [6], the smallness of the parameter d/w enables us to *split the problem into two simpler problems*: A one-dimensional problem across the thickness of the sample, and a problem for the infinitely thin strip. Namely, we first
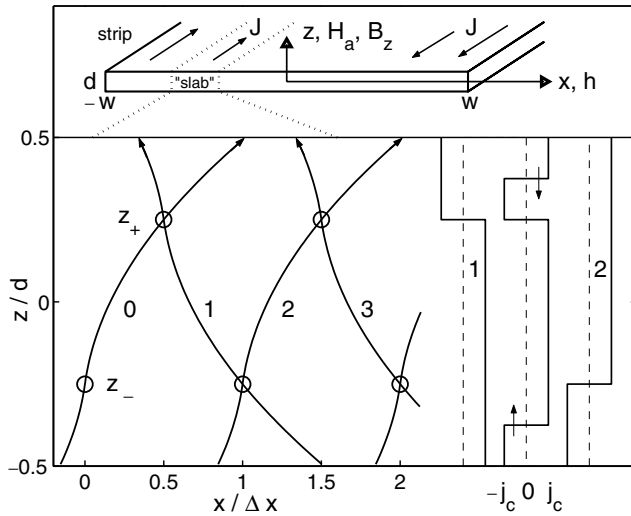
FIG. 1. Strip geometry and magnetic fields (upper inset). A flux line "walking" from left to right through a section of the strip ("slab") is shown at times $t\omega/\pi = 0, 1, 2, 3$. The swivel points of the line are marked by circles. Here $h/J_c = 0.6$, $J/J_c = 0.5$, yielding $z_+ = -z_- = 0.25d$ and $\Delta x = 0.35(\mu_0 J/B_z)d$. Δx is the shift per cycle, Eq. (1); x is measured from an arbitrary point of the "slab." The line shape consists of parabolas with $d^2x/dz^2 = \pm\mu_0 j_c/B_z$. The scheme at the right shows the current profiles across the slab at the extremal times 1 and 2 and at some intermediate time, where the two arrows indicate the penetration of the current-inversion front.

interpret a small section of the strip around an arbitrary point x (see Fig. 1) as an "infinite" slab of thickness d placed in a perpendicular dc magnetic field $B_z(x)$ and in a parallel ac field $h\cos\omega t$ and carrying a sheet current $J(x)$. The resulting dc electric field $E_y = E_y(J, B_z, h)$ for the slab we then use as the local electric field $E_y(x)$ for an infinitely thin strip, to calculate the temporal evolution of the sheet current $J(x)$ and induction $B_z(x)$ in this strip by the method of Ref. [7].

We begin with the analysis of the dc electric field E_y generated by the ac magnetic field, $h\cos\omega t$, in an infinite slab with sheet current J and with a constant and homogeneous magnetic field B_z. At sufficiently small ω [8], the problem may be considered quasistatically: We solve the critical state equation for the slab, $dB_x/dz = \mu_0 j_{cr}(z, t)$, at every moment of time t, and then find the shape of the flux lines at this moment from $dx/B_x = dz/B_z$ and the condition that the points on each line where j_{cr} changes its sign cannot move. The critical current density j_{cr} is always equal to j_c or $-j_c$, and its distribution over z is just the well-known distribution in a slab in a *parallel* magnetic field and with current J. In particular, at $t = 0$ and $t = 2\pi/\omega$ one has $j_{cr} = j_c$ for $z_- < z \leq d/2$, and $j_{cr} = -j_c$ for $-d/2 \leq z < z_-$ (see Fig. 1), while at $t = \pi/\omega$ one has $j_{cr} = -j_c$ for $z_+ < z \leq d/2$, and $j_{cr} = j_c$ for $-d/2 \leq z < z_+$ where $z_+ = -z_- = J/2j_c$. An essential finding of this analysis is that during one half cycle ($0 \leq t \leq \pi/\omega$) every flux line turns around a fixed

"swivel point" with the coordinate $z = z_+$, while during the second half cycle ($\pi/\omega \leq t \leq 2\pi/\omega$) the line turns around another fixed point with $z = z_-$. As a result, the line "walks" along x; see Fig. 1. The shift of the vortices during one full cycle is

$$\Delta x = \frac{2\mu_0 J}{j_c B_z}[h - h_p(J)], \qquad (1)$$

where $h_p(J) = (J_c - |J|)/2$ is the field of full penetration of *parallel* flux into a slab with current J, and $J_c = j_c d$ is the critical sheet current. Note that in deriving Δx we have assumed $h \geq h_p(J)$. Otherwise, the x component of the magnetic field does not penetrate completely into the slab, a section of the flux line does not sway and hence does not move at all, and thus $\Delta x = 0$. The dc electric field generated by the above shift of the vortex lines is $E_y = (\omega/2\pi)\Delta x B_z$, and hence we arrive at

$$\begin{aligned} E_y &= 0 \quad \text{for } h < h_p(J), \\ E_y &= \frac{\mu_0 \omega dJ}{\pi J_c}[h - h_p(J)] \quad \text{for } h \geq h_p(J). \end{aligned} \qquad (2)$$

The described mechanism for the generation of a dc electric field was proposed many years ago [8], and Eq. (2) is equivalent to the formula (2.8) of Ref. [8]. Equation (2) also coincides with the corresponding result for the slab obtained in our paper [4], though in Ref. [4] the dc magnetic field lies in the plane of the slab. This coincidence [9] is not surprising, since in both cases E_y may be interpreted as being generated by the transfer of the x component of the flux across the slab thickness d.

In the derivation of E_y we have assumed that J and B_z do not change during one cycle. This approximation is justified when the relaxation time of the profiles $J(x)$ and $B_z(x)$ considerably exceeds the period $2\pi/\omega$ of the ac field. As will be shown below, this condition is indeed fulfilled for thin samples, $d \ll w$, and thus the approximation is self-consistent.

We now consider the temporal evolution of the profiles $J(x)$ and $B_z(x)$ in the infinitely thin strip [7]. This evolution is caused by the slow "walking" of vortices toward the center of the strip and is given by the Maxwell equation

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}, \qquad (3)$$

with E_y specified by Eq. (2). Since $E_y = v_x B_z$, where v_x is the average velocity of the vortices, the above equation simply expresses the conservation law of their number inside the strip. On the other hand, the Biot-Savart law yields the following relation between the sheet current J and B_z:

$$\frac{B_z(x)}{\mu_0} = H_a + \frac{1}{2\pi}\int_{-w}^{w}\frac{J(u)du}{u - x}. \qquad (4)$$

Equations (2)–(4) together with the initial condition

$$J(x)|_{t=0} = -J_c x/|x|, \qquad (5)$$

are sufficient to determine the temporal evolution of J, E_y, and B_z in the strip, i.e., the functions $J(x,t)$, $E_y(x,t)$, and $B_z(x,t)$.

To proceed further, it is convenient to rearrange Eqs. (3) and (4) as follows: We invert [10] Eq. (4), differentiate both sides of the result with respect to t, insert Eq. (3), and arrive at an equation for $J(x,t)$:

$$\frac{\partial J(x,t)}{\partial t} = \frac{2}{\pi \mu_0} \int_{-w}^{w} \frac{du}{u-x} \left(\frac{w^2-u^2}{w^2-x^2}\right)^{1/2} \frac{\partial E_y(J)}{\partial u},$$
(6)

with $E_y(J)$ given by Eq. (2). The right-hand side of Eq. (6) is proportional to $(d/w)\omega$; this becomes evident if one introduces the dimensionless length x/w. Thus, the decrease of J/J_c during one cycle is determined by the small parameter d/w, and the above assumption that J and B_z are constant over one cycle is well justified.

The numerical method of solving Eq. (6) is well elaborated [7], and we use it to analyze the evolution of the sheet current $J(x,t)$. In Fig. 2 we show the time dependence of the magnetic moment [11] per unit length of the strip, $M(t) = \int_{-w}^{w} J(x,t)x\,dx$, at various amplitudes h of the ac magnetic field. If $h/J_c > 0.5$, then $|M(t)|$ decreases almost exponentially with time, and eventually the equilibrium state [12] is established since $J \to 0$ everywhere in the strip; when $h/J_c < 0.5$, the equilibrium state is not reached, Fig. 3. At large times t, the saturation value of the current in the sample, J_∞, can be found from the condition $E_y(J_\infty) = 0$, yielding

$$J_\infty = J_c - 2h.$$
(7)

In the special case $h = h^* = 0.5J_c$, Eq. (2) gives $E_y \propto J^2$, and it follows from Eq. (6) that $M \propto J \propto t^{-1}$. Thus,
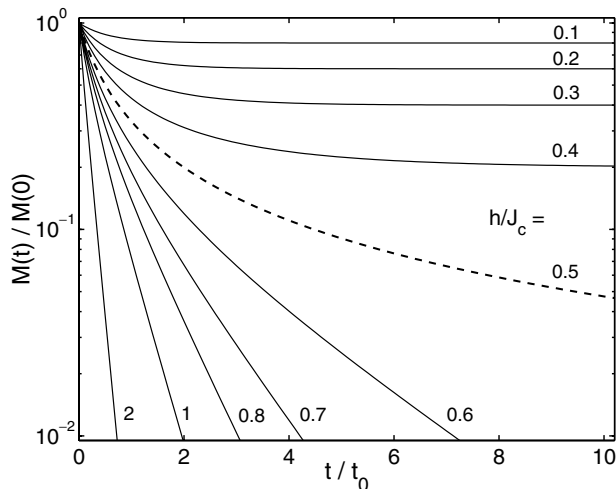
we conclude the following: When in an experiment the amplitude h is fixed while j_c increases due to a decrease of temperature T, then at some T one has $J_c = 2h$, and at lower T the vortex shaking will not lead to the equilibrium state. In other words, the greater is $j_c d$, the greater is the ac amplitude h required for the equilibrium state to be reached. This feature of the vortex-shaking process was indeed observed in the experiments [2,3].

We consider now more closely the practically important case $h > J_c/2$. When J decays, then, according to Eq. (2), the problem formally reduces to the Ohmic strip problem analyzed in Ref. [7], but with the resistivity now associated neither with free flux flow nor with thermally assisted flux flow, namely, $E/j = \rho = \mu_0 \nu d^2(2h/J_c - 1) = \text{const}$, $\nu = \omega/2\pi$. In this case the sheet current after some transient time tends to the following solution of Eq. (6):

$$J(x,t) = -CJ_c f(x/w) \exp(-t/\tau),$$
(8)

where C is a constant depending on the magnetic history, $f(\nu)$ is the normalized eigenfunction with lowest

FIG. 2. The time dependence of the magnetic moment $M(t)$ of the strip at various amplitudes h of the ac magnetic field: $h/J_c = 0.1, 0.2, \ldots, 0.8, 1, 2$. The dashed line ($h/J_c = 0.5$) separates the complete and incomplete relaxation of M. $M(0) = M_0 = -w^2 J_c$ is the magnetic moment of the strip in the fully penetrated critical state. The time unit $t_0 = (\pi w/\omega d)$ corresponds to $w/2d \gg 1$ cycles.
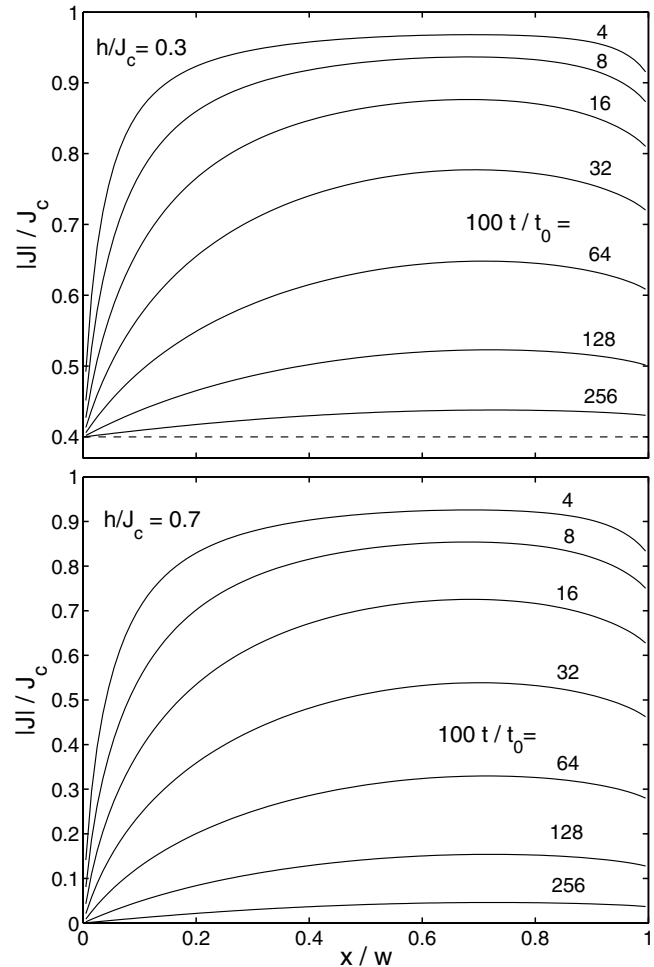

FIG. 3. Profiles of the sheet current $J(x,t)$ in the strip at various times t (in units $t_0/100$, $t_0 = \pi w/\omega d$) starting from Eq. (5). Top: $h/J_c = 0.3$; the dashed line shows J_∞, Eq. (7). Bottom: $h/J_c = 0.7$; exponential relaxation to $J = 0$.
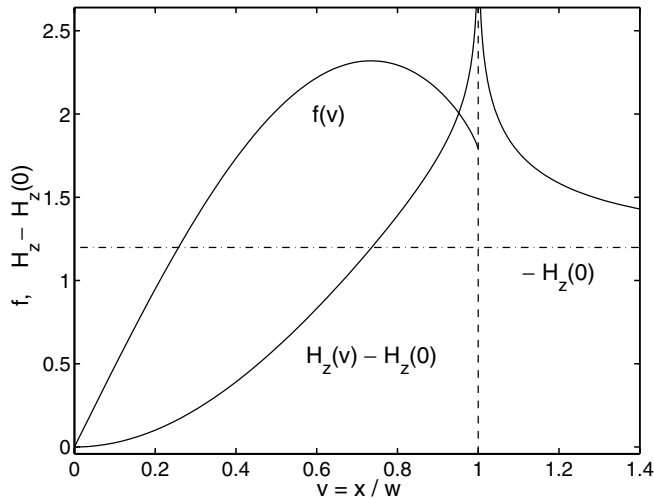
FIG. 4. The universal spatial profile $f(v)$ of the relaxing sheet current $J(x, t)$ in the strip at $t > \tau$, Eq. (8), and the magnetic field H_z generated by this current profile. The dash-dotted line indicates $-H_z(0)$. H_z is in units $J/f = CJ_c \exp(-t/\tau) = -M(t)/w^2$. At the edges H_z has a logarithmic infinity, which is cut off by the finite thickness d.

eigenvalue Λ of the integral equation,

$$\Lambda f(v) = -\frac{1}{\pi^2} \int_{-1}^{1} \frac{du}{u - v} \left(\frac{1 - u^2}{1 - v^2}\right)^{1/2} \frac{df(u)}{du}, \quad (9)$$

and the relaxation time τ is given by

$$\tau^{-1} = \Lambda \frac{\omega d}{w} \left(\frac{2h - J_c}{J_c}\right). \quad (10)$$

Numerical analysis [7] yields $\Lambda = 0.6386$. The odd function $f(v) = -f(-v)$ normalized to $\int_0^1 v f(v) \, dv = \frac{1}{2}$ is shown in Fig. 4. The magnetic moment per unit length of the strip, $M = \int_{-w}^{w} xJ \, dx$, is then $M = -Cw^2 J_c \exp(-t/\tau)$, while in the fully penetrated critical state one has $M = M_0 = -w^2 J_c$ and in the Meissner state [7] $M = -\pi w^2 H_a$.

Thus, we obtain that at $h > J_c/2$ the decay of the current becomes exponential in time, in agreement with the experimental data [1], and the spatial profile of the current tends to the universal function $f(x/w)$, which is independent of the superconducting parameters and dimensions of the strip. It also follows from Eq. (10) that the rate of the decay increases with ω and h. However, for the above quasistatic analysis to be valid, ω should not be too large [8], while the condition $(\tau\omega/2\pi) \gg 1$ leads to $(w/d)(J_c/h) \gg 1$, or $h \ll j_c w$. This puts an upper bound on h up to which the above formulas are applicable.

Interestingly, with Eqs. (7) and (10), measurements of the amplitude h^* at which $J_\infty = 0$, or of τ for $h > h^*$, allow one to obtain j_c at short times $t \sim \omega^{-1}$ where j_c is only weakly affected by thermally activated creep.

To conclude, we give a quantitative description of the vortex-shaking effect in a thin superconducting strip. It is shown that in the framework of the critical-state theory, the decay of the critical currents to zero, or the decrease of the magnetic moment of the strip to its equilibrium value, is due to the generation of a dc electric field by an ac magnetic field applied normal to the main dc magnetic field. This electric field originates only if the finite thickness of the strip is taken into account, and so the theoretical description of the vortex-shaking effect is, strictly speaking, a two-dimensional spatial problem. But the small value of the parameter d/w enabled us to simplify the problem and to solve it explicitly for ac amplitude $h \ll j_c w$. When the thin superconductor has a different shape, e.g., looks like a square platelet, there are regions where the ac magnetic field is parallel to the sheet current. In this case, the analysis of the problem becomes more complicated. Nevertheless, the obtained results explain the properties of the vortex-shaking effect observed in the experiments [1–3] and also shed light on the origin of similar intriguing effects [13–15] discovered earlier for slightly different geometries.

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