Phase Glass is a Bose Metal: A New Conducting State in Two Dimensions

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In the quantum rotor model with random exchange interactions having a nonzero mean, three phases, a (i) phase (Bose) glass, (ii) superfluid, and (iii) Mott insulator, meet at a bicritical point. We demonstrate that proximity to the bicritical point and the coupling between the energy landscape and the dissipative degrees of freedom of the phase glass lead to a metallic state at $T = 0$. Consequently, the phase glass is unique in that it represents a concrete example of a metallic state that is mediated by disorder, even in 2D. We propose that the experimentally observed metallic phase which intervenes between the insulator and the superconductor in a wide range of thin films is in actuality a phase glass.

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There is now a preponderance of experimental evidence [1–4] that the disorder or magnetic-field-induced destruction of superconductivity in a wide range of thin metal alloy films leads first to a metallic state with a nonzero conductivity as $T \to 0$. At sufficiently large values of the disorder or magnetic field, a transition to a true insulating state obtains. Within the standard bosonic description [5,6] of the insulator-superconductor transition (IST), the onset of an intervening metallic state is problematic because only two options are thought to exist for bosons: (i) localized in a Mott insulating state or (ii) condensed in a superfluid. In the former, the conductivity vanishes, whereas the latter exhibits resistanceless transport. Further, including degrees of freedom which lie outside the bosonic or phase-only models, for example, electronic excitations, is of no help as electrons are localized in 2D. Indeed, while the onset of the insulator in homogeneously disordered thin films is consistent [7–9] with the emergence of electronic excitations, the intervening metallic and the subsequent superconducting states appear to be inherently bosonic in origin.

Consequently, recent theoretical efforts [10–12] on the origin of the metallic state have focused strictly on bosonic models. Along these lines, we have shown [11] that the standard Mott insulating phase in a clean array of Josephson junctions has a nonzero conductivity as $T \to 0$. This result arises from the noncommutativity [13] of the frequency and temperature tending to zero limits of the conductivity in the vicinity of a quantum critical point, with $\omega = 0$, $T \to 0$ being the experimentally relevant limit for the dc conductivity. In the Mott insulator, quasiparticle excitations are gapped and obey a Boltzmann distribution. However, the collision time of such quasiparticles grows exponentially with the gap [11]. Since the conductivity is a product of the collision time and the quasiparticle density, the conductivity is necessarily finite in the limit $\omega < T$. This type of Bose metal is fragile [11], however, and suppressed by dissipation and disorder. In fact, in the presence of disorder, the nature of the superfluid-insulator transition changes dramatically. For example, several authors [6,14] have argued that, in the presence of on-site disorder, destruction of the superfluid may (in the presence of incommensuration) obtain through an intervening phase with gapless excitations referred to as a Bose glass. In analogy with the Fermi glass, Fisher *et al.* [6] proposed that the Bose glass is an insulator with variable range-hopping conductivity.

In this Letter we show explicitly that the glass phase which may interrupt the direct transition from a superfluid to a Mott insulator in the generally disordered case is in fact a metal that has a well-defined $T \to 0$ limit for the conductivity. Since the thermal average of the superconducting order parameter is nonzero but vanishes once averaged over disorder, we refer to the glass as a phase glass. We propose that the intervening metallic phase seen in the experiments is a phase glass. Our proof that such a phase possesses a nonzero conductivity as $T \to 0$ constitutes the first demonstration of a stable metallic state in 2D in the presence of disorder.

The starting point for our analysis is the charging model for an array of superconducting islands,

$$H = -E_C \sum_i \left(\frac{\partial}{\partial \theta_i}\right)^2 - \sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j), \quad (1)$$

with random Josephson couplings J_{ij} but fixed on-site energies E_C. The phase of each island is θ_i. Note that additional on-site disorder of the form $i v_j \partial/\partial\theta_j$ results in the equivalent particle-hole symmetric field theory provided that the distribution of on-site energies has zero mean. The nonzero mean case is irrelevant here as this corresponds to a density-driven IST [15]. Hence, our conclusions apply to the general disordered case. To incorporate ordered phases, we assume that the Josephson energies are random and characterized by a Gaussian distribution, $P(J_{ij}) = 1/\sqrt{2\pi J^2} \exp -(J_{ij} - J_0)^2/2J^2$, with nonzero mean, J_0. The negative Josephson couplings included in this distribution are essential to the physics of a disordered superconductor [14], particularly glassy ordering. We have studied the nonzero mean problem extensively [10] and established

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explicitly the existence of a bicritical point in which three phases meet, a Mott insulator, phase glass, and superconductor. To distinguish between the phases, it is expedient to introduce the set of variables $\mathbf{S}_i = (\cos\theta_i, \sin\theta_i)$ which allows us to recast the interaction term in the random Josephson Hamiltonian as a spin problem with random magnetic interactions, $\sum_{\langle i,j\rangle} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$. Let $\langle\cdots\rangle$ and $[\cdots]$ represent averages over the thermal degrees of freedom and over the disorder, respectively. In the superconductor not only $\langle S_{i\nu}\rangle$ but also $[\langle S_{i\nu}\rangle]$ acquire a nonzero value. In the phase (or spin) glass, however, $\langle S_{i\nu}\rangle \neq 0$ but $[\langle S_{i\nu}\rangle] = 0$, whereas in the paramagnet or Mott insulator $\langle S_{i\nu}\rangle = 0$.

The Landau theory for this problem is easily obtained [10] using replicas to average over the disorder and the identity $\ln[Z] = \lim_{n\to 0}([Z^n] - 1)/n$ to obtain the zero replica limit. The quartic and quadratic spin-spin interaction terms that arise from the disorder average can be decoupled by introducing the auxiliary real fields,

$$Q_{\mu\nu}^{ab}(\mathbf{k},\mathbf{k}',\tau,\tau') = \langle S_\mu^a(\mathbf{k},\tau) S_\nu^b(\mathbf{k}',\tau')\rangle \qquad (2)$$

and $\Psi_\mu^a(\mathbf{k},\tau) = \langle S_\mu^a(\mathbf{k},\tau)\rangle$, respectively. The superscripts represent the replica indices. A nonzero value of $\Psi_\mu^a(\mathbf{k},\tau)$ implies phase ordering in a charge $2e$ condensate. Hence, Ψ_μ^a couples to the charge degrees of freedom. For quantum spin glasses (SGs), it is the diagonal elements of the Q matrix $D(\tau - \tau') = \lim_{n\to 0}\frac{1}{Mn}\langle Q_{\mu\mu}^{aa}(\mathbf{k},\mathbf{k}',\tau,\tau')\rangle$ in the limit that $|\tau - \tau'| \to \infty$ that serve as the effective Edwards-Anderson spin-glass order parameter [16–18] within Landau theory. The free energy per replica,

$$\mathcal{F}[\Psi, Q] = \mathcal{F}_{\rm SG}(Q) + \sum_{a,\mu,k,\omega_n}(k^2 + \omega_n^2 + m^2)|\Psi_\mu^a(\mathbf{k},\omega_n)|^2 - \frac{1}{\kappa t}\int d^d x \int d\tau_1 d\tau_2 \sum_{a,b,\mu,\nu}\Psi_\mu^a(x,\tau_1)\Psi_\nu^b(x,\tau_2)Q_{\mu\nu}^{ab}(x,\tau_1,\tau_2) + U\int d\tau \sum_{a,\mu}[\Psi_\mu^a(x,\tau)\Psi_\mu^a(x,\tau)]^2, \qquad (3)$$

consists of a spin-glass part which is a third-order functional of the Q matrices discussed previously [10,16], the Ψ_μ^a terms that describe the charge $2e$ condensate, and the term which couples the charge and glassy degrees of freedom. The parameters, κ, t, and U are the standard coupling constants in a Landau theory and m^2 is the inverse correlation length. In the disordered phases, $\langle\Psi_\mu^a\rangle = 0$; hence, in the glassy phase, it is the fluctuations of the Ψ_μ^a field that survive. Our previous analysis [10] shows that the cross term we have retained here is the most dominant of the possible coupling terms near the bicritical point.

Our goal now is to calculate the charge transport in the glassy phase. Near the spin-glass/superconductor boundary, m^2 should be regarded as the smallest parameter. Hence, it is the fluctuations of Ψ_μ^a rather than those of Q^{ab} that dominate. Consequently, we adopt the most general mean-field ansatz [10,16] for the Q matrices

$$Q_{\mu\nu}^{ab}(\mathbf{k},\omega_1,\omega_2) = \beta(2\pi)^d\delta^d(k)\delta_{\mu\nu}[D(\omega_1)\delta_{\omega_1+\omega_2,0}\delta_{ab} + \beta\delta_{\omega_1,0}\delta_{\omega_2,0}q^{ab}]. \qquad (4)$$

The diagonal elements of the Q matrices describe the excitation spectrum. In the glassy phase, the spectrum is ungapped and given by $D(\omega) = -|\omega|/\kappa$. The linear dependence on $|\omega|$ arises because the correlation function $Q_{\mu\mu}^{aa}(\tau)$ decays as τ^{-2} [16,17]. This dependence results in a fundamental change in the dynamical critical exponent from $z = 1$ to $z = 2$ and the onset of overdamped dynamics, thereby eliminating the $\delta(\omega)$ term from the conductivity. In addition, without loss of generality, we work in the replica symmetric case, $q^{ab} = q$ for all a and b, since it was shown [10,16] that replica symmetry breaking vanishes as $T \to 0$ and our emphasis is on the low-temperature limit. Finally, because our focus is charge transport and the electromagnetic gauge couples only to the Ψ_μ^a field, we retain only those terms in the free energy in which at least one of the Ψ_μ^a fields is present. Substituting the Q-matrix ansatz [Eq. (4)] into Eq. (3) and introducing a one-component complex field $\psi^a = (\Psi_1^a, \Psi_2^a)$, we arrive at the following Gaussian theory:

$$\mathcal{F}_{\rm Gauss} = \sum_{a,\mathbf{k},\omega_n}(k^2 + \omega_n^2 + \eta|\omega_n| + m^2)|\psi^a(\mathbf{k},\omega_n)|^2 - \beta q \sum_{a,b,\mathbf{k},\omega_n}\delta_{\omega_n,0}\psi^a(\mathbf{k},\omega_n)[\psi^b(\mathbf{k},\omega_n)]^*, \qquad (5)$$

In the above action we introduced the effective dissipation $\eta = 1/(\kappa^2 t)$ and rescaled $q \to q\kappa t$. The associated Gaussian propagator is

$$G_{ab}^{(0)}(\mathbf{k},\omega_n) = G_0(\mathbf{k},\omega_n)\delta_{ab} + \beta G_0^2(\mathbf{k},\omega_n)q\delta_{\omega_n,0} \qquad (6)$$

in the $n \to 0$ limit [19] with $G_0(\mathbf{k},\omega_n) = (k^2 + \omega_n^2 + \eta|\omega| + m^2)^{-1}$. The first term in Eq. (6) is the standard Gaussian propagator in the presence of Ohmic dissipation. The Ohmic dissipative term in the free energy arises from the diagonal elements of the Q matrices. However, it is the q-dependent term in the Gaussian free energy, the last term in Eq. (5), that is new and changes fundamentally the form of the propagator. Because of the $\delta_{\omega_n,0}$ factor in the second term in the free energy, the propagator now contains a frequency-independent part, $\beta G_0^2(\mathbf{k},\omega_n = 0)q$. In the free energy, this term couples different components of the replicas and hence cannot be regrouped with the mass term, m^2. In fact, this term is a highly relevant perturbation in all dimensions. From simple tree-level scaling, $\psi = b^{(d+z+2)/2}\psi'$, $k = k'/b$, $\omega = \omega'/b^z$, we find that the term proportional to q in the free energy rescales as $q' = qb^{2+z}$. The dynamical exponent z is determined by the fact that the scaling dimension of η should remain unchanged. This gives at the tree level $z = 2$, as proposed previously [6] for the superfluid/Bose glass transition. Hence, $q' = qb^4$, implying that the coupling to the energy landscape of the phase glass is strongly relevant and ultimately responsible for the metallic phase.

To see how this comes about, we use the generalization [20] of the Kubo formula for the replicated action and write the conductivity to one-loop order per replica in the Gaussian approximation as

$$\sigma(i\omega_n) = \frac{2(e^*)^2}{n\hbar\omega_n} T \sum_{a,b,\omega_m} \int \frac{d^2k}{(2\pi)^2} [G_{ab}^{(0)}(\mathbf{k},\omega_m)\delta_{ab} - 2k_x^2 G_{ab}^{(0)}(\mathbf{k},\omega_m) G_{ab}^{(0)}(\mathbf{k},\omega_m + \omega_n)]. \tag{7}$$

The conductivity contains two types of terms. All terms not proportional to q have been evaluated previously [21] and vanish as $T \to 0$. The terms proportional to q^2 vanish in the limit $n \to 0$. The only terms remaining are proportional to q and yield, after an appropriate integration by parts,

$$\sigma(i\omega_n) = \frac{8qe^{*2}}{\hbar\omega_n} \int \frac{d^2k}{(2\pi)^2} k_x^2 G_0^2(\mathbf{k},0) \times [G_0(\mathbf{k},0) - G_0(\mathbf{k},\omega_n)].$$

The momentum integrations are straightforward and yield

$$\sigma(\omega = 0, T \to 0) = \frac{8e^2}{\hbar} \frac{q\eta}{2m^4}, \tag{8}$$

a temperature-independent value for the conductivity as $T \to 0$. The dependence on q and η implies that dissipation alone is insufficient to generate a metallic state. What seems to be the case is that a bosonic excitation moving in a dissipative environment in which many false minima exist does not localize because it takes an exponentially long amount of time to find the ground state. This is the physical mechanism that defeats localization in a glassy phase. Further, the conductivity scales as $1/m^4$ and hence diverges as the superconducting phase is approached. This is precisely what is seen experimentally [1–4].

That the conductivity plateaus in the phase glass regime does not appear to have been anticipated previously. We now appeal to much more general arguments to prove that the singular dependence of the conductivity on m^2 as $T \to 0$ survives even in the presence of the quartic interaction. At the tree level, a dynamical exponent of $z = 2$ renders the quartic interaction U marginally irrelevant. However, by considering the last term in Eq. (5) on equal footing with U in the one-loop renormalization group (RG) scheme, we reach the conclusion that the RG equations flow to strong coupling. The relevance of q at all dimensions manifests itself also by the increasing singularity of relevant contributions from higher-order diagrams in the perturbation series in U. We consider first the linear U correction. At this level, the self-energy is given by a standard tadpole diagram that arises from the couplings in the average $\langle\psi^a\psi^{*b}\sum_c \psi^c\psi^{*c}\psi^c\psi^{*c}\rangle$, yielding $\Sigma = U\int_\omega\int_{\mathbf{k}} G_{aa}^{(0)}(\mathbf{k},\omega_n)$. This diagram [22] is regularizable only once the term ω_n^2 is retained in the propagator. The first term in Eq. (6) leads at $T = 0$ to a standard mass renormalization and innocuous logarithmic corrections, while the last term is more singular, giving $\Sigma^{(1)} = Uq/(4\pi m^2)$. A similar analysis can be undertaken to find the first-order correction to the $T = 0$ conductivity. The relevant diagrams [23] can be readily generalized for a two-replica propagator given by Eq. (6). The straightforward evaluation of the contribution that does not vanish in the $T \to 0$ limit yields

$$\sigma^{(1)}(\omega = 0) = \frac{3e^2}{4h} \frac{U\eta q^2}{\pi m^6}, \tag{9}$$

suggesting that each subsequent order in the interaction leads to a more singular contribution to the self-energy. This points to a scaling function of the form $\sigma \approx (e^2/\hbar)(\eta q/m^4)\Phi(q/m^2)$, where $\Phi(y) \sim y^p$ for large y, which yields the critical behavior $\sigma \sim m^{-x}$ with $x = 4 + 2p$. The value of the exponent p cannot inferred at any finite order in perturbation theory.

Nonetheless, we assume that all of the most singular diagrams can be resummed. A simple inspection of the perturbation series suggests that the fully renormalized propagator,

$$G_{ab}(\mathbf{k},\omega_n) = \tilde{G}(\mathbf{k},\omega_n)\delta_{ab} + \beta q g(\mathbf{k})\delta_{\omega_n,0}, \tag{10}$$

can be broken into replica diagonal and off-diagonal pieces. Likewise, we define the self-energy associated with this propagator to be $\Sigma_{ab}(\mathbf{k},\omega_n) = \tilde{\Sigma}(\mathbf{k},\omega_n)\delta_{ab} + \beta q\theta(\mathbf{k})\delta_{\omega_n,0}$ which contains formally all interaction terms. From the Dyson equation $G_{ab} = G_{ab}^{(0)} + G_{ac}^{(0)}\Sigma_{cd}G_{db}$, in which the summation over the repeated indices is implied, we obtain $\tilde{G}(\mathbf{k},\omega_n) = [G_0^{-1}(\mathbf{k},\omega_n) - \tilde{\Sigma}(\mathbf{k},\omega_n)]^{-1}$ and $g(\mathbf{k}) = [1 + \theta(\mathbf{k})]/[G_0^{-1}(\mathbf{k},0) - \tilde{\Sigma}(\mathbf{k},0)]^2$. The renormalization of the interaction U leads to the appearance of the corresponding vertex function [19] $\Gamma(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k};\Omega_1,\Omega_2,\omega_n)$ which is connected to the self-energy by means of the standard Dyson equation. This vertex function enters the general expression for the conductivity represented diagrammatically in Fig. 1. Each solid line represents the renormalized propagator, G_{ab}, while the shaded region denotes the vertex function, $\Gamma(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k} = 0;\Omega_1,\Omega_2,\omega_n)$. We are interested here only in the static $T = 0$ conductivity. Once the first term in this diagrammatic expansion is integrated by parts, use of the standard Ward identity leads immediately to a cancellation of all diagrammatic contributions to σ in which the external frequency vanishes. As a consequence, we obtain the leading contribution to the conductivity in the limit that $\omega = 0$, $T \to 0$ simply from a Taylor expansion around $\omega = 0$. Using Eq. (10) for the renormalized Green function, we obtain the exact expression,

$$\sigma = \frac{32\pi e^2}{h} q \Big\{ 2 \int \frac{d^2k}{(2\pi)^2} k_x^2 g(\mathbf{k}) \Big(- \frac{\partial \tilde{G}(\mathbf{k}, |\Omega|)}{\partial |\Omega|} \Big|_{\Omega=0} \Big) - q \int \frac{d^2k_1 d^2k_2}{(2\pi)^4} k_{1x} k_{2x} g(\mathbf{k}_1) g(\mathbf{k}_2) \tilde{G}(\mathbf{k}_1, 0) \times \Big[2 \frac{\partial \tilde{G}(\mathbf{k}_2, |\Omega|)}{\partial |\Omega|} \Big|_{\Omega=0} \tilde{\Gamma}(\mathbf{k}_1, \mathbf{k}_2, 0) + \tilde{G}(\mathbf{k}_2, 0) \frac{\partial \tilde{\Gamma}}{\partial |\Omega|} (\mathbf{k}_1, \mathbf{k}_2, |\Omega|) \|_{\Omega=0} \Big] \Big\}, \tag{11}$$

for the temperature-independent part of the conductivity. Here $\tilde{\Gamma}(\mathbf{k}_1, \mathbf{k}_2, |\omega_n|) = \Gamma(\mathbf{k}_1, \mathbf{k}_2, 0; -\omega_n, \omega_n, \omega_n) + \Gamma(\mathbf{k}_1, \mathbf{k}_2, 0; 0, 0, \omega_n)$, and it is taken into account that the frequency dependence of all functions enters through $|\omega_n|$ due to the full particle-hole symmetry. In deriving Eq. (11), we assumed that (i) the infinite perturbation series in U is resummable in principle and (ii) that all propagators and the vertex function are analytical in $|\omega_n|$. The latter assumption seems reasonable, because the most singular contributions come from diagrams that do not contain frequencies at all.

We have demonstrated here that the sluggish phase dynamics in a phase glass leads ultimately to a metallic state in $d = 2$ for bosonic excitations. The strong divergence of the resultant conductivity on m^2 is consistent with the experiments that have observed a distinct plateauing of the resistivity at low temperatures which increases in magnitude [1] as the distance from the true superconducting phase is increased. The metallic $T = 0$ behavior obtains as a result of the coupling between the dissipative environment and the energy landscape of the phase glass. Further, since the dissipation inherent in a phase glass is independent of temperature, external dissipation arising from phonons is irrelevant as such coupling vanishes as $T \to 0$. Consequently, the metallic phase we have found here is robust to disorder and phonon scattering and in fact constitutes the first explicit demonstration of a metallic state in $d = 2$. Nonetheless, the theory presented here does not address the issue of whether or not the destruction of the superconducting phase occurs directly to a conventional insulator or a glassy phase. However, we suggest that only in the second scenario is the destruction of a 2D superconductor in the absence of a magnetic field consistent with the robustness of a metallic phase with respect to increasing disorder. We propose that aging and noise measurements as well as experiments sensitive to trapped flux should be performed in the intervening metallic regime to explore the glassy scenario suggested here. Clearly a promising extension of this work would be the fermionic case.

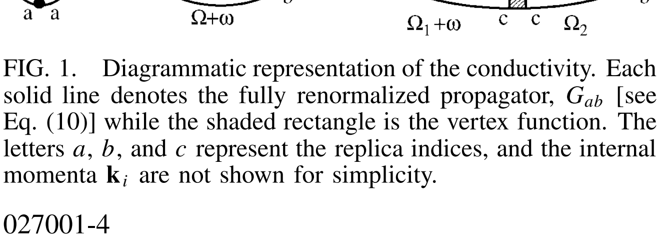

FIG. 1. Diagrammatic representation of the conductivity. Each solid line denotes the fully renormalized propagator, G_{ab} [see Eq. (10)] while the shaded rectangle is the vertex function. The letters a, b, and c represent the replica indices, and the internal momenta $\mathbf{k}_i$ are not shown for simplicity.

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