Rectification and Phase Locking for Particles on Symmetric Two-Dimensional Periodic Substrates

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We demonstrate a rectification phenomenon for overdamped particles interacting with a 2D *symmetric* periodic substrate when driven with a dc and a circular ac drive. As a function of longitudinal dc amplitude, the longitudinal velocity increases in a series of quantized steps distinct from Shapiro steps with transverse rectification occurring near these transitions. The rectification phenomenon is explained using symmetry arguments and a simple model.

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There has been great interest in dissipative nonequilibrium systems capable of producing a ratchet effect. For a particle moving in an asymmetric potential, a net dc drift in one direction can occur in the absence of a dc drive when an applied ac drive is combined with ac flashing of the potential. This ratchet or rectification phenomenon has been studied extensively in the context of biological motors [1], particle segregation [1,2], atom transport in optical lattices [3], and fluxon motion in superconductors or SQUIDs [4,5]. In these effectively one-dimensional (1D) systems the asymmetric potential is the key ingredient leading to rectification. In 2D systems, there are many new ways to introduce an asymmetry which are not accessible in 1D, such as the choice of a clockwise ac drive. There has been considerable work on separating different particle species through rectification in the flow of biomolecules driven with time-varying fields through 2D arrays of obstacles [6]. Particles transported through symmetrical periodic potentials under an ac drive exhibit a wide variety of nonlinear behavior, including phase locking that occurs when the external ac frequency matches the frequency of motion over the periodic potential. This produces the well-known Shapiro steps observed in the $V(I)$ curves of Josephson-junction (JJ) arrays [7]. In all these phase locking systems the ac and dc drives are in the *same* direction. Almost nothing is known about what type of phase locking can occur when the ac and dc drives are *not* in the same direction.

In this work we study rectification and phase locking for an overdamped particle moving in a 2D *symmetrical* periodic substrate under an applied longitudinal dc drive ($\mathbf{f}_{\rm dc}$) and two ac drives: $\mathbf{f}_{\rm ac}^x$ in the longitudinal direction 90° out of phase from $\mathbf{f}_{\rm ac}^y$ in the transverse direction. We find that the longitudinal velocity increases in a series of steps as $\mathbf{f}_{\rm dc}$ increases. Near the transition between two steps we find a rectification of the particle motion in the *transverse* direction. The steps correspond to drives at which the particle motion forms orbits commensurate with the substrate period. The rectification, which can be understood on symmetry grounds, is predominantly in one direction; however, we also observe a number of *reversals* of the rectification as well. Our results should be directly applicable to vortices in superconductors with periodic pinning, as well as colloids and biomolecules interacting with 2D arrays of obstacles.

As a model for vortices in superconductors or colloids in solution, we consider an overdamped particle moving in 2D interacting with a repulsive periodic substrate according to the equation of motion: $\mathbf{f}_i = \mathbf{f}_s + \mathbf{f}_{\rm dc} + \mathbf{f}_{\rm ac} = \eta \mathbf{v}_i$, with $\eta = 1$. The force from the substrate, a square array of side a, is $\mathbf{f}_s = -\nabla U(r)$; the form of $U(r)$ is discussed below. We consider a system of size $8a \times 8a$. The dc drive $\mathbf{f}_{\rm dc}$ is applied along the symmetry axis of the pinning array, in the longitudinal (x) direction. The ac drive is $\mathbf{f}_{\rm ac} = A\sin(\omega_A t)\hat{\mathbf{x}} + B\cos(\omega_B t)\hat{\mathbf{y}}$. Note that there is *no* dc driving component in the y or transverse direction. We fix $w_A/w_B = 1.0$ and $A = B$ and examine both the longitudinal time averaged particle velocity $\langle V_x \rangle$ and the transverse velocity $\langle V_y \rangle$. $\mathbf{f}_{\rm dc}$ is increased from 0 to 1.0 in increments of 0.0001, with 3×10^5 time steps spent at each drive to ensure a steady state.

To model specific systems, we consider periodic substrate potentials created by pinned particles, such as vortices in a periodic array of holes [8] or magnetic dots [9]. Once all the holes are filled with a vortex, any vortices in the interstitial regions between holes experience a smooth periodic substrate created by the interactions with the pinned vortices. For $\mathbf{f}_{\rm dc} = 0$, two crossed ac drives with small amplitudes A cause the vortex to move in a circle in the interstitial regions. At increasing A, there are stable vortex orbits which encircle one pinned vortex, then four, nine, and so on. We focus on A large enough to generate orbits encircling one or more pinned vortices. For the pinned particles, we use the potential for vortices in a thin film superconductor, $U(r) = -\ln(r)$, and employ a summation technique [10] for the long range interaction. We have also considered potentials for unscreened or screened charges with interaction of $U(r) = 1/r$ and $U(r) = e^{-\kappa r}/r$, respectively.

In Fig. 1(a) we show the longitudinal velocity $V_x/(a\omega)$ versus $\mathbf{f}_{\rm dc}$ for a particle with $A = B = 0.36$. For these parameters, at $\mathbf{f}_{\rm dc} = 0$ the particle encircles one pin for

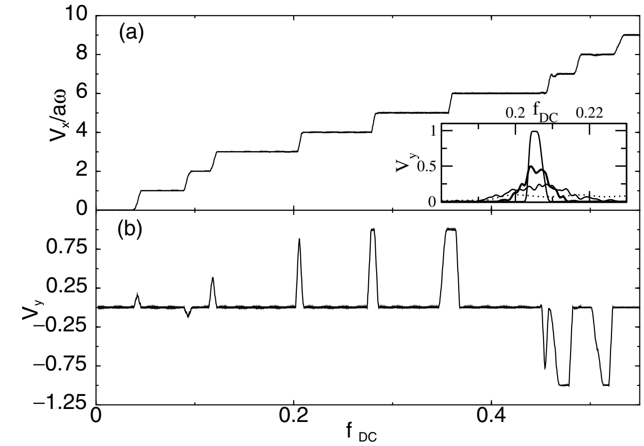

FIG. 1. (a) Longitudinal velocity $V_x/a\omega$ vs driving force $\mathbf{f}_{dc}$ which is applied in the x direction. (b) Corresponding transverse velocity $V_y/a\omega$ vs $\mathbf{f}_{dc}$. Inset to (a): simulated V_y for the system shown in the main figure at temperatures $T = 0.005$ (top), $T = 0.01$, $T = 0.03$, and $T = 0.05$ (bottom).

$0.29 < A < 0.4$. As shown, V_x increases in quantized steps with step heights of $\Delta V_x = a\omega$. We label a step n according to the value of V_x on the step, $V_x = na\omega$. The widths of the integer steps vary with A and ω in an oscillatory manner. There are also some fractional steps with heights $\Delta V_x = (p/q)a\omega$ where p and q are integers such that $p/q < 1.0$.

In Fig. 1(b) we show the transverse velocity $V_y/(a\omega)$ versus $\mathbf{f}_{dc}$. Since there is no net dc force in the transverse direction, $V_y = 0$ for most values of $\mathbf{f}_{dc}$. Near the steps in V_x, however, V_y is nonzero, indicating that rectification is occurring. The first rectification, in the positive y direction, occurs at the $n = 0 \rightarrow 1$ step, while at the $n = 1 \rightarrow 2$ step the rectification is in the *negative* y direction. For the higher steps, positive rectification regions with increasing widths appear, while near the $n = 7 \rightarrow 8$ step there are regions of negative rectification. Some of the rectification phases have very well defined heights of $\Delta V_y = \omega a$. These include the positive rectification phases at $3 \rightarrow 4$, $4 \rightarrow 5$, and $5 \rightarrow 6$, as well as the negative rectification phases at $6 \rightarrow 7$ and $7 \rightarrow 8$. On these rectification plateaus, we find that the particle moves in only one type of orbit. The other rectification phases do not have a well defined height, including $0 \rightarrow 1$, $2 \rightarrow 3$, as well as some portions of the $6 \rightarrow 7$ and $7 \rightarrow 8$ regions. In these phases, for any fixed $\mathbf{f}_{dc}$ the particle jumps intermittently between different rectifying orbits with transverse velocities $(p/q)a\omega$.

For all of the rectifying regions, if the polarity of the ac drive is reversed, V_x remains unchanged while V_y changes to $-V_y$. If the waiting time between dc drive increments is increased, the results are unchanged. The phases described here remain stable when the system is started from a fixed $\mathbf{f}_{dc}$ value and are not transient phenomena. As the system size varies, certain orbits become incommensurate with the system length and precess spatially; however, the velocity curves are not affected by the system size.

In Fig. 2 we illustrate representative nonrectifying particle orbits for V_x steps of $n = 0$, 1, 2, 4, 5, and 8. For $n = 0$ [Fig. 2(a)], $V_x = 0$, and the particle moves in a confined square orbit which encircles one pin. For $n = 1$ [Fig. 2(b)] there is a net motion in V_x, and the particle circles around one pin before moving over to the next plaquette. For $n = 2$ [Fig. 2(c)] the orbit does not encircle a pin but forms a small loop which repeats every second plaquette. For $n = 4$ and $n = 5$ [Figs. 2(d) and 2(e)], orbits similar to $n = 2$ occur, with the loop motion now repeating at every fourth or fifth plaquette, respectively. A similar process continues up through the $n = 7$ step. For the $n = 8$ step [Fig. 2(f)] and above, the particle is moving fast enough that the transverse width of the orbit is less than a, and no loops appear.

Figure 3 shows representative rectifying orbits. The particle orbits differ below and above a given step, as illustrated in Figs. 3(a) and 3(b) for the $n = 3 \rightarrow 4$ step. Below the step [Fig. 3(a)], the particle moves $3a$ in the x direction and a in the positive y direction in a single period. A loop forms when the particle moves in the y direction. Above the step [Fig. 3(b)], the particle moves $4a$ in the x direction but still only a in the y direction in one period; therefore, V_y does not change at the V_x jump. In Fig. 3(c), below the $n = 4 \rightarrow 5$ step, an orbit similar to that of Fig. 3(a) appears. The rectifying orbit above the $4 \rightarrow 5$ step resembles that of Fig. 3(b). In Fig. 3(d) ($\mathbf{f}_{dc} = 0.454$) we show the negative rectification phase below the $n = 7 \rightarrow 8$ step. Here the particle jumps a in the negative y direction every seventh plaquette through a small kink. We do not observe loops in the trajectories for the negatively rectifying phases. Figure 3(e) ($\mathbf{f}_{dc} = 0.465$) illustrates trajectories for a particle in the negative rectification region close to the $n = 7 \rightarrow 8$ step. The particle does not move in a specific orbit but

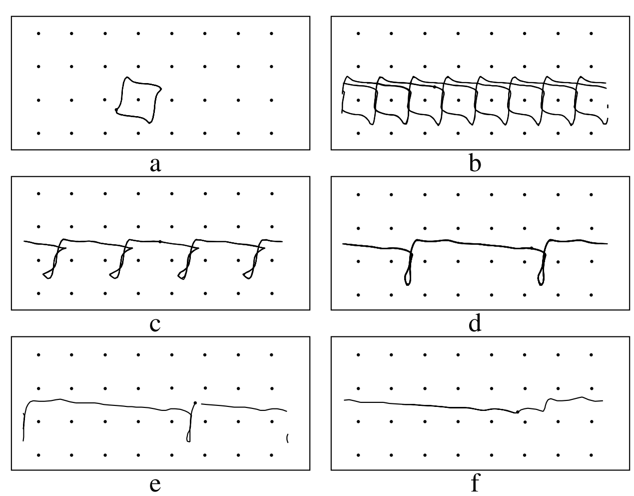

FIG. 2. Trajectories for fixed $\mathbf{f}_{dc}$ for different nonrectifying regions seen in Fig. 1. The black dots denote the location of the fixed particles or the potential maxima of the periodic substrate. Shown are steps with (a) $n = 0$, (b) $n = 1$, (c) $n = 2$, (d) $n = 4$, (e) $n = 5$, and (f) $n = 8$.

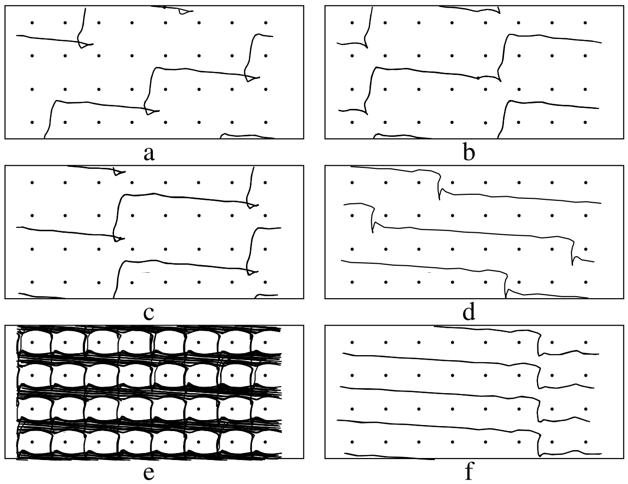

FIG. 3. Trajectories for fixed $\mathbf{f}_{dc}$ for different rectifying regions seen in Fig. 1. The black dots denote the location of the fixed particles or potential maxima of the periodic substrate. (a) Right before $n = 3 \rightarrow 4$ step, (b) right after $n = 3 \rightarrow 4$ step, (c) right before $n = 4 \rightarrow 5$ step, (d) negative rectification phase below the $n = 6 \rightarrow 7$ step ($\mathbf{f}_{dc} = 0.454$), (e) negative rectification region on the $n = 7$ step ($\mathbf{f}_{dc} = 0.465$), and (f) negative rectification region on $n = 8$ step ($\mathbf{f}_{dc} = 0.514$).

jumps intermittently over time between different orbits with $V_y = (p/q)a\omega$; however, V_x remains fixed. Similar intermittent trajectory patterns appear in the rectifying phases near $n = 0 \rightarrow 1$, $1 \rightarrow 2$, and $2 \rightarrow 3$. Intermittent patterns occur only on the lower steps; above the $n = 8$ step, only stable rectifying trajectories occur, as illustrated in Fig. 3(f) ($\mathbf{f}_{dc} = 0.514$).

Rectifying phases occur for any A large enough that the particle trajectory at $\mathbf{f}_{dc} = 0$ encircles more than one pin. We have measured $V_y/a\omega$ for the same system shown in Fig. 1, but with an orbit at $A = 0.42$ that encircles four pins. As in Fig. 1, steps in V_x and transverse rectification in V_y appear, but the step heights are now $\Delta V_x = 2a\omega$. We find that as A is further increased, orbits that stably encircle p^2 pins, with p integer, produce steps of height $\Delta V_x = pa\omega$. We obtain very similar results for substrates with a $1/r$ or $e^{-\kappa r}/r$ interaction.

We next consider the effects of finite temperature. We add a noise term f^T to the equation of motion with the property $\langle f^T(t)\rangle = 0.0$ and $\langle f^T(t)f^T(t')\rangle = 2\eta k_B T\delta(t - t')$. In the inset of Fig. 1(a) we plot V_y for $0.18 < f_{dc} < 0.22$ at increasing T. For low T there are still regions where V_y is near zero within our resolution. The value of V_y should not be zero but exponentially small for low temperatures. For higher T the maximum V_y decreases, the width of the V_y peaks are smeared, and the regions where $V_y = 0$ are lost. The transverse rectification still occurs well into the high T regime where the particle is diffusing about rapidly. We note that the $T = 0$ approximation should still be a good approximation for vortex motion in thin film superconductors for $T/T_c < 0.9$. We have also found that the effects of temperature can be reduced for stronger interactions between the particle and the substrate, which are increased for smaller a.

We now consider a more formal argument employing nonlinear maps to understand the nature of the longitudinal phase locking and transverse rectification. Define a map $(x, y) \rightarrow (x' + n_x a, y' + n_y a)$, from the position of the particle at the start of a period to that at the end, where we may restrict to $0 \leq x, y, x', y' \leq a$, with n_x, n_y integer. If there is a stable fixed point, $(x, y) = (x', y')$, then the particle translates by $(n_x a, n_y a)$ in time ω^{-1} and so has average velocity V_x, V_y quantized in multiples of $a\omega$, as found above. If the qth power of the map has a stable fixed point, there are instead steps of fractional heights $(p/q)a\omega$.

As $\mathbf{f}_{dc}$ increases, the periodic orbit becomes unstable, and a different periodic orbit with larger V_x appears. This new orbit will be the next stable periodic orbit at higher drive. The transition to the new orbit can occur in one of three ways. (1) If both periodic orbits are stable simultaneously, the particle velocity will depend on the initial conditions in the transition regime. This was not observed. (2) The second periodic orbit could become stable at the same time that the first orbit becomes unstable. This behavior, which gives rise to infinitely sharp jumps in V_x, is not generic and hence not expected. (3) There can be a finite range of drive containing no stable periodic orbits. Over this range, the average velocity is not quantized. If, however, some orbits are close to stable, the particle will spend long times in these orbits, giving rise to intermittent behavior. This behavior is consistent with what we observe.

The rectification can be understood on symmetry grounds. The dc drive breaks R_y, the reflection symmetry across the y axis, but preserves R_x, reflection across the x axis. The ac drive breaks both R_x and R_y but preserves the combined symmetry $R_x R_y$. The combined drives break all such symmetries, leading to rectification.

We now turn to a specific toy model illustrating some of these ideas. Consider a particle in a lattice of repulsive sites with $a = 1$, where the potential minima between repulsive sites are at integer x and y values. The y position of the particle is constrained to take only integer values, but the x position can be any real value. To model the translation of the particle through the lattice, the particle moves first (i) right, then (ii) down, then (iii) left, then (iv) up. (i) We apply the rule $x \rightarrow x + v_r$. (ii) If x is within 0.25 of an integer, x is set to that integer and y is decremented by one. (iii) Apply $x \rightarrow x - v_l$. (iv) As in (ii) except y is incremented by one. Here v_r and v_l are the velocity of the particle in the rightward and leftward parts of the cycle, respectively. In steps (ii) and (iv), the particle will move to a new y position only if it reaches the minima between sites at the correct phase of the driving period, when transverse motion is possible. In this case, the particle slips into the next row and the x coordinate of the particle is set to midway between the pinning sites. In Fig. 4 we show the time averaged velocities V_x and V_y

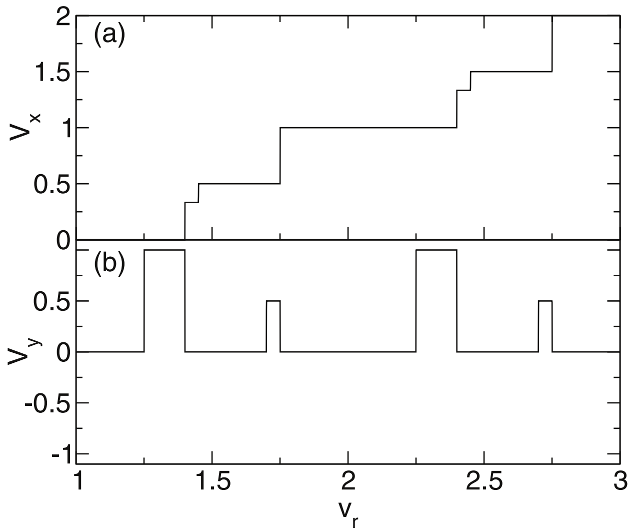

FIG. 4. (a) Time averaged longitudinal velocity V_x obtained from model. (b) Corresponding transverse velocity V_y.

obtained with this model for fixed $v_l = 1.15$ and increasing v_r, representing increasing $\mathbf{f}_{\rm dc}$. This simple model produces both plateaus and ratcheting behavior. Much of this behavior is specific to two or more dimensions. Consider a map $x \rightarrow x'$, subject to $x + a \rightarrow x' + a$ and $dx'/dx \geq 0$, true for overdamped motion in one dimension. It can be shown that it is not possible to have stable periodic orbits with different values of the current: Shapiro steps cannot exhibit jumps.

The ratcheting behavior occurs near transitions in V_x when the number of pinning centers the particle passes in one period changes, making it possible for the particle to interact asymmetrically with the pinning sites. For a clockwise orbit, the particle moves rapidly on the upper portion of the orbit and is likely to scatter off the pinning site below when the orbit does not quite match na. On the lower part of the orbit, however, the particle is moving more slowly and is likely to slip between the pinning sites above in spite of a small mismatch, tending to ratchet in the positive y direction.

The phases we have described should be experimentally observable for vortex motion in superconductors with a dc and crossed ac applied currents with periodic pinning arrays where vortices are located in the interstitial regions, as well as fluxon motion in 2D JJ arrays at rational filling fractions where vortex-vortex interactions are reduced, and in biomolecules moving through 2D arrays of posts [6]. In these active ratchets the rectified velocities can be controlled by $n\omega a$, unlike thermal ratchets which rely on Brownian motion. The longitudinal velocity steps and transverse rectification may also be observed in electrons undergoing classical cyclotron orbits in antidot arrays [11] for orbits where electrons encircle at least one antidot with an additional external dc drive.

In conclusion, we find that a novel form of rectification can occur for an overdamped particle driven by a dc and a circular ac drive in a system without an asymmetric potential. The longitudinal velocity increases in a series of steps of height $n\omega a$. Along these steps the particle moves in commensurate orbits. Near the steps, rectification in the transverse direction occurs. We have specifically demonstrated this model for vortices in superconductors with periodic pinning and overdamped charged particles such as colloids. Using symmetry arguments we explain the origin of the rectification phenomenon. In addition using nonlinear maps we show that the phase locking phenomenon is distinct from Shapiro steps found for 1D systems. With a simple toy model we have shown that the qualitative features of the phase locking and rectification can be captured indicating that the results are not specific to a particular system. Our results can be relevant to vortices in superconductors with periodic pinning arrays, colloids, and biomolecules moving through arrays, and classical electron motion in antidot arrays.

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[1] M. O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993); R. D. Astumian, Science **276**, 917 (1997); F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997); J. Rousselt *et al.,* Nature (London) **370**, 446 (1994).
[2] R. Bartussek *et al.,* Europhys. Lett. **28**, 459 (1994); P. Jung *et al.,* Phys. Rev. Lett. **76**, 3436 (1996); J. L. Mateos, Phys. Rev. Lett. **84**, 258 (2000).
[3] C. Mennerat-Robilliard *et al.,* Phys. Rev. Lett. **82**, 851 (1999).
[4] C. S. Lee *et al.,* Nature (London) **400**, 337 (1999); J. F. Wambaugh *et al.,* Phys. Rev. Lett. **83**, 5106 (1999); C. J. Olson *et al., ibid.* **87**, 177002 (2001).
[5] I. Zapata *et al.,* Phys. Rev. Lett. **77**, 2292 (1996); G. Carapella and G. Costabile *ibid.* **87**, 077002 (2001); S. Weiss *et al.,* Europhys. Lett. **51**, 499 (2000).
[6] W. D. Volkmuth and R. H. Austin, Nature (London) **358**, 600 (1992); T. A. J. Duke and R. H. Austin, Phys. Rev. Lett. **80**, 1552 (1998); D. Ertas *ibid.* **80**, 1548 (1998); J.-L. Viovy, Rev. Mod. Phys. **72**, 813 (2000); C.-F. Chou *et al.,* Electrophoresis **21**, 81 (2000).
[7] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982); S. P. Benz *et al.,* Phys. Rev. Lett. **64**, 693 (1990).
[8] A. T. Fiory, A. F. Hebard, and S. Somekh, Appl. Phys. Lett. **32**, 73 (1978); M. Baert *et al.,* Phys. Rev. Lett. **74**, 3269 (1995); K. Harada *et al.,* Science **274**, 1167 (1996).
[9] J. I. Martín *et al.,* Phys. Rev. Lett. **83**, 1022 (1999); M. J. Van Bael *et al., ibid.* **86**, 155 (2001).
[10] N. Grønbech-Jensen, Int. J. Mod. Phys. C **7**, 873 (1996).
[11] D. Weiss *et al.,* Phys. Rev. Lett. **66**, 2790 (1991); R. Fleischmann *et al., ibid.* **68**, 1367 (1992).