Forward Quark Jets from Protons Shattering the Color Glass Condensate

Adrian Dumitru and Jamal Jalilian-Marian
Physics Department, Brookhaven National Lab, Upton, New York 11973
(Received 4 April 2002; published 24 June 2002)

We consider the single-inclusive minijet cross section in pA at forward rapidity within the color glass condensate model of high energy collisions. We show that the nucleus appears black to the incident quarks except for very large impact parameters. A markedly flatter p_t distribution as compared to QCD in the dilute perturbative limit is predicted for transverse momenta about the saturation scale, which could be as large as $Q_s^2 \simeq 10$ GeV^2 for a gold nucleus boosted to rapidity ~ 10 (as at the BNL-RHIC).

DOI: 10.1103/PhysRevLett.89.022301 PACS numbers: 24.85.+p, 12.38.–t, 13.85.–t

QCD correctly predicted logarithmic violations of Bjorken scaling in deep-inelastic electron-proton scattering at very large Q^2, i.e., at very short distances [1]. Asymptotic freedom provides the theoretical basis for the successful applications of perturbative QCD to hard scattering, short distance phenomena. However, the region of QCD phase space where the field strengths are strong is largely unexplored. This is where one expects that cross sections become comparable to geometric sizes of hadrons and nuclei (the "black limit") and where the unitarity limit is reached. A perturbative QCD based mechanism for unitarization of cross sections is provided by gluon saturation effects [2,3]. A semiclassical approach to gluon saturation and QCD in the high energy limit (small x) was developed in [4–7] and applied to high energy heavy ion collisions at RHIC [8–11].

Large nuclei provide an ideal environment to study gluon saturation and unitarization effects since the gluon density per unit transverse area is larger by a factor of $A^{1/3}$ due to the Lorentz contraction of the nucleus. The scale associated with the high gluon density, the saturation scale Q_s, grows with energy and A and decreases with increasing impact parameter. At a resolution less than Q_s^2, the color field carries large occupation numbers, of order of the inverse QCD coupling constant $1/\alpha_s$. Thus, the gluons in the nuclear wave function at $Q^2 < Q_s^2$ condense. The local color charge density in the transverse plane is a stochastic variable which eventually has to be averaged over (see below). Also, the large x (hard) gluons evolve slowly and so appear frozen to the low x (soft) gluons. Therefore, the high gluon density state of high energy QCD at $Q^2 \lesssim Q_s^2$ is called a "color glass condensate" [7].

The Relativistic Heavy-Ion Collider (RHIC) at BNL will soon allow experimental study of proton-gold or deuteron-gold collisions at a center-of-mass energy of $\sqrt{s} \sim 200–300$ GeV. We suggest that the saturation regime of QCD can be probed at RHIC by measuring the inclusive cross section in p + Au (or d + Au) collisions (in this regard, see also [11,12]). In particular, in the forward region, i.e., close to the rapidity of the proton beam, the saturation scale Q_s can become quite large due to renormalization group evolution in rapidity [6,7]. Thus, we predict significant modifications of the p_t distribution of produced pions relative to leading twist perturbation theory at transverse momenta as large as several GeV. A modification of the *longitudinal* distribution of leading hadrons produced by electrons scattering inelastically from a black target has been predicted previously [13]. Here, we focus on the transverse distribution in the forward region from $p + A$ scattering, which will be analyzed experimentally in the near future at RHIC.

At large rapidity, we consider the quark-nucleus elastic and total scattering cross sections. (In turn, towards mid-rapidity gluon production becomes the dominant contribution to the cross section in the color glass condensate model [11].) We argue that the total quark-nucleus scattering cross section may be related to the single inclusive hadron (jet) cross section in proton-nucleus collisions by using the collinear factorization theorem on the proton side. Let p^μ (q^μ) be the momentum of the incoming (outgoing) quark. We assume the quark is moving along the left branch of the light cone such that $p^- \gg p^+ = p_t^2/2p^-$. The starting point is the scattering amplitude (for brevity, we do not write polarization indices explicitly)

$$\langle q(q)_{\rm out} \,|\, q(p)_{\rm in}\rangle = \langle {\rm out}|b_{\rm out}(q)b^\dagger_{\rm in}(p)\,|{\rm in}\rangle, \qquad (1)$$

which, using the LSZ formalism [14] can be written as

$$\langle {\rm out}|b_{\rm out}(q)b^\dagger_{\rm in}(p)\,|{\rm in}\rangle = -\frac{1}{Z_2}\int d^4x\, d^4y\, e^{-i(px-qy)}\bar{u}(q)[i\overrightarrow{\partial_y} - m]\langle {\rm out}|T\psi(y)\bar{\psi}(x)\,|{\rm in}\rangle[-i\overleftarrow{\partial_x} - m]u(p), \qquad (2)$$

where m is the quark mass and Z_2 is the fermion wave function renormalization factor. $u(p)$ is the quark spinor with momentum p. The fermion propagator G_F in the background of the classical color field is

$$\langle {\rm out}|T\psi(y)\bar{\psi}(x)\,|{\rm in}\rangle \equiv -i\langle {\rm out}\,|\,{\rm in}\rangle G_F(y,x). \qquad (3)$$

The amplitude then becomes

$$\langle q(q)_{\rm out}\,|\,q(p)_{\rm in}\rangle = \frac{i}{Z_2}\langle {\rm out}\,|\,{\rm in}\rangle \int d^4x\, d^4y\, e^{-i(px-qy)}\bar{u}(q)[i\overrightarrow{\partial_y} - m]G_F(y,x)[-i\overleftarrow{\partial_x} - m]u(p). \quad (4)$$

In momentum space, the fermion propagator G_F can be written as [4,15,16]

$$G_F(q,p) = (2\pi)^4\delta^4(q-p)G_F^0(p) - igG_F^0(q)\int \frac{d^4k}{(2\pi)^4}\not{A}(k)G_F(q+k,p), \quad (5)$$

where $\not{A} = A^\mu\gamma_\mu$ is the classical background color field, and G_F^0 is the free fermion propagator. It is useful to define the interaction part of the fermion propagator from (5) as

$$G_F(q,p) = (2\pi)^4\delta^4(q-p)G_F^0(p) + G_F^0(q)\tau(q,p)G_F^0(p). \quad (6)$$

Substituting (6) into the amplitude (4) leads to

$$\langle q(q)_{\rm out}\,|\,q(p)_{\rm in}\rangle = \bar{u}(q)\tau(q,p)u(p). \quad (7)$$

Here, we have set $Z_2 = 1$ and $\langle {\rm out}\,|\,{\rm in}\rangle = 1$ since we are working to leading order in α_s and our background field is time independent. This is a very simple relation between the amplitude for scattering of a quark or antiquark from the color glass condensate and the quark propagator in the background color field of the nucleus.

The explicit form of the quark propagator in the background of a classical color field was calculated in [4,15–17]. The interaction part, as defined in (6) is given by

$$\tau(q,p) = (2\pi)\delta(p^- - q^-)\gamma^-\int d^2z_t \times [V(z_t) - 1]e^{i(q_t-p_t)z_t}, \quad (8)$$

where

$$V(z_t) \equiv \hat{P}\exp\left[-ig^2\int_{-\infty}^{+\infty} dz^-\frac{1}{\partial_t^2}\rho_a(z^-,z_t)t_a\right], \quad (9)$$

and t_a are in the fundamental representation. Using (8) in the scattering amplitude (7) gives

$$\langle q(q)_{\rm out}\,|\,q(p)_{\rm in}\rangle = (2\pi)\delta(p^- - q^-)\bar{u}(q) \times \gamma^-u(p)\int d^2z_t[V(z_t) - 1] \times e^{i(q_t-p_t)z_t}. \quad (10)$$

The presence of the delta function in the amplitude is due to the target being (light cone) time independent which leads to conservation of the "minus" component of momenta. It can be factored out in the standard fashion,

$$\langle q(q)_{\rm out}\,|\,q(p)_{\rm in}\rangle = (2\pi)\delta(p^- - q^-)M(p,q), \quad (11)$$

which gives the cross section

$$d\sigma = \int\frac{d^4q}{(2\pi)^4}(2\pi)\delta(2q^+q^- - q_t^2)\theta(q^+) \times \frac{1}{2p^-}(2\pi)\delta(p^- - q^-)\,|M(p,q)|^2. \quad (12)$$

The local density of color charge in the nucleus, $\rho_a(x_t,x^-)$, is a stochastic variable which has to be averaged over. Commonly, one assumes a distribution of the charge sources which is local and Gaussian [4,11,16,18,19], such that the average of any operator O is

$$\langle O\rangle = \int \mathcal{D}\rho\, O[\rho]\exp(-{\rm tr}\rho^2/\mu^2). \quad (13)$$

$x^-\mu^2(z_t,x^-)$ denotes the gluon density per unit transverse area d^2z_t, and per unit of rapidity, dx^-/x^-, in the nucleus. When computing the total cross section, we will have to square the amplitude (10) before averaging over the color charge density ρ of the classical background field. On the other hand, for the case of elastic scattering, we first average the amplitude (10) over ρ and then square it [18,20]. In that way no color exchange occurs over a large distance in rapidity (from the projectile quark to the nucleus).

The averages of $V(z_t)$ and $V^\dagger(z_t)V(\bar{z}_t)$ are given by [16,19]

$$\langle V(z_t)\rangle_\rho = \exp\left[-\frac{g^4(N_c^2-1)}{4N_c}\chi\int d^2y_t\, G_0^2(z_t - y_t)\right], \quad (14)$$

and

$$\langle V^\dagger(z_t)V(\bar{z}_t)\rangle_\rho = \exp\left[-\frac{g^4(N_c^2-1)}{4N_c}\chi\int d^2y_t \times [G_0(z_t-y_t) - G_0(\bar{z}_t - y_t)]^2\right]. \quad (15)$$

We have defined $\chi(x^-) \equiv \int_{x^-}^{x_A^-} dz^-\,\mu^2(z^-)$, which is the density of color charge in the nucleus per unit transverse area *integrated* over longitudinal phase space (rapidity). $x_A^- \ll x_0^-$ is the coordinate of the nucleus, with $y_A = \log x_0^-/x_A^-$ its rapidity; while $x^- \ll x_0^-$ is the coordinate of the quark projectile, and $y = \log x^-/x_0^-$ is its rapidity. (x_0^- is a reference point, which we choose to be midrapidity.) $G_0(z_t - y_t)$ is the free propagator of static gluons

$$G_0(z_t - y_t) = -\int_{\Lambda_{\rm QCD}^2}\frac{d^2k_t}{(2\pi)^2}\frac{e^{ik_t(z_t-y_t)}}{k_t^2}. \quad (16)$$

The trace of the quark spinors in the squared amplitude is

$$\frac{1}{2}\sum_{\rm spins}|\bar{u}(q)\gamma^-u(p)|^2 = 4p^-q^-. \quad (17)$$

Averaging over the colors of the incoming quark is made trivial by the fact that (14) and (15) are diagonal in color space. For elastic scattering, we shall use (14) to color average the amplitude given by (10), and afterwards square it. The color averaging of the amplitude leads to a delta function of transverse momenta $(2\pi)^2\delta^2(p_t - q_t)$. This is due to the assumed translational invariance of the target in transverse space (we have assumed a large cylindrical target nucleus so that the color charge density is uniform) and should be understood as $\pi R_A^2 (2\pi)^2 \delta^2(p_t - q_t)$ in the squared amplitude. Using (10)–(12) finally leads to

$$\frac{d\sigma_{qA}^{el}}{d^2b} = [1 - e^{-\pi^2 Q_s^2/N_c \Lambda_{\rm QCD}^2}]^2, \qquad (18)$$

where we have introduced the saturation scale of the target, $Q_s^2 \equiv (N_c^2 - 1)\alpha_s^2 \chi/\pi$ (this is the same definition as in [10]).

To compute the total cross section for scattering of the quark on the nucleus, we first square the amplitude (10) and then average over the background field using (14) and (15). It leads to

$$\frac{d\sigma_{qA}^{\rm tot}}{d^2b} = \int \frac{d^2q_t}{(2\pi)^2} \int d^2r_t\, e^{-iq_t r_t} \times [e^{-(2\pi Q_s^2/N_c)\int d^2k_t/k_t^4 [1-\exp(ik_t r_t)]} - 2e^{-\pi^2 Q_s^2/N_c \Lambda_{\rm QCD}^2} + 1]. \qquad (19)$$

The integral over q_t just gives $\delta^2(r_t)$ which in turn can be used to perform the r_t integration. The final result for the total cross section is

$$\frac{d\sigma_{qA}^{\rm tot}}{d^2b} = 2[1 - e^{-\pi^2 Q_s^2/N_c \Lambda_{\rm QCD}^2}]. \qquad (20)$$

From Eqs. (18) and (20), we see that in the high energy limit ($Q_s \to \infty$) we have

$$\sigma_{qA}^{\rm tot} = 2\sigma_{qA}^{el} = 2\pi R_A^2 \qquad (21)$$

as a consequence of unitarity.

It is more interesting to consider the *differential* cross section $d\sigma_{qA}^{\rm tot}/d^2b\,dq^- d^2q_t$ when the quark momentum q_t is large ($q_t^2 \gg \Lambda_{\rm QCD}^2$). In this limit, one can convolute the quark-nucleus cross section with the quark distribution function in a proton and the quark fragmentation function into hadrons, thereby relating qA scattering to single inclusive hadron (jet) production in pA collisions [21]. The differential cross section is given by

$$\frac{d\sigma^{qA\to qX}}{dq^- d^2q_t d^2b} = \frac{\delta(p^- - q^-)}{(2\pi)^2} \int d^2r_t\, e^{-iq_t r_t} \times [e^{-(2\pi Q_s^2/N_c)\int d^2k_t/k_t^4[1-\exp(ik_t r_t)]}]. \qquad (22)$$

Note that we have dropped the last two terms in (19) since they do not contribute to the cross section at large q_t.

At very large transverse momentum, $q_t^2 \gg Q_s^2$, we can expand the V's in Eq. (15), keeping only the first nontrivial term. This corresponds to the dilute perturbative limit. Using (16) then gives the expected

$$\frac{d\sigma^{qA\to qX}}{d^2q_t d^2b} \sim \frac{Q_s^2}{q_t^4}. \qquad (23)$$

In the region $q_t^2 \sim Q_s^2 \gg \Lambda_{\rm QCD}^2$, in turn, we must resum higher twists, i.e., rescatterings in the nuclear field. We then obtain from (22)

$$\frac{d\sigma^{qA\to qX}}{d^2q_t d^2b} \sim \frac{1}{q_t^2}. \qquad (24)$$

The nonlinearities of the classical field flatten the differential cross section as compared to the perturbative cross section (23). This arises, in fact, because of the "saturation" of the gluon density in the nucleus at $Q^2 \lesssim Q_s^2$ as obtained here from the classical description of the semihard gluon field with large occupation numbers [4–7]. (The same effect arises for inclusive gluon production [11] and $q\bar{q}$ [16] photoproduction at central rapidity, where however Q_s^2 is much smaller.) The predicted *suppression* of the inclusive cross section should be easy to distinguish from p_t broadening, i.e., "initial-state" interactions of the beam quarks with spectators from the target [22], which *enhance* the cross section at semihigh p_t.

The single inclusive $p + A \to h + X$ cross section can now be obtained in principle by convoluting the total qA cross section with the quark distribution function of the proton at the factorization scale Q_f^2, which can, for example, be chosen to be q_t^2, and the quark-hadron (jet) fragmentation function $D_{q/h}(z, Q_f^2)$, where z is the ratio of hadron and quark momenta.

$$\frac{d\sigma^{pA\to hX}}{dyd^2k_t d^2b} = \int dx \frac{dz}{z^2} q(x, Q_f^2) \times \frac{d\sigma^{qA\to qX}}{dy_q d^2q_t d^2b} D_{q/h}(z, Q_f^2). \qquad (25)$$

Here, x denotes the fractional (light-cone) momentum carried by the quark, such that the light-cone coordinate of the incident proton is related to that of its quark by $x_p^- = xx^-$.

The qA cross sections depend on x through the saturation momentum of the nucleus. From the analysis of HERA DIS data, the saturation momentum scales like $Q_s^2(x^-/x_0^-)/Q_s^2(x_0^-) = (x_0^-/x^-)^\lambda = (x_0^- x/x_p^-)^\lambda$, with $\lambda \simeq 0.3$ [23]. For gold nuclei, it has been estimated that $Q_s^2(x_0^-) \approx 2$ GeV^2 at RHIC energy (100 A GeV gold beam, corresponding to $y_A = 5.4$) [10]. Therefore, near the rapidity of the incident proton, i.e., $y_p = \log x_p^-/x_0^- = -5.4$, we estimate that roughly $Q_s^2 \approx 10$ GeV^2. With $1/\Lambda_{\rm QCD}^2 \simeq 25$ GeV^{-2}, the exponent in Eqs. (18) and (20) is <-800. In fact, even at $b \simeq 6$ fm from the center of a gold nucleus, using a Gaussian density distribution for the nucleus we estimate that $Q_s^2(x_0^-, b \sim 6\ {\rm fm}) \simeq 0.75$ GeV^2, and therefore $Q_s^2(x_p^-, b \sim 6\ {\rm fm}) \simeq 3.8$ GeV$^2 \simeq 100\Lambda_{\rm QCD}^2$. At the edge of the acceptance of the BRAHMS spectrometer at RHIC, $y = \log x^-/x_0^- = -4$, one obtains $Q_s^2(x^-, b \sim 0) \simeq 6.6$ GeV^2. That is, $2\sigma_{qA}^{el} = \sigma_{qA}^{\rm tot} = 2\pi R_A^2$, with R_A the radius of the black disc formed by the nucleus (i.e., the distance from the center of the nucleus where Q_s^2 becomes of order $\Lambda_{\rm QCD}^2$), is quite close to the geometric cross section of the gold nucleus.

The quark distribution functions in the proton, and their fragmentation into pions will modify the transverse momentum dependence of the pion cross section (25) relative to the quark cross section (24). Nevertheless, that modification is the same for both the dilute perturbative estimate of the qA cross section (23) as well as for the resummed qA cross section from (24). Therefore, a modification of the p_t distribution from strong nonlinear color fields as compared to the dilute limit should hold independently of any scale dependence of the quark distribution and fragmentation functions. The big advantage of measurements in the forward region is that these effects extend much farther in p_t than in the central rapidity region, hopefully making it much less ambiguous to observe them experimentally.

We thank F. Gelis, S. Jeon, D. Kharzeev, Y. Kovchegov, A. Kovner, L. McLerran, J. Qiu, and R. Venugopalan for useful discussions. This work is supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886. J. J-M. is also supported in part by a LDRD and a PDF from BSA.

[1] D. J. Gross and F. Wilczek, Phys. Rev. D **8**, 3633 (1973); **9**, 980 (1974); A. H. Mueller, Phys. Rep. **73**, 237 (1981).
[2] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983).
[3] A. H. Mueller and J. Qiu, Nucl. Phys. **B268**, 427 (1986).
[4] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); **49**, 3352 (1994); **59**, 094002 (1999).
[5] Y. V. Kovchegov, Phys. Rev. D **54**, 5463 (1996); **55**, 5445 (1997).
[6] J. Jalilian-Marian, A. Kovner, L. McLerran, and H. Weigert, Phys. Rev. D **55**, 5414 (1997); J. Jalilian-Marian, A. Kovner, A. Leonidov, and H. Weigert, Nucl. Phys. **B504**, 415 (1997); Phys. Rev. D **59**, 014014 (1999); **59**, 034007 (1999) [**59**, 099903(E) (1999)]; J. Jalilian-Marian, A. Kovner, and H. Weigert, *ibid.* **59**, 014015 (1999); A. Ayala, J. Jalilian-Marian, L. D. McLerran, and R. Venugopalan, Phys. Rev. D **52**, 2935 (1995); A. Kovner and J. G. Milhano, Phys. Rev. D **61**, 014012 (2000); A. Kovner, J. G. Milhano, and H. Weigert, Phys. Rev. D **62**, 114005 (2000).
[7] E. Iancu, A. Leonidov, and L. D. McLerran, Nucl. Phys. **A692**, 583 (2001); E. Ferreiro, E. Iancu, A. Leonidov, and L. McLerran, hep-ph/0109115.
[8] A. Kovner, L. McLerran, and H. Weigert, Phys. Rev. D **52**, 3809 (1995);**52**, 6231 (1995).
[9] A. Krasnitz and R. Venugopalan, Phys. Rev. Lett. **86**, 1717 (2001); A. Krasnitz, Y. Nara, and R. Venugopalan, *ibid.* **87**, 192302 (2001).
[10] D. Kharzeev and M. Nardi, Phys. Lett. B **507**, 121 (2001); D. Kharzeev and E. Levin, *ibid.* **523**, 79 (2001).
[11] A. Dumitru and L. McLerran, Nucl. Phys. **A700**, 492 (2002); A. Dumitru and J. Jalilian-Marian, hep-ph/0111357.
[12] Y. V. Kovchegov and A. H. Mueller, Nucl. Phys. **B529**, 451 (1998); A. Accardi and D. Treleani, Phys. Rev. D **64**, 116004 (2001).
[13] L. Frankfurt, V. Guzey, M. McDermott, and M. Strikman, Phys. Rev. Lett. **87**, 192301 (2001).
[14] C. Itzykson and J-B. Zuber, *Quantum Field Theory* (McGraw-Hill, Singapore, 1980).
[15] A. Hebecker and H. Weigert, Phys. Lett. B **432**, 215 (1998).
[16] F. Gelis and A. Peshier, Nucl. Phys. **A697**, 879 (2002); hep-ph/0111227.
[17] I. Balitsky, Nucl. Phys. **B463**, 99 (1996).
[18] Y. V. Kovchegov and L. D. McLerran, Phys. Rev. D **60**, 054025 (1999) [**62**, 019901(E) (1999)]; Y. V. Kovchegov, *ibid.* **60**, 034008 (1999).
[19] A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001).
[20] A. Hebecker, Nucl. Phys. **B505**, 349 (1997); W. Buchmüller, M. F. McDermott, and A. Hebecker, Nucl. Phys. **B487**, 283 (1997) [**500**, 621(E) (1997)]; Phys. Lett. B **410**, 304 (1997).
[21] J. Qiu (private communication).
[22] G. T. Bodwin, S. J. Brodsky, and G. P. Lepage, Phys. Rev. Lett. **47**, 1799 (1981); Phys. Rev. D **39**, 3287 (1989); B. Z. Kopeliovich, J. Nemchik, A. Schäfer, and A. V. Tarasov, hep-ph/0201010.
[23] K. Golec-Biernat and M. Wüsthoff, Phys. Rev. D **59**, 014017 (1999); **60**, 114023 (1999).