

Nonlinear ac Resistivity in s -Wave and d -Wave Disordered Granular Superconductors

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We model s -wave and d -wave disordered granular superconductors with a three-dimensional lattice of randomly distributed Josephson junctions. The nonlinear ac resistivity ρ_2 of these systems was calculated using Langevin dynamical equations. The current amplitude dependence of ρ_2 at the peak position is found to be a power law characterized by exponent α , which is not universal but depends on the self-inductance and current regimes. In the weak current regime α is independent of the self-inductance and $\alpha = 0.5 \pm 0.1$ for both s - and d -wave materials. In accord with experiments, we find $\alpha \approx 1$ for some interval of inductance in the strong current regime.

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The symmetry of the superconducting pairing function has been of great interest lately. The gap of conventional superconductors has s -wave symmetry whereas there is now good evidence that the superconducting gap of the high- T_c cuprates has d -wave symmetry [1]. Granular superconductors are usually described as a random network of superconducting grains coupled by Josephson links [2,3]. In high- T_c ceramics, depending on the relative orientation of the d -wave superconducting grains, it is possible to have weak links with negative Josephson coupling [4], which are called π junctions. The existence of these π junctions may cause, e.g., the paramagnetic Meissner effect [4] observed at low magnetic fields [5].

Recently, Kawamura [6] proposed that a novel thermodynamic phase may occur in zero external magnetic field in unconventional superconductors. This phase is characterized by a broken time-reversal symmetry and is called the chiral glass phase. The frustration effect due to the random distribution of π junctions leads to a glass state of quenched-in "chiralities," which are local loop supercurrents circulating over grains and carrying a half-quantum of flux [7]. Evidence for the transition to chiral glass has been seen from experimental studies of the nonlinear ac magnetic susceptibility [8], the dynamic scaling [9], and the aging phenomenon [10]. The susceptibility measurements of Ishida *et al.* [11] do not, however, support the existence of the chiral glass.

In order to further probe the existence of the chiral glass phase Yamao *et al.* [12] have measured the ac linear resistivity ρ_0 and the nonlinear resistivity ρ_2 of ceramic superconductor $\text{YBa}_2\text{Cu}_4\text{O}_8$. ρ_2 is defined as the third coefficient of the expansion of the voltage $V(t)$ in terms of the external current $I_{\text{ext}}(t)$:

$$V = \rho_0 I_{\text{ext}} + \rho_2 I_{\text{ext}}^3 + \dots \quad (1)$$

When the sample is driven by an ac current $I_{\text{ext}}(t) = I_0 \sin(\omega t)$, one can relate ρ_2 to third harmonics $V'_{3\omega}$ in the following way:

$$\begin{aligned} \rho_2 &= -4V'_{3\omega}/I_0^3, \\ V'_{3\omega} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} V(t) \sin(3\omega t) d(\omega t). \end{aligned} \quad (2)$$

Yamao *et al.* have made two key observations. First, since the linear resistivity does not vanish at the peak position of ρ_2 they identify the transition as a transition to the chiral glass phase. The second interesting observation is the power law dependence of $|V'_{3\omega}(T_p)/(I_0)^3|$ (or of ρ_2) at its maximum position T_p on I_0 : $|V'_{3\omega}(T_p)/I_0^3| \sim I_0^{-\alpha}$. The experimental value of the power law exponent was $\alpha \approx 1.1$. Using the XY -like model for d -wave superconductors Li and Domínguez [13] were able to reproduce the experimental results of Yamao *et al.* [12] qualitatively. The quantitative agreement was, however, poor and the role of inductance was not explored. Namely, α was computed only for one value of dimensionless inductance $\tilde{L} = 1$ and with large error bars [13]: $\alpha = 1.1 \pm 0.6$.

The goal of this Letter is twofold. First, we calculate α with high accuracy for both s - and d -wave systems using the Langevin equations for the XY -like model with screening. Second, we try to answer the question if it is possible to discriminate between s - and d -pairing symmetry by measurements of α . We show that there are two distinct current regimes for α . In the weak current regime (WCR) (small I_0) this exponent does not depend on the inductance and $\alpha = 0.50 \pm 0.1$ for s - and d -wave ceramics. In the strong current regime (SCR) α depends on the screening. For small \tilde{L} we obtain $\alpha_{d\text{-wave}} > \alpha_{s\text{-wave}}$, possibly because in the weak screening limit the energy landscape of the d -wave case is more rugged than the s -wave case. As the self-inductance increases the number of energy local minima gets smaller [14] and the behavior of the two systems becomes more similar, with the values of α being almost the same. For the d -wave system in the SCR and with $1 < \tilde{L} \leq 5$ we find $\alpha \approx 1.0$ which agrees with the experimental value [12].

We consider the following “coarse grained” Hamiltonian [15–17]:

$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j - A_{ij}) + \frac{1}{2L} \sum_p \Phi_p^2, \quad (3)$$

where θ_i is the phase of the condensate of the grain at the i th site of a simple cubic lattice, J_{ij} denotes the Josephson coupling between the i th and j th grains, L is the self-inductance of a loop (an elementary plaquette), while the mutual inductance between different loops is neglected. The first sum is taken over all nearest-neighbor pairs and the second sum is taken over all elementary plaquettes on the lattice. Fluctuating variables to be summed over are the phase variables, θ_i , at each site and the gauge variables, $A_{ij} = \frac{2\pi}{\phi_0} \int_i^j \vec{A}(\vec{r}) d\vec{r}$, at each link. $\Phi_p = \frac{\phi_0}{2\pi} \sum_{\langle ij \rangle \in p} A_{ij}$ is the total magnetic flux threading through the p th plaquette, and ϕ_0 denotes the flux quantum. The effect of screening currents inside grains is not considered explicitly, since for large length scales they simply lead to a Hamiltonian \mathcal{H} with an effective self-inductance L [17].

For the d -wave superconductors we assume J_{ij} to be an independent random variable taking the values J or $-J$ with equal probability ($\pm J$ or bimodal distribution),

each representing 0 and π junctions. For the s -wave superconductors J_{ij} is always positive but distributed uniformly between 0 and $2J$. It should be noted that model (3) with uniform couplings was first studied by Dasgupta and Halperin [18]. Random π -junction models (in which J_{ij} is allowed to take negative values with certain probability) have also been adequate to explain several phenomena observed in high- T_c superconductors such as the anomalous microwave absorption [14,19], the compensation effect [20], the effect of applied electric fields in the apparent critical current [21], and the aging effect [22].

In order to study transport properties, we use the resistively shunted junction model [3]. Then in addition to the Josephson current one has the contribution of a dissipative Ohmic current due to an intergrain resistance R and the Langevin noise current. We have redefined notation: the site of each grain is at position $\mathbf{n} = (n_x, n_y, n_z)$ (i.e., $i \equiv \mathbf{n}$); the lattice directions are $\boldsymbol{\mu} = \hat{x}, \hat{y}, \hat{z}$; the link variables are between sites \mathbf{n} and $\mathbf{n} + \boldsymbol{\mu}$ (i.e., link $ij \equiv$ link $\mathbf{n}, \boldsymbol{\mu}$); and the plaquettes p are defined by the site \mathbf{n} and the normal direction $\boldsymbol{\mu}$ [i.e., plaquette $p \equiv$ plaquette $\mathbf{n}, \boldsymbol{\mu}$; for example, the plaquette \mathbf{n}, \hat{z} is centered at position $\mathbf{n} + (\hat{x} + \hat{y})/2$]. Then the gauge invariant phase differences $\theta_{\boldsymbol{\mu}}(\mathbf{n}) = \Delta_{\boldsymbol{\mu}}^+ \theta(\mathbf{n}) - A_{\boldsymbol{\mu}}(\mathbf{n})$ obey the following equations [3,15]:

$$\begin{aligned} \frac{\hbar}{2eR} \frac{d\theta_{\boldsymbol{\mu}}(\mathbf{n})}{dt} &= -\frac{2e}{\hbar} J_{\boldsymbol{\mu}}(\mathbf{n}) \sin\theta_{\boldsymbol{\mu}}(\mathbf{n}) - \delta_{\boldsymbol{\mu},y} J_{\text{ext}} - \frac{\hbar}{2eL} \Delta_{\boldsymbol{\nu}}^- [\Delta_{\boldsymbol{\nu}}^+ \theta_{\boldsymbol{\mu}}(\mathbf{n}) - \Delta_{\boldsymbol{\mu}}^+ \theta_{\boldsymbol{\nu}}(\mathbf{n})] - \eta_{\boldsymbol{\mu}}(\mathbf{n}, t), \\ \langle \eta_{\boldsymbol{\mu}}(\mathbf{n}, t) \eta'_{\boldsymbol{\mu}'}(\mathbf{n}', t') \rangle &= \frac{2kT}{R} \delta_{\boldsymbol{\mu}\boldsymbol{\mu}'} \delta_{\mathbf{n}\mathbf{n}'} \delta(t - t'), \end{aligned} \quad (4)$$

where $\eta_{\boldsymbol{\mu}}(\mathbf{n}, t)$ is the Langevin noise current. The forward difference operator is $\Delta_{\boldsymbol{\mu}}^+ \theta_{\boldsymbol{\nu}}(\mathbf{n}) = \theta_{\boldsymbol{\nu}}(\mathbf{n} + \boldsymbol{\mu}) - \theta_{\boldsymbol{\nu}}(\mathbf{n})$ and the backward operator is $\Delta_{\boldsymbol{\mu}}^- \theta_{\boldsymbol{\nu}}(\mathbf{n}) = \theta_{\boldsymbol{\nu}}(\mathbf{n}) - \theta_{\boldsymbol{\nu}}(\mathbf{n} - \boldsymbol{\mu})$. In what follows we consider currents normalized by $I_J = 2eJ/\hbar$, time by $\tau = \phi_0/2\pi JR$, voltages by RI_J , temperature by J/k_B , and inductance by $\phi_0/2\pi J$. Free boundary conditions and numerical integration are implemented in the same way as in [13,15]. Depending on values of I_0 and ω the number of samples used for the disorder averaging ranges between 5 and 800. The number of integration steps is chosen to be $10^5 - 5 \times 10^5$.

The temperature dependence of the *nonlinear* resistivity ρ_2 of the s -wave system for $I_0 = 0.1$ and for different values of ω is shown in the upper panel of Fig. 1. Similar to the d -wave case [13], there is no visible dependence on ω . As seen in the lower panel, as I_0 decreases peak values of ρ_2 tend to diverge. For $\tilde{L} = 1$ the peak is located at $T_p = 1.4$ and it coincides with the metal-superconductor transition at which thermodynamic quantities diverge and the *linear* resistivity ρ_0 vanishes. It should be noted that our disordered s -wave model is different from the gauge glass model [23] (in the latter case the screening spoils the transition to the superconducting state). Figure 2 shows the I_0 dependence of $\max|V'_{3\omega}/I_0^3|$ of the s -wave

samples ($\tilde{L} = 1$). Clearly, we have two distinct regimes for small and large currents. In the WCR ($I_0 \leq 0.1$) $\alpha = 0.50 \pm 0.04$ and $\alpha = 0.51 \pm 0.03$ for $l = 8$ and $l = 12$, respectively. In the second regime we obtain $\alpha = 1.0 \pm 0.05$ and $\alpha = 1.07 \pm 0.02$ for $l = 8$ and $l = 12$, respectively. Since within the error bars the finite system size effect is negligible, we consider only the system size $l = 8$.

Figure 3 shows the dependence of $\max|V'_{3\omega}/I_0^3|$ on I_0 for the d -wave case ($l = 8$ and $\omega = 0.001$). In the weak current part one has $\alpha = 0.51 \pm 0.03, 0.45 \pm 0.05, 0.48 \pm 0.05$, and 0.43 ± 0.06 for $\tilde{L} = 0.1, 1, 10$, and 20 , respectively. Clearly, within error bars α is not sensitive to the screening. In the SCR it becomes dependent on \tilde{L} : $\alpha = 1.8 \pm 0.16, 1.56 \pm 0.17, 0.97 \pm 0.02$, and 0.60 ± 0.02 for $\tilde{L} = 0.1, 1, 10$, and 20 , respectively. Figure 4 shows the results obtained in the SCR for s - and d -wave systems with different values of \tilde{L} . The power law region of the d -wave case is sensitive to the screening and is narrower than its s -wave counterpart.

The dependence of α on \tilde{L} in the SCR is shown in Fig. 5. Such a dependence may be understood taking into account the interplay between the thermal fluctuations and the rugged energy landscape. In the weak screening limit

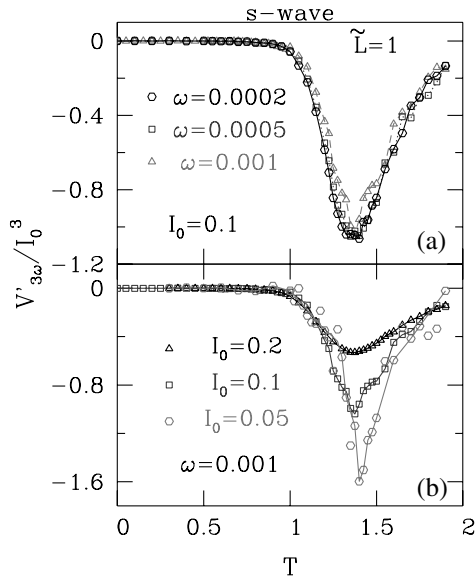


FIG. 1. (a) Upper panel: the temperature dependence of $V'_{3\omega}/I_0^3$ for the s -wave system. System size $l = 8$, $\tilde{L} = 1$, and $I_0 = 0.1$. The open triangles, squares, and hexagons correspond to $\omega = 0.001, 0.0005$, and 0.0002 , respectively. The peak is located at $T_p = 1.4$. (b) Lower panel: the same as in the upper panel but $\omega = 0.001$. The open triangles, squares, and hexagons correspond to $I_0 = 0.2, 0.1$, and 0.05 , respectively. The results are averaged over 15–40 samples.

the latter plays an important role and α of the d -wave system is bigger than that for the s -wave one. As \tilde{L} increases the thermal fluctuations take over and the opposite situation would happen. The pronounced difference between two types of symmetry is seen only in the weak screening region.

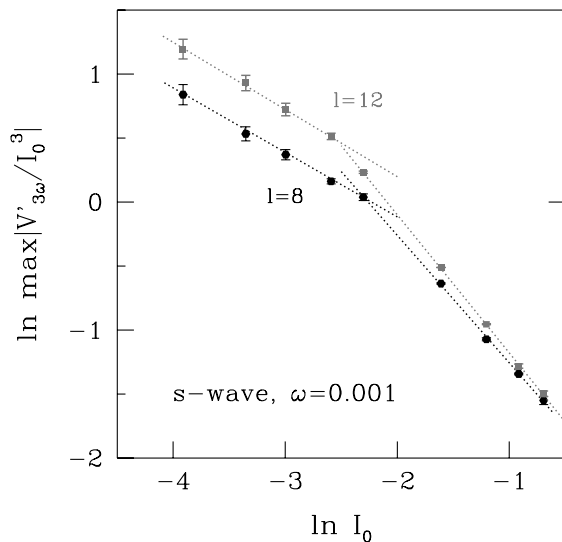


FIG. 2. The current dependence of $\max|V'_{3\omega}/I_0^3|$ for s -wave superconductors. We choose $\omega = 0.001$ and $\tilde{L} = 1$. In the WCR $\alpha = 0.50 \pm 0.04$ and 0.51 ± 0.03 for the system size $l = 8$ and 12 , respectively. In the SCR $\alpha = 1.0 \pm 0.05$ and $\alpha = 1.07 \pm 0.02$ for $l = 8$ and $l = 12$, respectively. The results are averaged over 5–800 samples.

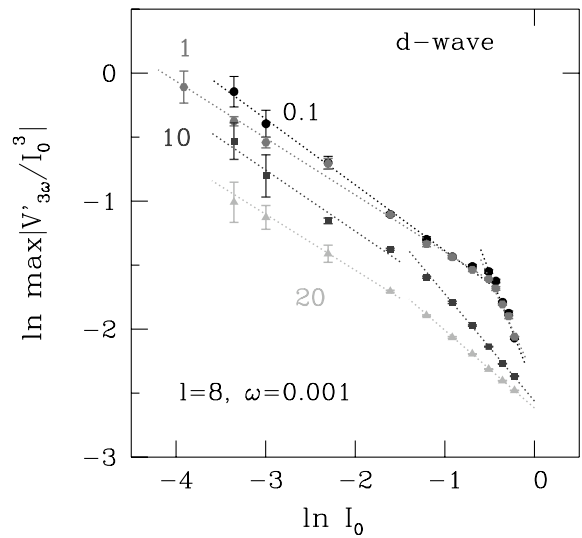


FIG. 3. The current dependence of $\max|V'_{3\omega}/I_0^3|$ for the d -wave system. We choose the system size $l = 8$, $\omega = 0.001$, and $\tilde{L} = 0.1, 1, 10$, and 20 (its values are shown next to the curves). For each inductance one has two distinct current regimes. The results are averaged over 10–800 samples.

It is tempting to interpret the two regimes for α as the WCR corresponding to the critical regime for $\rho_2(T_c, I_0)$ (since $I_0 \rightarrow 0$) and the SCR corresponding to a mean-field regime (away from criticality). If there is a continuous phase transition at a critical temperature $T_c = T_p$, then current-voltage scaling [24] predicts that $V \sim I^{(z+1)/(d-1)}$ at T_c , with z the dynamical exponent. Therefore, the nonlinear resistivity should be $\rho_2(T_c) \sim I_0^{(z+1)/(d-1)-3}$, and thus the expected WCR value is $\alpha = (5 - z)/2$ in $d = 3$.

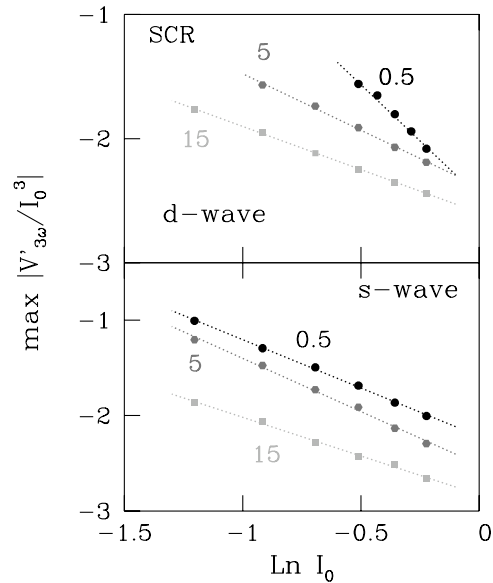


FIG. 4. The current and self-inductance dependence of $\max|V'_{3\omega}/I_0^3|$ for d - and s -wave systems in the SCR for $\tilde{L} = 0.5, 5$, and 15 (they are shown next to the curves). We choose the system size $l = 8$ and $\omega = 0.001$. The results are averaged over 5–10 samples.

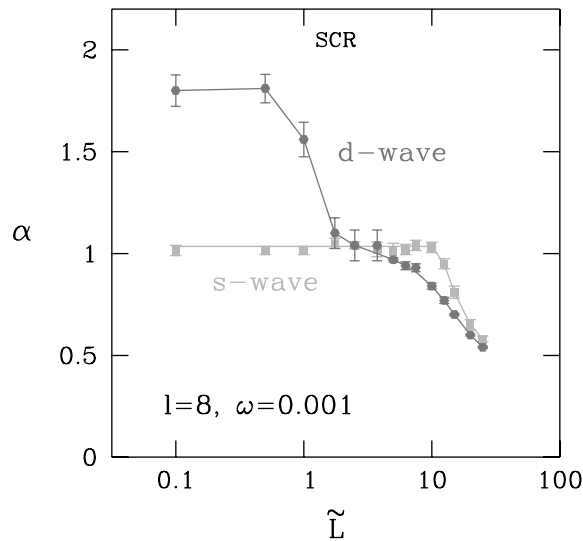


FIG. 5. Dependence of α on \tilde{L} in the SCR for s - and d -wave systems.

This predicts that a peak in $\rho_2(T)$ at T_c is possible if $z < 5$ (i.e., $\alpha > 0$). In our case, we obtain $\alpha \approx 0.5$ and therefore $z \approx 4$ for the disordered s -wave transition.

In the experiment of Ref. [12] the temperature T_p is merely an intergrain ordering transition temperature above which the thermoremanent magnetization disappears. In the previous simulations of [13] for the d -wave system, T_p is the temperature where there is an onset of positive magnetization, i.e., the paramagnetic Meissner effect starts to be observed, but it does not seem to correspond to a phase transition. The chiral glass phase transition temperature T_{cg} is found at a lower temperature, $T_{cg} < T_p$ (for $L = 1$, e.g., $T_{cg} \approx 0.29$ [7]). Kawamura [25] has found that $z \approx 6 > 5$ for the chiral glass transition, and thus no peak in $\rho_2(T)$ is expected for this transition according to the scaling argument. Therefore, the peak measured by Yamao *et al.* may not correspond to the chiral glass transition, but to the crossover we find at T_p for the d -wave case.

In order to compare our results with experiments we first show that Yamao *et al.* [12] performed measurements in the SCR. Since real current is $I = \frac{2eJ}{h}I_0$, $J \sim 10^2$ K and $I_0 \sim 10^{-1}$ we have $I \sim 10^{-2}$ mA. On the other hand, the current used in experiments $I \sim 10$ mA suggests that the experiments were performed in the SCR. As seen from Fig. 5, the value of α in the SCR for $1 < \tilde{L} < 5$ coincides with the experimental value [12]. This interval of inductance is realistic for ceramics [26] because typical values of \tilde{L} are bigger than 3. An accurate comparison between theory and experiments requires, however, the knowledge of \tilde{L} which is not known for the compound of $\text{YBa}_2\text{Cu}_4\text{O}_8$ studied in Ref. [12].

In conclusion, we have calculated the nonlinear ac resistivity exponent α for s and d -wave granular superconductors with high accuracy. Our results reveal two distinct current regimes. In the WCR α is independent of the screening strength and of types of pairing symmetry. In the opposite case this exponent depends on \tilde{L} . A difference between s - and d -wave symmetries in the nonlinear resistivity can be found only in samples with weak screening. The agreement between simulation and experimental results is possible for some interval of \tilde{L} .

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