One-Dimensional Disordered Density Waves and Superfluids: The Role of Quantum Phase Slips and Thermal Fluctuations

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The low temperature phase diagram of 1D disordered quantum systems such as charge or spin density waves, superfluids, and related systems is considered by a full finite-T renormalization group approach for the first time. At zero temperature the consideration of quantum phase slips leads to a new scenario for the unpinning (delocalization) transition. In the strong pinning limit the model is solved exactly. At finite T a rich crossover diagram with various scaling regions is found which reflects the zero temperature quantum critical behavior.

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The collective behavior of condensed modulated structures such as charge or spin density waves (CDWs/SDWs) [1,2], flux line lattices [3,4], and Wigner crystals [2] in random environments has been the subject of detailed investigations since the early 1970s. These are motivated by the drastic influence of disorder: without pinning CDWs would be ideal superconductors, whereas type-II superconductors would show finite resistivity. In three-dimensional systems the low temperature phase of these structures is determined by a zero temperature disorder fixed point, resulting in quasi-long-range order and glassy dynamics [3,4]. In two dimensions this fixed point is extended to a fixed line which terminates at the glass transition temperature [5,6]. In the low temperature phase, correlations decay slightly faster than a power law and the linear resistivity vanishes (for a recent review, see [4]). In one dimension the situation is different: the glass temperature is shifted to $T = 0$. Nevertheless, there remains a residual trace of disorder which is reflected in the low temperature behavior of spatial correlations and in the dynamics [7].

Clearly, at low temperatures also quantum fluctuations have to be taken into account. Disorder and quantum fluctuations in 1D CDWs at zero temperature have been considered previously (see, e.g., [8,9]) and an unpinning transition as a function of the interaction strength was found. Finite temperature effects were partially incorporated by truncating the renormalization group (RG) flow at the de Broglie wavelength of the phason excitations [9]. However, for a complete study of the thermal to quantum crossover, quantum and thermal fluctuations have to be considered on an equal footing [10] which is the first goal of this Letter. The second goal is the consideration of quantum phase slips which trigger their own quantum phase transition to a disordered phase and influence also the unpinning transition. Experimentally, 1D behavior can be seen in real materials, e.g., in whiskers [11], hairlike single crystal fibers, with a transverse extension smaller than the correlation length or in chainlike crystals with weak interchain coupling. In the latter case there is a large crossover length scale up to which 1D behavior can be observed [1,2]. The results obtained for the CDWs/SDWs have a number of further applications on disordered quantum systems: they relate, e.g., to the localization transition of Luttinger liquids [8,9], tunnel junction chains [12], superfluids [13], Josephson coupled chains of these systems if the coupling is treated in mean-field theory [8], and CDWs in a lattice potential. In most parts of this Letter we use the terminology of CDWs.

Well below the mean-field (MF) condensation temperature T_{MF} of the CDW, the electron density $\rho(x)$ can be written in the form [1]

$$\rho(x) = \rho_0(1 + Q^{-1}\partial_x\varphi) + \rho_1 \cos[p(\varphi + Qx)] + \dots, \tag{1}$$

where $Q = 2k_F$, k_F is the Fermi momentum, ρ_0 is the mean electron density, and ρ_1 is proportional to the amplitude of the complex (mean-field) order parameter $\Delta e^{i\varphi} \sim \langle b_Q + b_{-Q}^+\rangle$. b_k^+, b_k denote the phonon creation and annihilation operators, respectively. $\varphi(x)$ is a slowly varying phase variable, and $p = 1$ for CDWs and $p = 2$ for SDWs. Neglecting fluctuations in Δ, the Hamiltonian of the CDW is given by

$$\hat{\mathcal{H}} = \int_0^L \bigg\{ \frac{c}{2}\bigg[\bigg(\frac{v}{c}\bigg)^2 \hat{P}^2 + (\partial_x\hat{\varphi})^2\bigg] + \sum_i U_i \rho(x)\delta(x - x_i) + W\cos\bigg(\frac{q\pi}{\hbar}\int^x dy\,\hat{P}(y)\bigg)\bigg\} dx, \tag{2}$$

where $[\hat{P}(x), \hat{\varphi}(x')] = \frac{\hbar}{i}\delta(x - x')$. $c = \frac{\hbar v_F}{2\pi} f$ denotes the elastic constant, v_F denotes the Fermi velocity, v denotes the effective velocity of the phason excitations, and $f(T)$ denotes the condensate density [1]. Note that $f(T)$ and $\Delta(T)$ vanish at T_{MF}, whereas v remains finite. The third term results from the effects of impurities with random potential strength $U_i = \pm U_{\mathrm{imp}}$ and positions x_i and includes a forward and a backward scattering term proportional to ρ_0 and ρ_1, respectively. We will assume that the mean impurity distance l_{imp} is large compared

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with the wave length of the CDW and that the disorder is weak; i.e., $1 \ll l_{\rm imp}Q \ll c/(U_{\rm imp}\rho_1)$. In this case the Fukuyama-Lee (FL) length $L_{\rm FL} = [c/(Up^2)]^{2/3}$ is large compared to the impurity distance; here $U = U_{\rm imp}\rho_1/\sqrt{l_{\rm imp}}$. The fourth term in (2) describes the influence of quantum phase slips by $\varphi = \pm q\pi$ and is discussed further below [13]. The model (2) includes the four dimensionless parameters $t = T/\pi\Lambda c$, $K = \hbar v/\pi c$, $u^2 = U^2/\Lambda^3\pi c^2$, and $w = W/\pi c\Lambda^2$, which measure the strength of the thermal, quantum, and disorder fluctuations, and the probability of phase slips, respectively. $\Lambda = \pi/a$ is a momentum cutoff with the lattice constant a. Although for CDWs and SDWs K-values of the order 10^{-1} and 1, respectively, have been discussed at $T = 0$ [14], the expressions relating K and t to the microscopic theory lead to the conclusion that both diverge by approaching $T_{\rm MF}$, whereas the ratio K/t remains finite. The classical region of the model is given by $K \ll t$ which can be rewritten as the condition that the thermal de Broglie wavelength $\lambda_T = K/t\Lambda$ of the phason excitations is small compared to a.

In order to determine the phase diagram we adopt a standard Wilson-type renormalization group calculation, which starts from a path integral formulation of the partition function corresponding to the Hamiltonian (2) with $u, w \ll 1$. We begin with the renormalization of the disorder term and put $w = 0$ for the moment. The system is transformed into a translationally invariant problem using the replica trick. Going over to dimensionless spatial and imaginary time variables, $\Lambda x \to x$ and $\Lambda v\tau \to \tau$, the replicated action is given by $[\sigma = (u\rho_0\Lambda/\rho_1 Q)^2]$

$$\frac{S^{(n)}}{\hbar} = \frac{1}{2\pi K}\sum_{\alpha,\beta}\int_0^{L\Lambda} dx \int_0^{K/t} d\tau \Big\{\Big[(\partial_x\varphi_\alpha)^2 + (\partial_\tau\varphi_\alpha)^2\Big]\delta_{\alpha\beta} - \frac{1}{2K}\int_0^{K/t} d\tau' [u^2 \cos p(\varphi_\alpha(x,\tau) - \varphi_\beta(x,\tau')) + \sigma\partial_x\varphi_\alpha(x,\tau)\partial_x\varphi_\beta(x,\tau')]\Big\}. \quad (3)$$

Integrating over the high momentum modes of $\varphi(x,\tau)$ in a momentum shell of infinitesimal width $1/b \le |q| \le 1$ but arbitrary frequencies and rescaling $x \to x' = x/b$, $\tau \to \tau' = \tau/b$, we obtain the following renormalization group flow equations (up to one loop):

$$\frac{dK}{dl} = -\frac{1}{2}p^4u^2KB_0\Big(p^2K, \frac{K}{2t}\Big)\coth\frac{K}{2t}, \quad (4)$$

$$\frac{du^2}{dl} = \Big[3 - \frac{p^2K}{2}\coth\frac{K}{2t}\Big]u^2, \qquad \frac{dt}{dl} = t, \quad (5)$$

$$B_i(\nu, y) = \int_0^y d\tau \times \int_0^\infty \frac{dx g_i(\tau,x)\cosh(y-\tau)(\cosh y)^{-1}}{[1 + (\frac{y}{\pi})^2(\cosh\frac{\pi x}{y} - \cos\frac{\pi\tau}{y})]^{\nu/4}}, \quad (6)$$

where $l = \ln b$ and $g_0(\tau, x) = \delta(x)\tau^2$. Note that $B_0(p^2K, \frac{K}{2t}) \to 0$ for $K \to 0$.

The equation for the flow of σ is more involved and is not discussed here, since it does not feed back into the other flow equations. Indeed, we can get rid of the forward scattering term by rewriting $\hat\varphi(x) = \hat\varphi_b(x) + \varphi_f(x)$ with $\varphi_f(x) = \int_0^x dy\, c(y)$, $\langle c(x)\rangle = 0$, and $\langle c(x)c(x')\rangle = \frac{\pi}{2}\sigma\delta(x - x')$. The phase correlation function $C(x,\tau) = \langle[\varphi(x,\tau) - \varphi(0,0)]^2\rangle = C_b(x,\tau) + C_f(x)$ has therefore always a contribution $C_f(x) \sim |x|/\xi_f$ with $\xi_f^{-1} \sim \sigma(l = \log|x|)$. Since all further remarks about phase correlations refer to $C_b(x,\tau)$ we drop the subscript b.

There is no renormalization of t (i.e., c) because of a statistical tilt symmetry [15]. The special case $t = 0$ was previously considered in [9] (with $p = \sqrt{2}$). The flow equation for K obtained in [9] for $w = t = 0$ deviates slightly from (4), which can be traced back to the different RG procedures. The critical behavior is, however, the same: there is a Kosterlitz-Thouless (KT) transition [16] at K_u between a disorder dominated, pinned and a free, unpinned phase which terminates in the fixed point $K_u^* = 6/p^2$. u_0 denotes the bare value of the disorder and K_u is given by $u^2 = \frac{K_u^*}{p^2\eta}(\frac{K_u - K_u^*}{K_u^*} - \log\frac{K_u}{K_u^*})$ with $\eta = B_0(p^2K_u^*, \infty)$. In the pinned phase the parameters K, u flow into the classical, strong disorder region: $K \to 0, u \to \infty$. The correlation function $C(x,0) \sim |x|/\xi_u$ increases linearly with $|x|$. Integration of the flow equations gives for small initial disorder and $K \ll K_u$ an effective correlation length $\xi_u \approx \Lambda^{-1}(\Lambda L_{\rm FL})^{(1-K/K_u)^{-1}}$ at which u becomes of the order unity. Close to the transition line ξ_u shows KT behavior. For $K \ge K_u$, ξ_u diverges and $C(x,\tau) \sim K(l = \log|z|)\log|z|$ where $|z| = \sqrt{x^2 + \tau^2}$. Note that $K(l)$ saturates on large scales at a value $K_{\rm eff}(u_0)$.

For large values of u our flow equations break down, but we can find the asymptotic behavior in this phase by solving the initial model in its *strong pinning limit* $U_{\rm imp} \gg 1, K = 0$ exactly. A straightforward, but somewhat clumsy, calculation yields for the pair correlation function: $C(x,\tau) = \frac{2\pi}{p\alpha}(1 - \frac{\alpha}{\sinh\alpha})|Qx|$, where $\alpha = \pi/pQl_{\rm imp}$. The connection to the weak pinning model follows by choosing $l_{\rm imp} \approx L_{\rm FL}$.

At *finite temperatures* thermal fluctuations destroy the quantum interference effects which lead to the pinning of the CDW at $t = 0$. In the region $K < K_u$, u first increases and then decreases under the flow. ξ can be found approximately by integrating the flow equations until the maximum of $u(l)$ and $t(l)/[1 + K(l)]$ is of order one. This can be done in full generality only numerically (see Fig. 1). It is however possible to discuss several special cases analytically. The zero temperature correlation

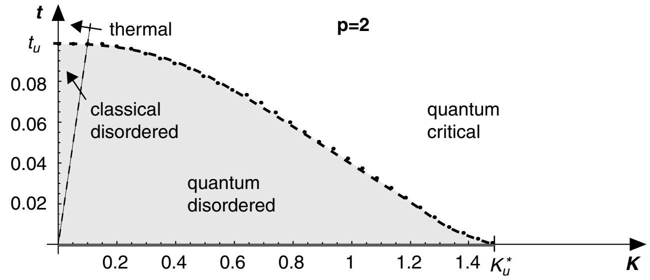

FIG. 1. The low temperature crossover diagram of a one-dimensional CDW. The amount of disorder corresponds to a reduced temperature $t_u \approx 0.1$. In the classical and the quantum disordered region, respectively, essentially the $t = 0$ behavior is seen. The straight line separating them corresponds to $\lambda_T \approx a$. In the quantum critical region the correlation length is given by λ_T. Pinning (localization) occurs only for $t = 0, K < K_u^*$.

length can still be observed as long as it is smaller than λ_T which rewrites for K not too close to K_u^* as $t \lesssim t_K \approx K t_u^{(1-K/K_u)^{-1}}$, $t_u \approx (\Lambda L_{\mathrm{FL}})^{-1}$. We call this domain the *quantum disordered region*. For $K \geq K_u$ the correlation length ξ is given by λ_T which is larger than given by purely thermal fluctuations. For scales smaller than λ_T, $C(x, \tau)$ still increases as $\sim \log|z|$ with a continuously varying coefficient $K_{\mathrm{eff}}(u_0)$. In this sense one observes *quantum critical behavior* in that region, despite the fact that the correlation length is now finite for all values of K [10]. In the *classical disordered region* $t_K < t < t_u$ the correlation length is roughly given by L_{FL} as follows from previous studies [6,7]. In the remaining region $t_u \lesssim t$ we adopt an alternative method by mapping the (classical) one-dimensional problem onto the Burgers equation with noise [17]. In this case the RG procedure applied to this equation becomes trivial since there is only a contribution from a single momentum shell and one finds for the correlation length $\xi^{-1} \approx \frac{\pi}{2} f(T) t [1 + (2\pi/p)^2 (t_u/t)^3]\Lambda$. The phase diagram depicted in Fig. 1 is the result of the numerical integration of our flow equations and shows indeed the various crossovers discussed before.

So far the phase field has been considered to be single valued. Taking into account also amplitude fluctuations of the order parameter, the phase may change by multiples of 2π by orbiting (in space and imaginary time) a zero of the amplitude. Such vortices correspond to *quantum phase slips* described by the last term in (2) (with $q = 2$), which we discuss here under equilibrium conditions. This operator superposes two translations of φ by $\pm q\pi$ left from x; i.e., it changes coherently the phase by $\pm q\pi$ in a macroscopic region. For vanishing disorder the model can be mapped on a sine-Gordon Hamiltonian for the θ field (with K replaced by K^{-1}) by using the canonical transformation $\hat{P} = -\frac{\hbar}{\pi}\partial_x \hat{\theta}$ and $-\frac{\hbar}{\pi}\partial_x \hat{\varphi} = \hat{\Pi}$. To see the connection to space-time vortices one rewrites the action of interacting vortices as a classical 2D Coulomb gas which is subsequently mapped to the sine-Gordon model [18]. The initial value w_0 of w is proportional to the fugacity $w_0 \approx e^{-S_{\mathrm{core}}/\hbar}$ of the space-time vortices which may be not negligible close to T_{MF}, where the action $S_{\mathrm{core}} \approx \hbar/(\pi K)$ of the vortex core is small. Performing a calculation analogous to the one above (but with $u = 0$) the RG flow equations read

$$\frac{dK}{dl} = -\frac{\pi}{2}\frac{q^4 w^2}{K^3} B_2\left(\frac{q^2}{K}, \frac{K}{2t}\right) \coth\frac{K}{2t}, \tag{7}$$

$$\frac{dt}{dl} = \left[1 - \frac{\pi}{2}\frac{q^4 w^2}{K^4} B_1\left(\frac{q^2}{K}, \frac{K}{2t}\right) \coth\frac{K}{2t}\right] t, \tag{8}$$

$$\frac{dw}{dl} = \left[2 - \frac{q^2}{4K}\coth\frac{K}{2t}\right] w, \tag{9}$$

where $B_{1,2}$ are given in (6) with $g_1 = 2\tau^2 \cos x$ and $g_2 = (x^2 + \tau^2)\cos x$. From (7)–(9) we find that for $t = u = 0$ quantum phase slips become relevant (i.e., w grows) for $K > K_w$ with $K_w^* = q^2/8$ ($q = 2$ for CDWs). In this region, vortices destroy the quasi-long-range order of the CDW; $C(x, \tau) \sim |z|/\xi_w$. The transition is of KT type with a correlation length ξ_w $[w(\log \xi_w) \approx 1]$ diverging at $K_w + 0$ [13]. At finite temperatures w first increases but then decreases and flows into the region of large t and small w. Thus quantum phase slips become irrelevant at finite temperatures. This can be understood as follows: at finite t the 1D quantum sine-Gordon model can be mapped on the Coulomb gas on a torus of perimeter K/t since periodic boundary conditions apply now in the τ direction. Whereas the entropy of two opposite charges increases for separation $L \gg K/t$ as $\log(LK/t)$, their action increases linearly with L. Thus the charges remain bound. The one-dimensional Coulomb gas has indeed only an insulating phase [19].

It is now interesting to consider the combined influence of disorder and phase slips. To this end we write an approximate expression for the action of a single vortex in a region of linear extension L as

$$\frac{S_{\mathrm{vortex}} - S_{\mathrm{core}}}{\hbar} = \left(\frac{q^2}{4K} - 2\right)\log L - \frac{u(L)}{K}. \tag{10}$$

For very low $K(<K_u, K_w)$ where $u(L) \approx u_0 L^{3/2}$ the disorder always favors vortices on the scale of the effective Fukuyama-Lee length ξ_u. These vortices will be pinned in space by disorder. On the other hand, for very large values of $K(>K_u, K_w)$ phase vortices are not influenced by disorder since $u(L)$ is renormalized to zero. In the remaining region we have to distinguish the cases $K_u \gtrless K_w$. For $K_w < K < K_u$ (i.e., $qp < 4\sqrt{3}$) and $u_0 = 0$ the phase correlations are lost on the scale of the KT correlation length ξ_w of the vortex unbinding transition. Not too close to this transition $\xi_w \Lambda \approx e^{(S_{\mathrm{core}}/2\hbar)(1-K_v/K)^{-1}}$ holds. Switching on the disorder, u will be renormalized by strong phase fluctuations which lead to an exponential decay of $u \sim u_0 e^{-\mathrm{const}\times L/\xi_w}$ such that disorder is irrelevant for the vortex gas as long as $\xi_w \lesssim \xi_u$. We expect that the relation $\xi_w \approx \xi_u$ determines the position of the phase boundary between a pinned low K phase, where vortices are favored by the disorder, and an unpinned high K phase,

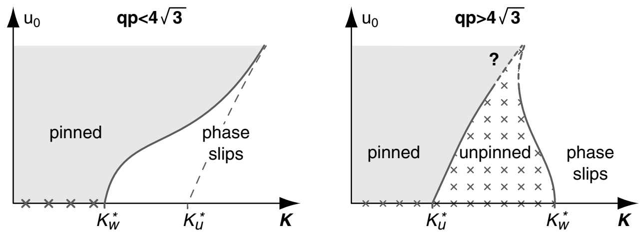

FIG. 2. $T = 0$ phase diagram for a CDW with quantum phase slips. If $qp < 4\sqrt{3}$ there is a single transition between a low K pinned and a high K unpinned phase. In both phases the correlation length is finite. If $qp > 4\sqrt{3}$ these two phases are separated by a third phase in which phase slips are suppressed and $C(x, \tau) \sim \log|z|$. Both transitions disappear at finite t.

where vortices are induced by quantum fluctuations. This line terminates in K_w for $u_0 \to 0$ (see Fig. 2). If $S_{\rm core}$ is large, ξ_w will be large as well and $\xi_w \approx \xi_u$ will be reached only for $K \approx K_u^*$. For moderate values of $S_{\rm core}$, the unpinning transition may be lowered considerably by quantum phase slips. In the opposite case $K_u < K < K_w$ (i.e., $qp > 4\sqrt{3}$) phase fluctuations renormalize weak disorder to zero such that vortices are still suppressed until K reaches K_w where vortex unbinding occurs. In this case two sharp phase transitions have to be expected.

Our flow equations describe also the effect of a *commensurate lattice potential* on the CDW: if the wavelength π/k_F of the CDW modulation is commensurate with the period a of the underlying lattice such that $\pi/k_F = n/(qa)$ with n, q integer, an Umklapp term $w \cos q\varphi$ appears in the Hamiltonian [1]. We obtain the results in this case from (7)–(9) (and the conclusions derived from them) if we use the replacements $K \to K^{-1}, t \to t/K^2$, and $w \to w/K^2$. Thus the lattice potential is relevant for $K < K_w$ with $K_w^* = 8/q^2$.

Next we consider the application of the results obtained so far to a *one-dimensional Bose fluid*. Its density operator is given by Eq. (1) if we identify $Q/\pi = \rho_0 = \rho_1$ ($p = 2$). $\partial_x \varphi$ is conjugate to the phase θ of the Bose field [20]. With the replacements $K \to K^{-1}$, $t \to t/K^2$, and $w = 0$, (3) describes the action of the 1D superfluid in a random potential. v denotes the phase velocity of the sound waves with $v/\pi K = \rho_0/m$ and $\pi v K = \kappa/\pi^2\rho_0^2$, where κ is the compressibility. The transition between the superfluid and the localized phase occurs for $K_u^* = 2/3$ [9]. Thermal fluctuations again suppress the disorder and destroy the superfluid localization transition in 1D. In contrast to CDWs here the θ field may have vortex-like singularities in space-time and the flow equations (7)–(9) apply again. The vortex unbinding transition appears at K_w with $K_w^* = 1/2, (q = 2)$. If both w and u are nonzero we can use the canonical transformation to rewrite the vortex contribution in the form $w \cos(q\varphi)$. For $K < K_u, K_w$ both perturbations are irrelevant and the system is superfluid. For $K_w < K < K_u$ the decay of u is stopped due to the suppression of the φ fluctuations which in turn are due to w. An Imry-Ma argument shows furthermore that the $q\varphi = \pi(n + 1/2)$ state is destroyed on the scale $\xi \approx \xi_w/u^2(\log\xi)$ by arbitrarily weak disorder, i.e., vortices become irrelevant above this scale. On larger scales one can expect that quantum fluctuations wash out the disorder, and the system is still superfluid. Finally, at $K > K_u, K_w$ both perturbations are relevant and superfluidity is destroyed.

To conclude we have shown that in 1D CDWs/SDWs and superfluids disorder driven zero temperature phase transitions are destroyed by thermal fluctuations leaving behind a rich crossover behavior. Quantum phase slips in CDWs and superfluids lead to additional phase transitions and shift the unpinning transition in CDWs to smaller K values. Coulomb hardening and dissipative quantum effects will be discussed in a forthcoming publication [21].

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