QCD-Like Behavior of High-Temperature Confining Strings

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We show that, contrary to previous string models, the high-temperature behavior of the recently proposed confining strings reproduces exactly the correct large-N QCD result, a *necessary* condition for any string model of confinement.

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Although fundamental strings [1] can be quantized only in critical dimensions, strings in four space-time dimensions are of great interest since there is a large body of evidence [2], recently confirmed by numerical tests [3], that they can describe the confining phase of non-Abelian gauge theories. However, a consistent quantum theory describing these strings has not yet been found: the simplest model, the Nambu-Goto string, can be quantized only in space-time dimension $D = 26$ or $D \leq 1$ because of the conformal anomaly; it is inappropriate to describe the expected smooth strings dual to QCD [4], since large Euclidean world sheets are crumpled. In the rigid string [5], the marginal term proportional to the square of the extrinsic curvature, introduced to cure this problem, turns out to be infrared irrelevant and, thus, unable to prevent crumpling.

Both these models also fail to describe the correct high-temperature behavior of large-N QCD [6]. As shown in [7], the deconfining transition in QCD is due to the condensation of Wilson lines, and the partition function of QCD flux tubes can be continued above the deconfining transition; this high-temperature continuation can be evaluated perturbatively. So, any string theory that is equivalent to QCD *must* reproduce this behavior. However, the Nambu-Goto action has the wrong temperature dependence, while the rigid string has the correct high-temperature behavior but with a wrong sign and an imaginary part signaling a world-sheet instability [6].

Recently, two new models have been proposed: the first one, the confining string [8], is based on an induced string action explicitly derivable for compact QED [9] and for Abelian-projected SU(2) [10]; a second one, originally proposed in [11], is based on a five-dimensional, curved space-time string action with the quarks living on a four-dimensional horizon [12]. The formulation of the string theory in the five-dimensional curved space-time is closely related to the AdS/CFT (anti–de Sitter/conformal field theory) correspondence [13]. In fact, with a special choice of the metric in the curved space one recovers the AdS_5 space, thereby providing a string theory description of a conformal gauge theory [13]. Although very interesting results on gauge/strings duality have been obtained in this framework [2,13], even at finite temperature [14], this correspondence is not directly relevant to the problem of confinement unless the conformal symmetry of the gauge theory is broken.

The confining string action possesses, in its world-sheet formulation, a nonlocal action with a negative stiffness [9,15] that can be expressed as a derivative expansion of the interaction between surface elements. To perform an analytic analysis of the geometric properties of these strings, this expansion must be truncated: this clearly makes the model nonunitary, but in a spurious way. Moreover, since the stiffness is negative, a stable truncation must, at least, include a sixth-order term in the derivatives [16]. In [16,17] it was shown that, in the large-D approximation, this model has an infrared fixed point at zero stiffness, corresponding to a tensionless smooth string whose world sheet has Hausdorff dimension 2, exactly the desired properties to describe QCD flux tubes. As first noticed in [18], the long-range orientational order in this model is due to an antiferromagnetic interaction between normals to the surface, a mechanism confirmed by numerical simulations [19]. Moreover, it was shown in [17] that this infrared fixed point does not depend on the truncation and is present for all ghost- and tachyon-free truncations and that the effective theory describing the infrared behavior is a conformal field theory with central charge $c = 1$.

In this Letter we study the high-temperature behavior of the string model defined in [16]. We show that this model has a high-temperature behavior that agrees in temperature dependence, *sign, and reality properties* with the large-N QCD result [6]. This result depends entirely on the higher order term and is totally independent of the stiffness. Finite-temperature confining strings in (2+1) dimensions and in the presence of D0 branes have been studied in [20].

In Euclidean space, the action proposed in [16] is

$$S = \int d^2\xi \, \sqrt{g} \, g^{ab} \mathcal{D}_a x_\mu \Big(t - s\mathcal{D}^2 + \frac{1}{M^2} \mathcal{D}^4 \Big) \mathcal{D}_b x_\mu \,, \tag{1}$$

where $\mathcal{D}_a$ are covariant derivatives with respect to the induced metric $g_{ab} = \partial_a x_\mu \partial_b x_\mu$ on the surface $\mathbf{x}(\xi_0, \xi_1)$. The first term in the bracket provides a bare surface tension $2t$, while the second accounts for the rigidity, with a stiffness parameter s that we set to its fixed-point value $s = 0$. In the third term, M is a new mass scale. Since this term contains the square of the gradient of the extrinsic curvature matrices, it suppresses the formation of spikes on the world sheet. In the large-D approximation it generates a string tension proportional to M^2, which takes control of the fluctuations where the orientational correlation dies off. To perform the large-D analysis we introduce a Lagrange multiplier [21] λ^{ab} that forces the induced metric $\partial_a x_\mu \partial_b x_\mu$ to be equal to the intrinsic metric g_{ab}, extending the action (1) to

$$S \to S + \int d^2\xi \sqrt{g}\,[\lambda^{ab}(\partial_a x_\mu \partial_b x_\mu - g_{ab})]. \quad (2)$$

We parametrize the world sheet in a Gauss map by $x_\mu(\xi) = [\xi_0, \xi_1, \phi^i(\xi)], i = 2, \dots, D-2$. The value of the periodic coordinate ξ_0 is $-\beta/2 \le \xi_0 \le \beta/2$ with $\beta = 1/T$ and T the temperature. Note that at high temperatures ($\beta \ll 1$) , the scale M^2 can be temperature dependent. This is not unusual in closed string theory as has been shown by Atick and Witten [22]. The value of ξ_1 is $-R/2 \le \xi_1 \le R/2$; $\phi^i(\xi)$ describe the $D-2$ transverse fluctuations. We look for a saddle-point solution with a diagonal metric $g_{ab} = \mathrm{diag}(\rho_0, \rho_1)$, and a Lagrange multiplier of the form $\lambda^{ab} = \mathrm{diag}(\lambda_0/\rho_0, \lambda_1/\rho_1)$. The action then becomes

$$\begin{aligned} S &= S_0 + S_1, \\ S_0 &= A_{\mathrm{ext}}\sqrt{\rho_0\rho_1}\left[t\left(\frac{\rho_0+\rho_1}{\rho_0\rho_1}\right) + \lambda_0\left(\frac{1-\rho_0}{\rho_0}\right) + \lambda_1\left(\frac{1-\rho_1}{\rho_1}\right)\right], \\ S_1 &= \int d^2\xi \sqrt{g}\,\partial_a\phi^i\left[g^{ab}\left(t + \frac{1}{M^2}\mathcal{D}^4\right) + \lambda^{ab}\right]\partial_b\phi^i, \end{aligned} \quad (3)$$

where $\beta R = A_{\mathrm{ext}}$ is the extrinsic, projected area in coordinate space, and S_0 is the tree-level contribution. Integrating over the transverse fluctuations in the one-loop term S_1, we obtain, in the limit $R \to \infty$,

$$S_1 = \frac{D-2}{2}R\sqrt{\rho_1}\sum_{n=-\infty}^{+\infty}\int\frac{dp_1}{2\pi}\ln\left[t(\omega_n^2 + p_1^2) + p_1^2\lambda_1 + \omega_n^2\lambda_0 + \frac{1}{M^2}(\omega_n^2 + p_1^2)^3\right], \quad (4)$$

where $\omega_n = \frac{2\pi}{\beta\sqrt{\rho_0}}n$. At high temperatures, satisfying

$$(M^2\beta^2)(t\beta^2) \ll 1, \quad (5)$$

the sixth-order term in the derivatives dominates in the one-loop term S_1 when $n \neq 0$. Using analytic regularization $\int_{\mathrm{reg}} dx \ln(x^2 + a^2) = 2\pi a$, and analytic continuation of the formula $\sum_{n=1}^{\infty} n^{-z} = \zeta(z)$, for the Riemann zeta function, with $\zeta(-1) = -1/12$, we obtain for the $n \neq 0$ contribution

$$\frac{D-2}{2}R\sqrt{\rho_1}\sum_{n=-\infty}^{+\infty}\int\frac{dp_1}{2\pi}\ln\frac{(\omega_n^2 + p_1^2)^3}{M^2} = \frac{D-2}{2}\sqrt{\frac{\rho_1}{\rho_0}}\,12\pi\,\frac{R}{\beta}\sum_{n=1}^{+\infty}\sqrt{n^2} = -\frac{D-2}{2}\sqrt{\frac{\rho_1}{\rho_0}}\,\frac{\pi R}{\beta}. \quad (6)$$

For $n = 0$, instead, we rewrite

$$\ln\left[p_1^2(t + \lambda_1) + \frac{1}{M^2}p_1^6\right] = \ln\frac{p_1^2}{M^2} + \ln[(p_1^2 + iM\sqrt{\lambda_1 + t})(p_1^2 - iM\sqrt{\lambda_1 + t})].$$

The integral over p_1 of the first term of the above equation is zero in analytic regularization, while for the second term we obtain

$$\frac{D-2}{2}R\sqrt{\rho_1}\int\frac{dp_1}{2\pi}\,2\,\mathrm{Re}\ln(p_1^2 + iM\sqrt{\lambda_1 + t}) = \frac{D-2}{2}R\sqrt{\rho_1}\sqrt{2M}\,(\lambda_1 + t)^{1/4}. \quad (7)$$

The action $S = (S_0 + S_1)$ then becomes

$$S = S_0 + \frac{D-2}{2}R\sqrt{\rho_1}\left[\sqrt{2M}\,(\lambda_1 + t)^{1/4} - \frac{\pi}{\beta\sqrt{\rho_0}}\right]. \quad (8)$$

The factor $\frac{D-2}{2}$ in $(S_0 + S_1)$ ensures that, for large D, the fields $\rho_0, \rho_1, \lambda_0$, and λ_1 are extremal and thus satisfy the four-gap equations:

$$\frac{1-\rho_0}{\rho_0} = 0, \quad (9)$$

$$\frac{1}{\rho_1} = 1 - \frac{D-2}{2}\frac{1}{4\beta}\frac{\sqrt{2M}}{(\lambda_1 + t)^{3/4}}, \quad (10)$$

$$\left[\frac{1}{2}(t - \lambda_1) + \frac{1}{2\rho_1}(\lambda_1 + t) - t - \lambda_0\right] + \frac{D-2}{2}\frac{\pi}{2\beta^2} = 0, \quad (11)$$

$$(t - \lambda_1) - \frac{1}{\rho_1}(\lambda_1 + t) + \frac{D-2}{2}\frac{1}{\beta}\left[\sqrt{2M}\,(\lambda_1 + t)^{1/4} - \frac{\pi}{\beta}\right] = 0. \quad (12)$$

Substituting (12) into (8) and using $\rho_0 = 1$ from (9) we obtain a simplified form of the effective action:

$$S^{\text{eff}} = A_{\text{ext}} \mathcal{T} \sqrt{\frac{1}{\rho_1}}, \qquad (13)$$

with $\mathcal{T} = 2(\lambda_1 + t)$ representing the physical string tension.

By inserting (10) into (12), we obtain an equation for $(\lambda_1 + t)$ alone:

$$(\lambda_1 + t) - \frac{D-2}{2}\frac{5}{8\beta}\sqrt{2M}(\lambda_1 + t)^{1/4} + \frac{D-2}{2}\frac{\pi}{2\beta^2} - t = 0. \qquad (14)$$

Without loss of generality we set

$$(\lambda_1 + t)^{1/4} = \frac{\sqrt{2M}}{\gamma}, \qquad (15)$$

where γ is a dimensionless parameter. It is possible to show that, at high temperatures, when

$$t\beta^2 \ll \frac{D-2}{2}, \qquad (16)$$

we can completely neglect t in (14). Indeed, as we now show, λ_1 is proportional to $(D-2)/\beta^2$. Note that this is compatible with the condition (5) used before. We can thus rewrite (14) as

$$\lambda_1 - \frac{D-2}{2}\frac{5}{8\beta}\gamma\lambda_1^{1/2} + \frac{D-2}{2}\frac{\pi}{2\beta^2} = 0. \qquad (17)$$

We now restrict ourselves to the regime

$$G(\gamma, D) = \frac{25}{64}\gamma^2\left(\frac{D-2}{2}\right)^2 - 2\pi\frac{D-2}{2} > 0, \qquad (18)$$

for which (17) admits two real solutions:

$$(\lambda_1^1)^{1/2} = \frac{5}{16\beta}\gamma\frac{D-2}{2} + \frac{1}{2\beta}\sqrt{\frac{25}{64}\gamma^2\left(\frac{D-2}{2}\right)^2 - 2\pi\frac{D-2}{2}}, \qquad (19)$$

$$(\lambda_1^2)^{1/2} = \frac{5}{16\beta}\gamma\frac{D-2}{2} - \frac{1}{2\beta}\sqrt{\frac{25}{64}\gamma^2\left(\frac{D-2}{2}\right)^2 - 2\pi\frac{D-2}{2}}. \qquad (20)$$

In both cases λ_1 is proportional to $(D-2)/\beta^2$, which justifies neglecting t in (14) and implies that the scale M^2 must be chosen proportional to $1/\beta^2$. Moreover, since the physical string tension is real we are guaranteed that $M^2 > 0$, as required by the stability of our model. Any complex solutions for $\mathcal{T}$ would have been incompatible with the stability of the truncation.

Let us start by analyzing the first solution (19). By inserting (19) in (11), we obtain the following equation for ρ_1:

$$\frac{1}{\rho_1} = 1 - \frac{4}{5 + \sqrt{25 - \frac{128\pi}{\gamma^2\frac{D-2}{2}}}}. \qquad (21)$$

Owing to the condition (18), $1/\rho_1$ is positive and, since λ_1^2 is real, the squared free energy is also positive:

$$F^2(\beta) \equiv \frac{S_{\text{eff}}^2}{R^2} = \frac{1}{\beta^2}\left(\frac{5}{16}\gamma\frac{D-2}{2} - \frac{1}{2}\sqrt{G(\gamma,D)}\right)^4\left(1 - \frac{4}{5 + \sqrt{25 - \frac{128\pi}{\gamma^2\frac{D-2}{2}}}}\right). \qquad (22)$$

In this case the high-temperature behavior is the same as in QCD, but the sign is wrong, exactly as for the rigid string. There is, however, a crucial difference: (22) is real, while the squared free energy for the rigid string is imaginary, signaling an instability in the model.

If we now look at the behavior of ρ_1 at low temperatures, below the deconfining transition [17], we see that $1/\rho_1$ is positive. The deconfining transition is indeed determined by the vanishing of $1/\rho_1$ at $\beta = \beta_{\text{dec}}$. In the case of (19) this means that $1/\rho_1$ is positive below the Hagedorn transition, touches zero at β_{dec}, and remains positive above it. Exactly the same will happen also for F^2, which is positive below β_{dec}, touches zero at β_{dec}, and remains positive above it. This solution thus describes an unphysical "mirror" of the low-temperature behavior of the confining string, without a real deconfining Hagedorn transition. For this reason we discard it.

Let us now study the solution (20). Again, by inserting (20) in (11), we obtain for ρ_1 the equation

$$\frac{1}{\rho_1} = 1 - \frac{4}{5 - \sqrt{25 - \frac{128\pi}{\gamma^2\frac{D-2}{2}}}}. \qquad (23)$$

In this case, when

$$\gamma > 4\sqrt{\frac{\pi}{3}}\left(\frac{D-2}{2}\right)^{-1/2}, \qquad (24)$$

$1/\rho_1$ becomes negative. The condition (24) is consistent with (18) and will be taken to fix the values of the range of parameter γ that enters in (15). We restrict ourselves to those that satisfy (24). Since $\rho_0 = 1$ and λ_1 is real and proportional to $1/\beta^2$, we obtain the following form of the squared free energy:

$$F^2(\beta) = -\frac{1}{\beta^2}\left(\frac{5}{16}\gamma\frac{D-2}{2} - \frac{1}{2}\sqrt{G(\gamma,D)}\right)^4 \times\left(\frac{4}{5 - \sqrt{25 - \frac{128\pi}{\gamma^2\frac{D-2}{2}}}} - 1\right). \qquad (25)$$

In the range defined by (24) this is *negative*. For this solution, thus, both $1/\rho_1$ and F^2 pass from positive values at low temperatures to negative values at high temperatures, exactly as one would expect for a string model undergoing the Hagedorn transition at an intermediate temperature. In fact, this is also what happens in the rigid string case, but there, above the Hagedorn transition, there is a second transition above which, at high temperature, λ_1 becomes large and essentially imaginary, giving a positive squared free energy. This second transition is absent in our model.

A consistency check is made to look if the two solutions (19) and (20), together with (15) and (24), are compatible with the validity range (5) of our high-temperature approximation. Ignoring numerical factors and subleading terms in $\frac{D-2}{2}$, (5) becomes

$$t\beta^2 \ll \frac{D-2}{2}, \tag{26}$$

which is exactly the condition (16).

Let us now compare the result (25) with the corresponding one for large-N QCD [7]:

$$F^2(\beta)_{\rm QCD} = -\frac{2g^2(\beta)N}{\pi^2\beta^2}, \tag{27}$$

where $g^2(\beta)$ is the QCD coupling constant. First of all, let us simplify our result by choosing large values of γ:

$$\gamma \gg \sqrt{\frac{128\pi}{25}}\left(\frac{D-2}{2}\right)^{-1/2}.$$

In this case (25) reduces to

$$F^2(\beta) = -\frac{1}{\beta^2}\frac{8\pi^3}{125}\frac{D-2}{\gamma^2}. \tag{28}$$

This corresponds *exactly* to the QCD result (27) with the identifications

$$g^2 \propto \frac{1}{\gamma^2},$$
$$N \propto D-2.$$

The weak β dependence of the QCD coupling $g^2(\beta)$ can be accommodated in the parameter γ. Note that our result is valid at large values of γ, i.e., small values of g^2, as it should be for QCD at high temperatures [23]. Note also the interesting identification between the order of the gauge group and the number of transverse space-time dimensions. Moreover, since the sign of λ_1 does not change at high temperatures, the field x_μ is not unstable. The contrary happens in the rigid string case [6], where the change of sign of λ_1 gives rise to a world-sheet instability.

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