Critical Rotation of a Harmonically Trapped Bose Gas

P. Rosenbusch,[1] D. S. Petrov,[2,3] S. Sinha,[1] F. Chevy,[1] V. Bretin,[1] Y. Castin,[1] G. Shlyapnikov,[1,2,3] and J. Dalibard[1]
[1]*Laboratoire Kastler Brossel,* 24 rue Lhomond, 75005 Paris, France*
[2]*FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands*
[3]*Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia*
(Received 30 January 2002; published 7 June 2002)

We study experimentally and theoretically a cold trapped Bose gas under critical rotation, i.e., with a rotation frequency close to the frequency of the radial confinement. We identify two regimes: the regime of explosion where the cloud expands to infinity in one direction, and the regime where the condensate spirals out of the trap as a rigid body. The former is realized for a dilute cloud, and the latter for a condensate with the interparticle interaction exceeding a critical value. This constitutes a novel system in which repulsive interactions help in maintaining particles together.

DOI: 10.1103/PhysRevLett.88.250403 PACS numbers: 03.75.Fi, 32.80.Lg

The rotation of a macroscopic quantum object is a source of spectacular and counterintuitive phenomena. In superfluid liquid helium contained in a cylindrical bucket rotating around its axis z, one observes the nucleation of quantized vortices for a sufficiently large rotation frequency Ω [1]. A similar phenomenon occurs in a Bose-Einstein condensate confined in a rotating harmonic trap [2–5]. In particular, vortices are nucleated when the rotation resonantly excites surface modes of the condensate. This occurs for particular rotation frequencies in the interval $0 < \Omega \leq \omega_\perp/\sqrt{2}$, where $\omega_\perp$ is the trap frequency in the xy plane perpendicular to the rotation axis z.

Several theoretical studies have recently considered the critical rotation of the gas, i.e., $\Omega \sim \omega_\perp$, which presents remarkable features [6–14]. From a classical point of view, for $\Omega = \omega_\perp$ the centrifugal force compensates the harmonic trapping force in the xy plane. Hence the motion of a single particle of mass m in the frame rotating at frequency Ω is simply due to the Coriolis force $2m\dot{\mathbf{r}} \times \mathbf{\Omega}$. This force is identical to the Lorentz force acting on a particle of charge q in the magnetic field $\mathbf{B} = 2(m/q)\mathbf{\Omega}$. The analogy between the motion of charged particles in a magnetic field and neutral particles in a rotating frame also holds in quantum mechanics. In this respect, a quantum gas of atoms confined in a harmonic trap rotating at the critical frequency is analogous to an electron gas in a uniform magnetic field. One can then expect [8,10] to observe phenomena related to the quantum Hall effect.

This paper presents an experimental and theoretical study of the dynamics of a magnetically trapped rubidium (^{87}Rb) gas stirred at a frequency close to $\omega_\perp$. We show that the single particle motion is dynamically unstable for a window of frequencies Ω centered around $\omega_\perp$. This result entails that the center of mass of the atom cloud (without or with interatomic interactions) is destabilized, since its motion is decoupled from any other degree of freedom for a harmonic confinement. This also implies that a gas of noninteracting particles "explodes," which we indeed check experimentally. When one takes into account the repulsive interactions between particles, which play an important role in a ^{87}Rb condensate, one would expect naively that this explosion is enhanced. However, we show experimentally that this is not the case, and repulsive interactions can "maintain the atoms together." This has been predicted for a Bose-Einstein condensate in the strongly interacting, Thomas-Fermi (TF), regime [9]. Here we derive the minimal interaction strength which is necessary to prevent the explosion. This should help studies of the quantum Hall related physics in the region of critical rotation.

Consider a gas of particles confined in an axisymmetric harmonic potential $V_0(\mathbf{r})$, with frequency ω_z along the trap axis z, and $\omega_\perp$ in the xy plane. To set this gas into rotation, one superimposes a rotating asymmetric potential in the xy plane. In the reference frame rotating at an angular frequency Ω around the z axis, this potential reads $V_1(\mathbf{r}) = \epsilon m \omega_\perp^2 (Y^2 - X^2)/2$, where $\epsilon > 0$. The rotating frame coordinates X, Y are deduced from the laboratory frame coordinates x, y by a rotation at an angle Ωt.

For a noninteracting gas, the equation of motion for each particle reads:

$$\ddot{X} - 2\Omega\dot{Y} + [\omega_\perp^2(1 - \epsilon) - \Omega^2]X = 0, \qquad (1)$$

$$\ddot{Y} + 2\Omega\dot{X} + [\omega_\perp^2(1 + \epsilon) - \Omega^2]Y = 0, \qquad (2)$$

while the motion along z is not affected by the rotation. One deduces from this set of equations that the motion in the xy plane is dynamically unstable if the stirring frequency Ω is in the interval $[\omega_\perp\sqrt{1-\epsilon}, \omega_\perp\sqrt{1+\epsilon}]$. In particular, for $\Omega = \omega_\perp$ and $\epsilon \ll 1$ the quantity $X + Y$ diverges as $\exp(\epsilon\omega_\perp t/2)$, whereas $X - Y$ remains finite.

To test this prediction we use a ^{87}Rb cold gas in a Ioffe-Pritchard magnetic trap, with frequencies $\omega_x = \omega_y = 2\pi \times 180$ Hz and $\omega_z = 2\pi \times 11.7$ Hz. The initial temperature of the cloud precooled using optical molasses is 100 μK. The gas is further cooled by radio-frequency evaporation. For the first set of experiments we stop the evaporation before the Bose-Einstein condensation is reached. The resulting sample contains 10^7 atoms at a temperature $T \sim 5$ μK. It is dilute, with a central density

$\sim 10^{12}$ cm^{-3}, and atomic interactions can be neglected (mean-field energy $\ll k_B T$). The second set of experiments corresponds to a much colder sample ($T < 50$ nK), i.e., to a quasipure condensate with 10^5 atoms.

After evaporation, the atomic cloud is stirred during a time t by a focused laser beam of wavelength 852 nm and waist $w_0 = 20\ \mu$m, whose position is controlled using acousto-optic modulators [2]. The beam is switched on abruptly, and it creates a rotating optical-dipole potential which is nearly harmonic over the extension of the cloud. Then we switch off the magnetic field and the stirrer, allow for a 25 ms free fall, and image the absorption of a resonant laser beam propagating along the z axis. We measure in this way the transverse density profile of the sample, that we fit assuming a Gaussian shape for the noncondensed cloud, and a parabolic TF shape for the condensate. We extract from the fit the long and short diameters in the plane $z = 0$, and the average position of the cloud. The latter gives access to the velocity of the center-of-mass of the cloud before time of flight.

The center-of-mass displacement as a function of the stirring time t is shown in Fig. 1. We choose here $\epsilon = 0.09$ and $\Omega = \omega_\perp$, so that the motion predicted by Eqs. (1) and (2) is dynamically unstable. To ensure reliable initial conditions, we deliberately offset the center of the rotating potential by a few micrometers with respect to the atom cloud [15]. We find the instability for the center-of-mass motion both for the noncondensed cloud (Fig. 1a) and for the quasipure condensate (Fig. 1b). The center-of-mass displacement increases exponentially, with an exponent consistent with the measured ϵ.

We consider now the evolution of the size of the atom cloud as a function of t (Fig. 2). The noncondensed cloud exhibits the behavior expected from the single particle dynamics, i.e., the "explosion" in the $X = Y$ direction. The cloud becomes more and more elliptical in the xy plane. The long radius increases with time, while the short one remains approximately constant (Fig. 2a). On the opposite, we find that the condensate remains circular (Fig. 2b), with no systematic increase in size. We then obtain the following counterintuitive result: for a significant repulsive interaction the atoms remain in a compact cloud, while they fly apart if the interaction is negligible. We observe this stability of the shape of the condensate rotating at the critical velocity for $\epsilon \leq 0.2$. Above this value of ϵ we find that the atomic cloud rapidly disintegrates. For $\epsilon \approx 0.3$, after a stirring time of 50 ms, we observe several fragments in the time-of-flight picture.

We now perform a theoretical analysis of how the interparticle interaction stabilizes a rotating condensate. To this end we use the 2D (x, y) time-dependent Gross-Pitaevskii (GP) equation for an idealized cylindrical trap ($\omega_z = 0$). In the rotating frame the GP equation reads

$$i\partial_t \psi = \frac{1}{2}[-\Delta + (1-\epsilon)X^2 + (1+\epsilon)Y^2 + 2g|\psi|^2 - 2\Omega \hat{L}]\psi, \quad (3)$$

where $\hat{L}$ is the z component of the angular momentum operator. In Eq. (3) the coordinates are given in units of the initial harmonic oscillator length $\sqrt{\hbar/m\omega_\perp}$, and the frequencies in units of $\omega_\perp$. The condensate wave function $\psi(X, Y, t)$ is normalized to unity, and the effective coupling constant is $g = 4\pi a \tilde{N}$, with a being the positive scattering length, and $\tilde{N}$ the number of particles per unit axial length. The effective coupling g depends on density and characterizes the ratio of the mean-field interparticle interaction to the radial frequency $\omega_\perp$. Our experimental conditions correspond to $g \sim 130$.

Since the trapping potential is harmonic, the average center-of-mass motion of the condensate is described by the classical equations (1) and (2) and is decoupled from the evolution of the condensate wave function in the center-of-mass reference frame [15]. We shall therefore restrict to wave functions ψ centered at $x = y = 0$ for all times.

We start with a variational analysis of the steady state of the condensate in the rotating frame. Since the experiment is restricted to stirring times shorter than typical vortex nucleation times [2], we use a simple vortex-free Gaussian ansatz for the condensate wave function [16]:

$$\psi(X, Y) \propto \exp(i\alpha XY - \beta X^2/2 - \gamma Y^2/2). \quad (4)$$

We extremize the GP energy functional with respect to the real parameters α, β, γ. Extremizing with respect to the phase parameter α gives $\alpha = \Omega(\gamma - \beta)/(\gamma + \beta)$. As β and γ should be finite and positive this sets the constraint

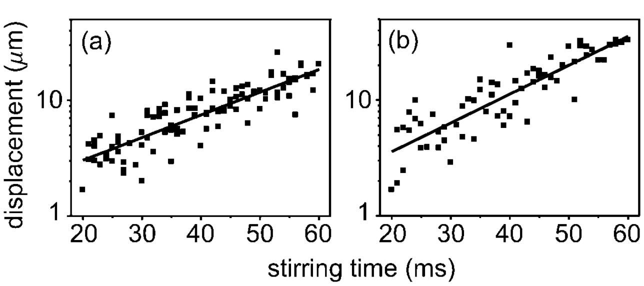

FIG. 1. Center-of-mass displacement after free expansion (log-scale) vs stirring time for $\Omega = \omega_\perp$ and $\epsilon = 0.09$. (a) Noncondensed cloud with 10^7 atoms, $T = 5\ \mu$K. (b) Condensate with 10^5 atoms. Solid line: exponential fit to the data.

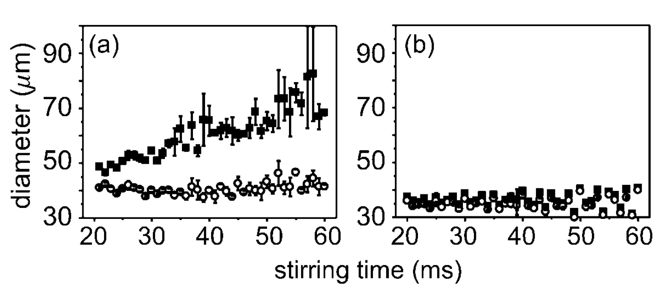

FIG. 2. Long (■) and short (∘) diameters of the atom cloud vs stirring time, for $\Omega = \omega_\perp$. (a) Noncondensed cloud. (b) Quasipure condensate (same parameters as in Fig. 1).

$\alpha^2 < \Omega^2$. Extremizing over β and γ and expressing β/γ in terms of α, we obtain a closed equation for α:

$$(\epsilon/\Omega)[\alpha^2 + 2\alpha\Omega(\Omega^2 - 1)/\epsilon + \Omega^2] - (g/2\pi)\sqrt{1 - \alpha^2/\Omega^2}[\alpha^3 + (1 - 2\Omega^2)\alpha - \Omega\epsilon] = 0. \quad (5)$$

In the noninteracting case ($g = 0$) the Gaussian ansatz is exact, and α is the root of a quadratic equation. This ansatz also captures the scaling properties of the rotating condensate in the regime of strong interactions. In the TF limit ($g \to \infty$) the first line of (5) can be neglected and we recover the cubic equation for α derived in [9].

For $g = 0$ the parameter α is complex in the interval of rotation frequencies $\sqrt{1-\epsilon} < \Omega < \sqrt{1+\epsilon}$, and there is no steady state solution for the condensate wave function at these Ω [9]. For a finite g the lower border Ω_- of this frequency interval remains equal to $\sqrt{1-\epsilon}$ irrespective of the value of g. The upper border, dashed curves in Fig. 3, decreases with increasing g at a given ϵ. For small anisotropy $\epsilon < 1/5$ it reaches the lower border at a critical coupling strength. For larger g the steady states exist at any Ω. If $\epsilon > 1/5$, the upper border never reaches $\Omega_- = \sqrt{1-\epsilon}$ and for any g one has an interval of Ω where steady state solutions are absent. The $\epsilon = 1/5$ threshold was derived for the TF limit in [9].

For $g \gg 1$, $\epsilon \ll 1$, and $\Omega = 1$, one of the three roots of (5) is $\alpha \simeq -\epsilon$, corresponding to a quasicircular cloud ($\beta \sim \gamma$). The result of Fig. 2b essentially corresponds to this steady state, apart from oscillations due to the abrupt switching of the stirring potential.

To describe the time-dependent solution of Eq. (3) and account for this abrupt switching, we have developed an approximate scaling approach. We assume (and later on check) that the evolution of the condensate shape is well described by dilations with factors $b_u(t)$ and $b_v(t)$ along the axes $\tilde{X}$ and $\tilde{Y}$, rotating at an angular frequency $\dot{\phi}(t)$ with respect to the laboratory frame x, y. To determine $b_u(t)$, $b_v(t)$, and $\phi(t)$, we write the wave function as

$$\psi(\tilde{X}, \tilde{Y}, t) = (b_u b_v)^{-1/2}\chi(u, v, t)\exp\{i\Phi(\tilde{X}, \tilde{Y}, t)\}, \quad (6)$$

where we have set $u = \tilde{X}/b_u$, $v = \tilde{Y}/b_v$, and

$$\Phi(\tilde{X}, \tilde{Y}, t) = \tilde{\alpha}(t)\tilde{X}\tilde{Y} + (\dot{b}_u/2b_u)\tilde{X}^2 + (\dot{b}_v/2b_v)\tilde{Y}^2, \quad (7)$$

with $\tilde{\alpha} = -\dot{\phi}\tanh\xi$ and $\xi(t) = \ln(b_v/b_u)$. Then the GP equation reduces to the following equation for $\chi(u, v, t)$:

$$i\left[\partial_t - \frac{\dot{\phi}}{\cosh\xi}(u\partial_v - v\partial_u)\right]\chi = \left[-\frac{\partial_u^2}{2b_u^2} - \frac{\partial_v^2}{2b_v^2} + \frac{1}{2}(\nu_u^2 b_u^2 u^2 + \nu_v^2 b_v^2 v^2) + \frac{g|\chi|^2}{b_u b_v}\right]\chi. \quad (8)$$

The "frequencies" ν_u and ν_v are given by

$$\nu_{u,v}^2 = 1 \mp \epsilon\cos(2\Omega t - 2\phi) + \tilde{\alpha}^2 \mp 2\dot{\phi}\tilde{\alpha} + \ddot{b}_{u,v}/b_{u,v}. \quad (9)$$

In Eq. (8) we took into account that u, v are eigenaxes of the condensate, which requires the absence of terms proportional to uv and is provided by the relation

$$\epsilon\sin(2\Omega t - 2\phi) - \dot{\tilde{\alpha}} - \tilde{\alpha}(\dot{b}_u/b_u + \dot{b}_v/b_v) + \dot{\phi}\dot{\xi} = 0. \quad (10)$$

We now replace the left-hand side (lhs) of Eq. (8) by $\tilde{\mu}\chi$, where $\tilde{\mu}$ follows from the normalization condition for χ. The solution of the resulting equation is a function of $b_{u,v}$, ϕ, and $\nu_{u,v}$. We then require that (6) is a relevant scaling transform, i.e., that the function $\chi(u, v, t)$ is most similar to the initial function $\chi(u, v, 0)$. More precisely we set the averages $\langle u^2\rangle_t$, $\langle v^2\rangle_t$ equal to their values at $t = 0$. This fixes ν_u and ν_v in terms of b_u, b_v, ϕ. The solution of Eqs. (9) and (10) then gives the desired scaling parameters.

The omitted lhs of Eq. (8) only insignificantly changes ν_u and ν_v. It vanishes in both the TF regime and for an ideal-gas condensate. For the TF limit our procedure gives the same results as the scaling approach of [17] and the function $|\chi|^2$ remains an inverted parabola.

For an abrupt switching of the rotating potential we use the initial conditions $b_{u,v}(0) = 1$, $\dot{b}_{u,v}(0) = \phi(0) = \dot{\phi}(0) = 0$. We find two types of solution: (i) oscillating functions $b_{u,v}(t)$; (ii) one of the scaling parameters eventually grows exponentially. Case (ii) describes an infinite expansion of the condensate in one direction, similarly to the expansion of the ideal gas under rotation.

For a given ϵ we obtain the upper (Ω_+) and lower (Ω_-) instability borders in the $\Omega - g$ space (see Fig. 3). The lower border is always equal to $\sqrt{1-\epsilon}$. The upper border $\Omega_+(g)$ decreases with increasing g, and for $\epsilon < 0.17$ it reaches Ω_- at a critical value of the coupling strength. For $\epsilon > 0.17$ we have $\Omega_+ > \Omega_-$ at any g.

The obtained results can be understood by comparing the frequency Ω_q of the rotating quadrupole mode of the condensate with the rotation frequency $\Omega \sim 1$ of the

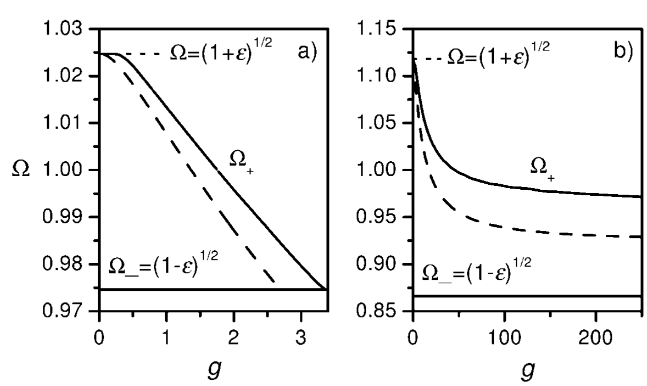

FIG. 3. Solid curves: upper (Ω_+) and lower (Ω_-) borders of the instability region vs g, for an abrupt switching of the rotation with $\epsilon = 0.05$ (a) and $\epsilon = 0.25$ (b). Dashed curves: upper border of the region where stationary states are absent.

perturbation $V_1(\mathbf{r})$. In the absence of interaction one has $\Omega_q = 1$, and the corresponding resonance leads to the condensate explosion. The interactions reduce the frequency of the rotating quadrupole mode ($\Omega_q = 1/\sqrt{2}$ in the TF limit), suppressing the resonance at $\Omega \approx 1$: the deformation of the condensate induced by V_1 remains small, at least for ϵ smaller than the detuning from the resonance. For larger ϵ the condensate explodes.

In the presence of interactions ($1/\sqrt{2} < \Omega_q < 1$), one could expect naively that the explosion occurs for a resonant drive with $\Omega \sim \Omega_q$, even for small ϵ. This is not the case because of a nonlinear character of the dynamics. As the system starts to elongate under the action of the resonant excitation, it becomes closer to an ideal gas, for which $\Omega_q = 1$. The gas is then driven away from the resonance and its deformation stops. This explains why the lower instability border Ω_- is independent of g.

The scaling method can also be used to identify stationary solutions, by setting $\dot{\phi} = \Omega$ and constant $b_{u,v}$ in Eqs. (9) and (10). The results nearly coincide with the ones from the Gaussian ansatz. The existence of these solutions can also be explored using an adiabatic switching of the rotating potential. As shown in Fig. 3 the domain of instability for an abrupt switching of the rotation includes the domain for the absence of stationary solutions.

Figure 4 shows the minimum coupling strength $g_c(\epsilon)$ required for the stability of the shape of the condensate at $\Omega = 1$. TF condensates remain stable for an abrupt switching of the rotation if $\epsilon \lesssim 0.28$. This explains the destruction of the condensate in our experiment at $\epsilon \approx 0.3$.

To summarize, our analysis shows that, thanks to repulsive interactions, the condensate can preserve its shape and size for any rotation frequency in the instability window of the center-of-mass motion, $\sqrt{1-\epsilon} < \Omega < \sqrt{1+\epsilon}$. This occurs for small ϵ ($\epsilon \lesssim 0.17$ for TF condensates). In this case the condensate spirals out of the trap as a rigid body after the rotation is switched on. For larger ϵ there are rotation frequencies in the center-of-mass instability window at which even TF condensates explode. One can think of observing related effects in rotating ion clouds in electromagnetic traps (see [18] and references therein).

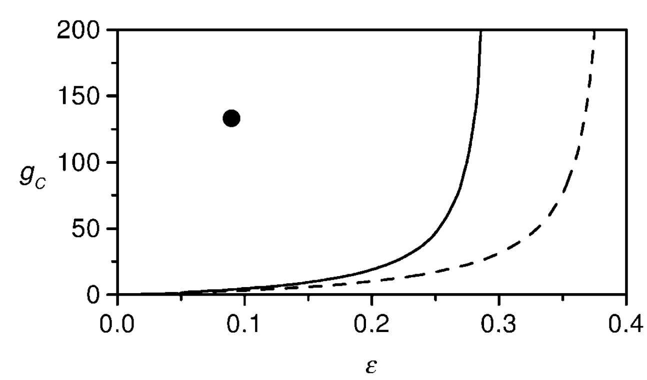

FIG. 4. Critical coupling strength g_c vs ϵ for $\Omega = 1$. The solid and dashed curves correspond to an abrupt and adiabatic switching of the rotation, respectively. The black disk corresponds to the experimental situation of Fig. 2b.

Although our picture properly describes the experiment, we do not deal here with the ground state of the system. On a longer time scale, the gas will evolve to a more complex state, for instance to a multivortex state (with possible quantum melting) or to a quantum-Hall-like state, discussed for the axially symmetric case [6,8,10–14]. A natural extension is to include a rotating anisotropy ϵ, a necessary ingredient for most present experiments. In particular, for $\Omega = \omega_\perp\sqrt{1-\epsilon}$, one reaches a one-body Hamiltonian corresponding to an unbound motion (with a gauge field) in the X direction, similar to a quantum Hall fluid in a narrow channel. We believe that the study of many body aspects of this regime will bring in new features of quantum mesoscopic physics.

We thank K. Madison for his help in early experiments and we acknowledge fruitful discussions with J. Bollinger, S. Stringari, and D. Wineland. P. R. acknowledges support by the Alexander von Humboldt-Stiftung and by the EU, Contract No. HPMF CT 2000 00830. D. P. and G. S. acknowledge support from the Dutch Foundations NWO and FOM, and from the Russian Foundation for Basic Research. This work was partially supported by the Région Ile de France, CNRS, Collège de France, DRED and INTAS.

*Unité de Recherche de l'Ecole normale supérieure et de l'Université Pierre et Marie Curie, associée au CNRS.

[1] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, United Kingdom, 1991).
[2] K. W. Madison *et al.,* Phys. Rev. Lett. **84**, 806 (2000).
[3] J. R. Abo-Shaeer *et al.,* Science **292**, 467 (2001).
[4] P. C. Haljan *et al.,* Phys. Rev. Lett. **87**, 210403 (2001).
[5] E. Hodby *et al.,* Phys. Rev. Lett. **88**, 010405 (2002).
[6] D. A. Butts and D. S. Rokhsar, Nature (London) **397**, 327 (1999).
[7] B. Mottelson, Phys. Rev. Lett. **83**, 2695 (1999); A. D. Jackson *et al.,* Phys. Rev. Lett. **86**, 945 (2001).
[8] N. K. Wilkin and J. M. F. Gunn, Phys. Rev. Lett. **84**, 6 (2000); N. R. Cooper, N. K. Wilkin, and J. M. F. Gunn, Phys. Rev. Lett. **87**, 120405 (2001).
[9] A. Recati, F. Zambelli, and S. Stringari, Phys. Rev. Lett. **86**, 377 (2001).
[10] B. Paredes *et al.,* Phys. Rev. Lett. **87**, 010402 (2001).
[11] T. L. Ho, Phys. Rev. Lett. **87**, 060403 (2001).
[12] A. L. Fetter, Phys. Rev. A **64**, 063608 (2001).
[13] U. R. Fischer and G. Baym, cond-mat 0111443.
[14] J. Sinova *et al.,* cond-mat 0201020.
[15] The rotation axis offset adds to Eqs. (1) and (2) a small uniform force rotating at frequency Ω. This has a negligible consequence on the center-of-mass instability and it does not affect the relative motion between particles.
[16] V. Perez-Garcia *et al.,* Phys. Rev. Lett. **77**, 5320 (1996).
[17] P. Storey and M. Olshanii, Phys. Rev. A **62**, 033604 (2000); S. Sinha and Y. Castin, Phys. Rev. Lett. **87**, 190402 (2001).
[18] X.-P. Huang *et al.,* Phys. Rev. Lett. **80**, 73 (1998).