Long-Range Magnetic Interaction due to the Casimir Effect

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The zero-point quantum fluctuations of the electromagnetic field in vacuum are known to give rise to a long-range attractive force between metal plates (Casimir effect). For ferromagnetic layers separated by vacuum, it is shown that the interplay of the Casimir effect and of the magneto-optical Kerr effect gives rise to a long-range magnetic interaction. The Casimir magnetic force is found to decay as D^{-1} in the limit of short distances, and as D^{-5} in the limit of long distances. Explicit expressions for realistic systems are given in the large- and small-distance limits. An experimental test of the Casimir magnetic interaction is proposed.

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Since the discovery of magnets by the ancient Greeks, long-range magnetic interactions have been an object of fascination. It is usually considered that there exists essentially two types of magnetic interactions between magnetic moments or magnetized bodies: (i) the dipole-dipole magnetostatic interaction, and (ii) the electron-mediated exchange interaction. The latter have recently received renewed attention, with the discovery of spectacular oscillatory behavior of the interlayer exchange coupling between ferromagnetic layers separated by a nonmagnetic metal spacerl [1], due to a spin-dependent quantum size effect [2].

For the case of two uniformly magnetized ferromagnetic plates (of infinite lateral extension) held parallel to each other in vacuum, the two above-mentioned magnetic coupling mechanisms yield a magnetic interaction which decreases exponentially with interplate distance D: (i) the stray field due to a uniformly magnetized plate decreases exponentially (with a characteristic decay length of the order of the interatomic distance) with the distance from the plate, and so does also the interplate dipolar interaction; (ii) the interplate exchange interaction also decays exponentially with D, since it is mediated by electrons tunneling through vacuum between the two plates. The aim of this Letter is to point out the existence of a novel, so far overlooked, mechanism of magnetic interaction between magnetized (i.e., ferro- or ferrimagnetic) bodies. This interaction arises from the Casimir effect, and gives rise to a long-range (i.e., with power-law decay) magnetic interaction; at sufficiently large distance D it is therefore the dominant mechanism of magnetic interaction between the two ferromagnetic plates.

As Casimir pointed out in a seminal paper [3], the zero-point quantum fluctuations of the electromagnetic (EM) field in vacuum leads to observable effects: changing the boundary condition of the EM field (e.g., by moving a body with respect to another) yields a finite change of the (infinite) zero-point energy of the system, and therefore results in an observable force. In particular, Casimir predicted the existence of a long-range attractive force between mirrors in vacuum. The Casimir effect is currently attracting considerable interest [4,5], in particular with respect to micromechanical devices, and has deep implications in many fields of physics.

The boundary condition of the EM field can, however, also be modified without any mechanical displacement, but rather by changing the order parameter of a collective ordering phenomenon such as ferromagnetism. When the two mirrors are ferromagnetic, the magneto-optical Kerr effect influences the boundary condition of the EM field, so that the Casimir effect manifests as an energy difference (per unit area) $\Delta\mathcal{E} \equiv \mathcal{E}_{\mathrm{AF}} - \mathcal{E}_{\mathrm{FM}}$ between the configurations in which the two mirrors have their magnetizations antiparallel (AF) or parallel (FM) to each other, i.e., as a magnetic interaction. Alternatively, the above effect manifests as a dependence of the (mechanical) Casimir force (per unit area) among the mirrors upon the relative orientation of their magnetizations: $\Delta\mathcal{F} \equiv \mathcal{F}_{\mathrm{AF}} - \mathcal{F}_{\mathrm{FM}} = -d\Delta\mathcal{E}/dD$. The *Casimir magnetic force* $\Delta\mathcal{F}$ has been calculated for ferromagnetic mirrors described by a Drude model: in the limit of very large distance, the magnetic force decays as D^{-5}; in the limit of small distances, it decays as D^{-1}; in the intermediate regime, it decays as D^{-4}. For equivalent ferromagnetic materials on both sides, the Casimir magnetic interaction is always antiferromagnetic. For realistic systems, the explicit expression of the Casimir magnetic force is found to be

$$\Delta\mathcal{F} \approx \frac{-3\zeta(3)}{16\pi^3}\frac{\hbar c^2}{D^5}\frac{\theta_A\theta_B}{\sqrt{\sigma_A\sigma_B}}, \tag{1}$$

in the limit of large distance, where $\sigma_{A(B)}$ and $\theta_{A(B)}$ are, respectively, the dc conductivity and anomalous Hall angle of the ferromagnetic plate A (B), and

$$\Delta\mathcal{F} \approx \frac{-1}{4\pi^2}\frac{\hbar}{c^2 D} \times \int_0^{+\infty} \frac{\omega^2 \varepsilon_{xy}^A(i\omega)\varepsilon_{xy}^B(i\omega)}{[1+\varepsilon_{xx}^A(i\omega)][1+\varepsilon_{xx}^B(i\omega)]}\, d\omega, \tag{2}$$

in the limit of short distances, where $\varepsilon_{xx}^{A(B)}(i\omega)$ and

$\varepsilon_{xy}^{A(B)}(i\omega)$ are, respectively, the diagonal and off-diagonal elements of the dielectric tensor of the ferromagnetic plate A (B), evaluated at imaginary frequency $i\omega$.

The Casimir interaction energy (per unit area) between two mirrors can be conveniently expressed in terms of their reflection coefficients as [6,7]

$$\mathcal{E} = \int_0^{+\infty} d\omega \, \frac{\hbar}{2} \coth\left(\frac{\hbar\omega}{k_B T}\right) \frac{1}{(2\pi)^2} \int d^2\mathbf{k}_\parallel \times \frac{1}{\pi} \operatorname{Im} \operatorname{Tr} \ln(1 - \mathsf{R}_A \mathsf{R}_B e^{2ik_\perp D}), \tag{3}$$

where $\mathbf{k}_\parallel$ and $k_\perp$ are the components of the wave vector parallel and perpendicular to the mirrors, respectively. The above expression is completely analogous to the one derived independently for the case of interactions mediated by fermions (electrons) [2]. The 2×2 matrix of reflection coefficients on mirror A (B) is given by

$$\mathsf{R}_{A(B)} \equiv \begin{pmatrix} r_{ss}^{A(B)} & r_{sp}^{A(B)} \\ r_{ps}^{A(B)} & r_{pp}^{A(B)} \end{pmatrix}, \tag{4}$$

where the index s (p) corresponds to a polarization with the electric field perpendicular (parallel) to the incidence plane. The off-diagonal matrix elements r_{sp} and r_{ps} are responsible for the magneto-optical effects. With the usual convention that the s axis remains unchanged upon reflection, one has $r_{sp} = r_{ps}$, and, for perpendicular incidence, $r_{ss} = -r_{pp}$. Performing the change of variables $(\omega, \mathbf{k}_\parallel) \to (\omega, k_\perp)$ and using complex plane integration methods, one can rewrite Eq. (3) at $T = 0$ as [7]

$$\mathcal{E} = \frac{\hbar}{4\pi^2} \int_0^{+\infty} k_\perp \, dk_\perp \int_0^{k_\perp c} d\omega \times \operatorname{Re} \operatorname{Tr} \ln[1 - \mathsf{R}_A(i\omega, ik_\perp) \mathsf{R}_B(i\omega, ik_\perp) e^{-2k_\perp D}], \tag{5}$$

where the reflection coefficients are evaluated at imaginary values of the frequency and normal wave vector.

For a mirror magnetized along its normal (pointing outwards), the reflection coefficients are given by [8]

$$r_{ss}(i\omega, ik_\perp) = \frac{k_\perp c - \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}}{k_\perp c + \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}},$$
$$r_{pp}(i\omega, ik_\perp) = \frac{\varepsilon_{xx}(i\omega) k_\perp c - \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}}{\varepsilon_{xx}(i\omega) k_\perp c + \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}}, \tag{6a}$$

$$r_{sp}(i\omega, ik_\perp) = r_{ps}(i\omega, ik_\perp) = \frac{-k_\perp c \omega \varepsilon_{xy}(i\omega)}{\{k_\perp c + \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}\}\{\varepsilon_{xx}(i\omega) k_\perp c + \sqrt{\omega^2[\varepsilon_{xx}(i\omega) - 1] + (k_\perp c)^2}\}}. \tag{6b}$$

For the sake of simplicity, the arguments $(i\omega, ik_\perp)$ will be omitted below. If the magnetization points inward, then the sign of r_{sp} and r_{ps} is reversed.

Since the magneto-optical reflection coefficients r_{sp} are usually much smaller than 1 and the usual reflection coefficients r_{ss} and r_{pp}, one can expand the Casimir magnetic energy to lowest order in the magneto-optical coefficients, yielding

$$\Delta\mathcal{E} = \frac{-\hbar}{\pi^2} \int_0^{+\infty} k_\perp \, dk_\perp \int_0^{k_\perp c} d\omega \times \operatorname{Re}\left[\frac{r_{sp}^A r_{sp}^B e^{-2k_\perp D}}{(1 - r_{ss}^A r_{ss}^B e^{-2k_\perp D})(1 - r_{pp}^A r_{pp}^B e^{-2k_\perp D})}\right]. \tag{7}$$

Equation (7) together with Eqs. (6a) and (6b) allow to calculate the Casimir magnetic energy and force.

In order to illustrate the above result, let us calculate the Casimir magnetic interaction for the case of (equivalent) ferromagnetic mirrors with a dielectric tensor approximated by a Drude model:

$$\varepsilon_{xx}(i\omega) = 1 + \frac{\omega_p{}^2 \tau}{\omega(1 + \omega\tau)},$$
$$\varepsilon_{xy}(i\omega) = \frac{\omega_p{}^2 \omega_c \tau^2}{\omega(1 + \omega\tau)^2}. \tag{8}$$

The plasma frequency ω_p is given by $\omega_p{}^2 \equiv 4\pi n e^2/m^\star$; $\omega_c \equiv eB_{\text{eff}}/m^\star c$ is the cyclotron frequency, where B_{eff} is the effective magnetic field experienced by conduction electrons as a result of the combined effect of the exchange and spin-orbit interactions; τ is the relaxation time. It is assumed that $\omega_c\tau \ll 1 \ll \omega_p\tau$, which constitutes the usual situation.

One can distinguish three different regimes: (i) $D \gg c\tau$, (ii) $c/\omega_p \ll D \ll c\tau$, and (iii) $D \ll c/\omega_p$. In regime (i) (i.e., at long distances), the integral in Eq. (7) is dominated by the range $\omega \leq k_\perp c \approx c/D \ll 1/\tau$, for which one has

$$\varepsilon_{xx}(i\omega) \approx \frac{\omega_p{}^2\tau}{\omega} \gg 1, \qquad \varepsilon_{xy}(i\omega) \approx \frac{\omega_p{}^2\omega_c\tau^2}{\omega}, \tag{9}$$

so that

$$r_{ss} \approx -r_{pp} \approx -1, \qquad r_{sp} \approx -\frac{\omega_c}{\omega_p}\sqrt{\omega\tau}. \tag{10}$$

One then finds that

$$\Delta\mathcal{E} \approx \frac{-3\zeta(3)}{16\pi^2} \frac{\hbar c^2}{D^4} \frac{\omega_c{}^2\tau}{\omega_p{}^2},$$
$$\Delta\mathcal{F} \approx \frac{-3\zeta(3)}{4\pi^2} \frac{\hbar c^2}{D^5} \frac{\omega_c{}^2\tau}{\omega_p{}^2}, \tag{11}$$

where $\zeta(x) \equiv \sum_{n=1}^{\infty} n^{-x}$ is the Riemann zeta function, with $\zeta(3) \approx 1.202\ldots$.

In the intermediate-distance regime (ii) ($c/\omega_p \ll D \ll c\tau$), the integral in Eq. (7) is dominated by the range $1/\tau \ll \omega \leq k_\perp c \approx c/D \ll \omega_p$, for which one has

$$\varepsilon_{xx}(i\omega) \approx \frac{{\omega_p}^2}{\omega^2} \gg 1, \qquad \varepsilon_{xy}(i\omega) \approx \frac{{\omega_p}^2 \omega_c}{\omega^3}, \quad (12)$$

so that

$$r_{ss} \approx -r_{pp} \approx -1, \qquad r_{sp} \approx -\frac{\omega_c}{\omega_p}. \quad (13)$$

One then finds that

$$\Delta\mathcal{E} \approx -\frac{1}{24}\frac{\hbar c}{D^3}\frac{{\omega_c}^2}{{\omega_p}^2}, \qquad \Delta\mathcal{F} \approx -\frac{1}{8}\frac{\hbar c}{D^4}\frac{{\omega_c}^2}{{\omega_p}^2}. \quad (14)$$

In the short-distance regime (iii) ($D \ll c/\omega_p$), one needs to consider separately the range with $\omega \leq k_\perp c \ll \omega_p$, for which the reflection coefficients are given by Eqs. (13), and the range with $\omega_p \ll \omega \leq k_\perp c$. For $\omega \gg \omega_p$, ε_{xy} is given by Eq. (12) and

$$\varepsilon_{xx}(i\omega) - 1 \approx \frac{{\omega_p}^2}{\omega^2} \ll 1, \quad (15)$$

so that, for $k_\perp c \gg \omega_p$,

$$|r_{ss}| \ll 1, \quad r_{pp} \ll 1, \quad r_{sp} \approx -\frac{{\omega_p}^2 \omega_c}{2k_\perp c(2\omega^2 + {\omega_p}^2)}. \quad (16)$$

One then finds that

$$\Delta\mathcal{E} \approx -\frac{1}{16\pi\sqrt{2}}\frac{\hbar}{c^2}\ln\left(\frac{c}{\omega^\star D}\right){\omega_c}^2\omega_p, \quad (17a)$$

$$\Delta\mathcal{F} \approx -\frac{1}{16\pi\sqrt{2}}\frac{\hbar}{c^2 D}{\omega_c}^2\omega_p, \quad (17b)$$

where $\omega^\star$ is a cutoff frequency of the order of the plasma frequency ω_p.

For realistic systems, it is in general necessary to perform a detailed calculation. However, in the limit of large and small distances, explicit expressions can be obtained. At large distances (i.e., for $D \gg c/\tau$), the Casimir magnetic force is essentially determined by the dielectric tensor $\varepsilon_{ij}(i\omega) = \delta_{ij} + 4\pi\sigma_{ij}(i\omega)/\omega$ at low imaginary frequency. In this regime, one can safely approximate the conductivity tensor $\sigma_{ij}(i\omega)$, in the above expression, by its dc value: $\sigma_{xx}(i\omega) \approx \sigma$, $\sigma_{xy}(i\omega) \approx \sigma\theta$, where σ is the dc conductivity, and θ is the anomalous Hall angle of the ferromagnetic mirror. Proceeding as for the Drude model, one then obtains

$$\Delta\mathcal{E} \approx \frac{-3\zeta(3)}{64\pi^3}\frac{\hbar c^2}{D^4}\frac{\theta_A\theta_B}{\sqrt{\sigma_A\sigma_B}}, \quad (18)$$

from which Eq. (1) follows immediately.

For short distances, the Casimir magnetic interaction is dominated by imaginary wave vectors $ik_\perp$ with $\omega^\star \ll k_\perp c \approx c/D$, where the cutoff frequency $\omega^\star$ is of the order of the plasma frequency, or the typical frequency of interband transitions. In this regime, one has

$$|r_{ss}| \ll 1, \quad r_{pp} \ll 1, \quad r_{sp} \approx -\frac{\omega\varepsilon_{xy}(i\omega)}{2k_\perp c[1 + \varepsilon_{xx}(i\omega)]}. \quad (19)$$

One eventually obtains

$$\Delta\mathcal{E} \approx \frac{-1}{4\pi^2}\frac{\hbar}{c^2}\ln\left(\frac{c}{\omega^\star D}\right) \times \int_0^{+\infty}\frac{\omega^2\varepsilon_{xy}^A(i\omega)\varepsilon_{xy}^B(i\omega)}{[1 + \varepsilon_{xx}^A(i\omega)][1 + \varepsilon_{xx}^B(i\omega)]}d\omega, \quad (20)$$

from which Eq. (2) follows immediately.

Let us now discuss whether the novel Casimir magnetic interaction can be observed experimentally. Obviously, the regime of potential experimental interest is the short-distance limit. To obtain a rough estimate of the magnitude of the effect, it is sufficient to approximate the (magneto-)optical absorption spectrum by a single absorption line at frequency ω_0 containing all the spectral weight, i.e., we write

$$\mathrm{Im}\varepsilon_{xx}(\omega) \approx \omega_0\varepsilon_{xx}^{\mathrm{eff}}\delta(\omega - \omega_0), \quad (21a)$$

$$\mathrm{Re}\varepsilon_{xy}(\omega) \approx \omega_0\varepsilon_{xy}^{\mathrm{eff}}\delta(\omega - \omega_0). \quad (21b)$$

This is expected to be a good approximation in the limit of small distances (i.e., high frequencies), where the details of the (magneto-)optical spectra should not matter too much. The dielectric tensor at the imaginary frequency is then obtained from the causality relations

$$\varepsilon_{xx}(i\omega) = 1 + \frac{2}{\pi}\int_0^{+\infty}d\omega'\,\frac{\omega'\,\mathrm{Im}\varepsilon_{xx}(\omega')}{\omega'^2 + \omega^2}, \quad (22a)$$

$$\varepsilon_{xy}(i\omega) = \frac{2}{\omega\pi}\int_0^{+\infty}d\omega'\,\frac{\omega'^2\,\mathrm{Re}\varepsilon_{xy}(\omega')}{\omega'^2 + \omega^2}, \quad (22b)$$

and one eventually obtains

$$\Delta\mathcal{E} \approx \frac{-1}{16\pi^3}\frac{\hbar}{c^2}\ln\left(\frac{c}{\omega_0 D}\right)\frac{{\omega_0}^3{\varepsilon_{xy}^{\mathrm{eff}}}^2}{(1 + \varepsilon_{xx}^{\mathrm{eff}}/\pi)^{3/2}}, \quad (23a)$$

$$\Delta\mathcal{F} \approx \frac{-1}{16\pi^3}\frac{\hbar}{c^2 D}\frac{{\omega_0}^3{\varepsilon_{xy}^{\mathrm{eff}}}^2}{(1 + \varepsilon_{xx}^{\mathrm{eff}}/\pi)^{3/2}}. \quad (23b)$$

By simple inspection of the (magneto-)optical absorption spectra of transition metal ferromagnets [9], one finds that the model parameters assume the typical values $\omega_0 \approx 6 \times 10^{15}\ \mathrm{s}^{-1}$, $\varepsilon_{xx}^{\mathrm{eff}} \approx 10$, and $\varepsilon_{xy}^{\mathrm{eff}} \approx 1.5 \times 10^{-2}$. Experimentally, it is usually not convenient to maintain two plates accurately parallel to each other, so that a configuration with a planar mirror and a lens-shaped mirror is usually adopted. Even in this configuration, the parasitic magnetostatic interaction can be made as small as needed by taking a uniformly magnetized plate of sufficiently low thickness and sufficiently large lateral extension. The net resulting Casimir magnetic force ΔF (not to be confused with the Casimir magnetic force *per unit area* $\Delta\mathcal{F}$) is then obtained by means of the "proximity force theorem" [10]: $\Delta F = 2\pi R \Delta \mathcal{E}(D)$, where R is the curvature radius of the lens-shaped mirror and D is the shortest distance. For $R = 100\ \mu$m and $D \leq c/\omega_0 \approx 50$ nm, one finds that $|\Delta F| \approx 10$ fN, with only a weak (logarithmic) dependence upon D.

The Casimir magnetic force ΔF is several orders of magnitude weaker than the nonmagnetic Casimir force F ($|F| \approx 0.5$ nN, for $D = 50$ nm); however, it can be detected independently of F by using a resonant differential method, as done in "magnetic resonant force microscopy" [11,12]: in this approach, one measures the mechanical force between two magnetic samples, one of them being fixed and magnetically hard, the other one being magnetically soft and attached to a high-Q mechanical resonator (of resonance frequency ω_r) consisting of a microcantilever. By modulating the magnetization of the soft sample by means of an ac magnetic field (the hard sample remaining unchanged) at $\omega = \omega_r$, one can detect the magnetic force among the two samples (i.e., the difference $\Delta F \equiv F_{\mathrm{AF}} - F_{\mathrm{FM}}$) with a considerably higher sensitivity than allowed by a dc measurement of F_{AF} or F_{FM} separately. In addition, when measuring the nonmagnetic Casimir effect, great care has to be taken to eliminate parasitic electrostatic interactions, which is done automatically in the approach proposed here. A detailed discussion of the sensitivity limitations of magnetic resonant force microscopy is given in Ref. [12]: it is limited on one hand by the sensitivity in measuring the deflection of the cantilever, which yields $|\Delta F_{\min}| \geq k\delta x/Q$ (where k is the cantilever spring constant, Q is the quality factor, and δx is the deflection sensitivity), and by the thermomechanical noise on the other hand, which yields $|\Delta F_{\min}| \geq \sqrt{4k_B T \Delta\nu k/(\omega_r Q)}$, for a bandwidth $\Delta\nu$. For the cantilever used in Ref. [11] ($k = 1$ mN/m, $Q = 3000$, $\omega_r = 1.4$ kHz), a force sensitivity of 0.5 fN at room temperature was obtained, to be compared with the sensitivity of at best 1 pN reported in Ref. [4] for the dc measurement of the nonmagnetic Casimir effect. Prospects for further improvement in the force sensitivity in magnetic resonant force microscopy up to $\approx 3 \times 10^{-17}$ N/$\sqrt{\mathrm{Hz}}$ at room temperature and $\approx 4 \times 10^{-18}$ N/$\sqrt{\mathrm{Hz}}$ at 4.2 K are discussed in Ref. [12]. Indeed, force sensitivity in the attoNewton (10^{-18} N) range has recently been demonstrated [13].

The cantilever used in Ref. [11] consisted of a 50-μm-long, 5-μm-wide, and 90-nm-thick Si beam terminated by a square paddle of 30 μm side length. Such a cantilever would be appropriate for the proposed experiment. By depositing a droplet of polymer on the paddle, it would be possible to produce a lens-shaped substrate of suitable curvature radius ($\approx$100 μm), which could then be covered by evaporation with a thin ($\approx$10 nm) layer of a soft ferromagnet such as permalloy. Great care has to be taken to minimize the parasitic magnetostatic interaction: the hard magnetic plate should be as wide, thin, and magnetically uniform as possible. One should therefore choose a material with a high coercivity and 100% remanence. A thin ($\approx$10 nm) layer of CoPt alloy or a Co/Pt multilayer would be suitable. For a plate radius of 1 cm, the parasitic magnetostatic force can be estimated to be below 1 attoNewton, which is sufficient for the present purpose. Finally, the parasitic force due to the interaction of the soft ferromagnet with the ac field would yield a signal at 2ω and would therefore be filtered out by lock-in detection.

To conclude, the above discussion suggests that the experimental test of the novel Casimir magnetic interaction would indeed be possible by using currently available techniques.

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