Computational Capacity of the Universe

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All physical systems register and process information. The laws of physics determine the amount of information that a physical system can register (number of bits) and the number of elementary logic operations that a system can perform (number of ops). The Universe is a physical system. The amount of information that the Universe can register and the number of elementary operations that it can have performed over its history are calculated. The Universe can have performed 10^{120} ops on 10^{90} bits (10^{120} bits including gravitational degrees of freedom).

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A recent paper by the author [1] put bounds on the amount of information processing that can be performed by physical systems. In particular, the Margolus-Levitin theorem [2] implies that the number of elementary logical operations per second that a physical system can perform is limited by the system's energy, and the amount of information that the system can register is limited by its maximum entropy [1,3]. As shown in [1], these bounds are actually attained by existing quantum computers [4–7]. The Universe is a physical system. This Letter applies these bounds to quantify the amount of information processing that can have been performed by the Universe as a whole since the big bang. In particular, the Universe can be shown to have the capacity to perform a maximum of $(t/t_P)^2 \approx 10^{120}$ elementary quantum logic operations on $(t/t_P)^{3/4} \approx 10^{90}$ bits registered in quantum fields [with a potential for $(t/t_P)^2 \approx 10^{120}$ bits if gravitational degrees of freedom are taken into account]. Here, $t \approx 10^{10}$ yr is the age of the Universe and $t_P = \sqrt{G\hbar/c^5} = 5.391 \times 10^{-44}$ sec is the Planck time—the time scale at which gravitational effects are of the same order as quantum effects. If the Universe is closed, then these numbers represent the amount of elementary logic operations (ops) and bits available in the entire Universe, so that the total number of ops that can be performed over the entire lifetime T of a closed Universe is $\approx (T/t_P)^2$. If the Universe is open, and infinite in extent, then these numbers give the amount of computation that can have been performed within the part of the Universe with which we are causally connected, i.e., the part within the horizon.

These numbers of ops and bits can be interpreted in three distinct ways: (i) They give upper bounds to the amount of computation that can have been performed by all the matter in the Universe since the Universe began. (ii) They give lower bounds to the number of ops and bits required to simulate the entire universe on a quantum computer [8–10]. (iii) If one chooses to regard the Universe as performing a computation, these numbers give the numbers of ops and bits in that computation.

To calculate the number of bits available and the number of ops that can have been performed over the history of the Universe requires a model of that history. The standard big-bang model will be used here [11]. In this model, the Universe began $\approx 10^{10}$ yr ago in what resembled a large explosion (the big bang). Since the big bang, the Universe has expanded to its current size. The well-established inflationary scenario will be used to investigate computation in the first fraction of a second [12]. For the sake of compactness, the effects of possible extra dimensions, pre-big-bang physics, sub-Planck scale physics, etc., will not be considered here. In other words, we will content ourselves with evaluating the number of ops and number of bits available in the part of the Universe that is accessible by observation (the part within the horizon) and for which well-established physical models exist. The techniques of Ref. [1] could also be used to calculate computational capacities in more speculative models, as long as those models obey the laws of quantum mechanics.

Now let us calculate the computational capacity of the Universe. For clarity of exposition, the calculation will not explicitly keep track of factors of $1/2, \pi$, etc., that only affect the final results by an order of magnitude or so [i.e., a factor of 2π will be ignored; a factor of $(2\pi)^6$ will not be]. The expression $\approx$ will be used to indicate equality to within such factors. In other words, $X \approx Y$ is equivalent to $\log X = \log Y + O(1)$.

For most of its history, the Universe has been matter dominated—most of the energy is in the form of matter. As will be seen below, most of the computation that can have taken place in the Universe occurred during the matter-dominated phase. Accordingly, begin with the computational capacity of the matter-dominated Universe, and then work back through the radiation-dominated and inflationary universes.

First, investigate the number of elementary logic operations that can have been performed. The maximum number of operations per second that can be performed by a physical system is proportional to its energy [1,2]. This result follows from the Margolus-Levitin theorem, which states that the minimum time required for a physical system to move from one state to an orthogonal state is given by $\Delta t = \pi\hbar/2E$, where E is the average energy of the system above its ground state [2]. Since quantum logic operations involve flipping bits and moving from one state to an orthogonal state, the Margolus-Levitin theorem also gives the limit on how fast one can perform a quantum logic operation given energy E. Note that while energy must be invested in the spin-field interaction to flip the bit, it need not be dissipated [1–3]. The Margolus-Levitin bound also holds for performing many logic operations in parallel. If energy E is divided up among N quantum logic gates, each gate operates N times more slowly than a single logic gate operating with energy E, but the maximum total number of operations per second remains the same.

Now apply these results to the Universe as a whole. In the matter-dominated Universe, the energy within a comoving volume is approximately equal to the energy of the matter within that volume and remains approximately constant over time. (A comoving volume is one that is at rest with respect to the microwave background and one that expands as the Universe expands.) Since the energy remains constant, the number of ops per sec that can be performed by the matter in a comoving volume remains constant as well. The total volume of the Universe within the particle horizon is $\approx c^3t^3$, where t is the age of the Universe. The particle horizon is the boundary between the part of the Universe about which we could have obtained information over the course of the history of the Universe and the part about which we could not. In fact, the horizon is currently somewhat further than ct away, due to the ongoing expansion of the Universe, but in keeping with the approximation convention adopted above we will ignore this factor along with additional geometric factors in estimating the current volume of the Universe.

The total number of ops per sec that can be performed in the matter-dominated Universe is therefore $\approx \rho c^2 \times c^3t^3/\hbar$, where ρ is the density of matter and ρc^2 is the energy density per unit volume. Since the number of ops per sec in a comoving volume is constant, and since the Universe has been matter dominated for most of its history, we have

$$\text{No. ops} \approx \rho c^5 t^4/\hbar. \qquad (1)$$

Insertion of current estimates for the density of the Universe $\rho \approx 10^{-27}$ kg/m^3 and the age of the Universe $t \approx 10^{10}$ yr, we see that the Universe could have performed $\approx 10^{120}$ ops in the course of its history. (Including all the factors of 2π, etc., in fact yields a slightly larger number of ops: $10^{123} \approx 10^{120+O(1)}$.)

A more revealing form for the number of ops can be obtained by noting that our Universe is close to its critical density. If the density of the Universe is greater than the critical density, it will expand to a maximum size and then contract. If the density is less than or equal to the critical density, it will expand forever. Because of the expansion of the Universe, a galaxy at distance R is moving away with velocity HR, where H is the Hubble constant. At the critical density, the kinetic energy $mH^2R^2/2$ of this galaxy is equal to its gravitational energy $4\pi Gm\rho_c R^3/3R$, so that $\rho_c = 3H^2/8\pi G \approx 1/Gt^2$. So for a matter-dominated Universe at its critical density, constant, the total number of ops that can have been performed within the horizon at time t is

$$\text{No. ops} \approx \rho_c c^5 t^4/\hbar \approx t^2c^5/G\hbar = (t/t_P)^2. \qquad (2)$$

A matter-dominated Universe whose density is higher than the critical density is closed [11]: it is spatially finite, expanding to a maximum length scale $a_{\max}$ over a time $T = \pi a_{\max}/2c$, and temporally finite, recontracting to a singularity over a time $2T$. The energy available for computation is $Mc^2 = 3\pi c^4 a_{\max}/4G$. As a result, the total number of ops that can be performed over the entire history of a closed, matter-dominated Universe is

$$2TMc^2/\hbar = (3\pi^2/4)(a_{\max}/\ell_P)^2 = 3(T/t_p)^2,$$

where $\ell_p = t_p c = 1.616 \times 10^{-35}$ m is the Planck length.

It is instructive to compare the total number of operations that could have been performed using all the matter in the Universe with the number of operations that have been performed by conventional computers. The actual number of elementary operations performed by all human-made computers is of course much less than this number. Because of Moore's law, about half of these elementary operations have been performed in the last two years. Let us estimate the total number of operations performed by human-made computers, erring on the high side. With $\approx 10^9$ computers operating at a clock rate of $\approx 10^9$ Hz performing $\approx 10^5$ elementary logical operations per clock cycle over the course of $\approx 10^8$ sec, all the human-made computers in the world have performed no more than $\approx 10^{31}$ ops over the last two years, and no more than approximately twice this amount in the history of computation.

What is the Universe computing? In the current matter-dominated Universe most of the known energy is locked up in the mass of baryons. If one chooses to regard the Universe as performing a computation, most of the elementary operations in that computation consist of protons, neutrons (and their constituent quarks and gluons), electrons, and photons moving from place to place and interacting with each other according to the basic laws of physics. In other words, to the extent that most of the Universe is performing a computation, it is "computing" its own dynamical evolution [13]. Only a small fraction of the Universe is performing conventional digital computations.

Now calculate the number of bits that can be registered by the Universe. The amount of information, measured in bits, that can be registered by any physical system is equal to the logarithm to the base 2 of the number of distinct quantum states available to the system given its overall energy, volume, electric charge, etc. [2]. In other words, $I = S/k_B \ln 2$, where S is the maximum entropy of the system and $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.

To calculate the number of bits that can be registered by the Universe requires a calculation of its maximum entropy, a calculation familiar in cosmology. The maximum entropy in the matter-dominated Universe would be obtained by converting all the matter into radiation. (Luckily for us, we are not at maximum entropy yet.) The energy per unit volume is ρc^2. The conventional equation for blackbody radiation can then be used to estimate the temperature T that would be obtained if that matter were converted to radiation at temperature T: $\rho c^2 = (\pi^2/30\hbar^3 c^3)(k_B T)^4 \sum_\ell n_\ell$. Here ℓ labels the species of effectively massless particles at temperature T (i.e., $m_\ell c^2 \ll k_B T$), and n_ℓ counts the number of effective degrees of freedom per species: $n_\ell =$ (number of polarizations) $\times$ (number of particles/antiparticles) $\times$ 1 (for bosons) or 7/8 (for fermions). Solving for the temperature for the maximum entropy state gives $k_B T = (30\hbar^3 c^5 \rho/\pi^2 \sum_\ell n_\ell)^{1/4}$. The maximum entropy per unit volume is $S/V = 4\rho c^2/3T$. The entropy within a volume V is then $S = (4k_B/3)(\pi^2 \sum_\ell n_\ell/30)^{1/4}(\rho c/\hbar)^{3/4} V^{1/4}$. The entropy depends only weakly on the number of effectively massless particles.

Using the formula $I = S/k_B \ln 2$ and substituting $V \approx c^3 t^3$ for the volume of the Universe gives the maximum number of bits available for computation:

$$I \approx (\rho c^5 t^4/\hbar)^{3/4} = (\text{No. ops})^{3/4}. \quad (3)$$

The Universe could currently register $\approx 10^{90}$ bits. To register this amount of information requires every degree of freedom of every particle in the Universe.

The above calculation estimated only the number of ops that could be performed and the amount of information that could be stored by matter and energy and did not take into account information that might be stored and processed on gravitational degrees of freedom. The energy available in the gravitational field is equal in magnitude and opposite in sign to the energy in the matter fields [11]. Applying the Margolus-Levitin theorem to the number of ops that can have been performed by this energy yields the same number of ops that can be performed by the matter fields. Similarly, the Bekenstein bound [14] together with the holographic principle [15–18] implies that the maximum amount of information that can be registered by any physical system, including gravitational ones, is equal to the area of the system divided by the the square of the Planck length, $\ell_P^2 = \hbar G/c^3$. This limit is, in fact, attained by black holes and other objects with event horizons. Applying the Bekenstein bound and the holographic principle to the Universe as a whole implies that the maximum number of bits that could be registered by the Universe using matter, energy, and gravity is $\approx c^2 t^2/\ell_P^2 = t^2/t_P^2$. That is, the maximum number of bits using gravitational degrees of freedom as well as conventional matter and energy is equal to the maximum number of elementary operations that could be performed in the Universe, $\approx 10^{120}$.

Not surprisingly, existing human-made computers register far fewer bits. Overestimating the number of bits registered in 2002, as above for the number of ops, yields $\approx 10^9$ computers, each registering at $\approx 10^{12}$ bits, for a total of $\approx 10^{21}$ bits.

Using the same methods, one can calculate the number of ops and the number of bits available in the radiation-dominated and inflationary universes. Despite the radically different forms of matter, formulas (2) and (3) can be shown to hold for the radiation-dominated Universe, and formula (2) holds for the inflationary Universe. The number of available bits in the inflationary Universe is dominated by the number of bits in the horizon radiation and is equal to $\approx (t/t_P)^2$. Of course, the character of the "computation" in these universes is very different from the matter-dominated Universe. In particular, the matter in the radiation-dominated Universe is primarily at thermal equilibrium, making it a hostile environment for complex processes such as life. The inflationary Universe is divided into causally noncommunicating sectors: the primary computational process in the inflationary Universe is "bit creation"—the production of large quantities of spatial volume and of free energy that will later on be used for more complicated computation in the matter-dominated Universe [16–18]. A more full account of the details of information processing in the radiation-dominated and inflationary universes can be found in [19].

Before concluding, it is worth noting that the number of bits and the number of operations possible in the Universe are related to the Eddington-Dirac large number hypothesis. Three quarters of a century ago, Eddington noted that two large numbers that characterize our Universe happen to be approximately equal [20]. In particular, the ratio between the electromagnetic force by which a proton attracts an electron and the gravitational force by which a proton attracts an electron is $\alpha = e^2/Gm_e m_p \approx 10^{40}$. Similarly, the ratio between the size of the Universe and the classical size of an electron is $\beta = ct/(e^2/m_e c^2) \approx 10^{40}$. The fact that these two numbers are approximately equal is currently regarded by most researchers as a coincidence. (A third large number, the square root of the number of baryons in the Universe, $\gamma = \sqrt{\rho c^3 t^3/m_p}$, is also $\approx 10^{40}$. This is not a coincidence given the values of α and β: $\alpha\beta \approx \gamma^2$ in a Universe near its critical density $\rho_c \approx 1/Gt^2$.)

The astute reader may have noted that the number of operations that can have been performed by the Universe is approximately equal to the Eddington-Dirac large number cubed. In fact, the number of ops is necessarily approximately equal to $\beta\gamma^2 \approx \alpha\beta^2 \approx 10^{120}$. This relation holds true whether or not $\alpha \approx \beta \approx \gamma$ is a coincidence. In particular,

$$\beta\gamma^2 = (\rho c^5 t^4/\hbar)(\hbar c/e^2)(m_e/m_p) = \text{No. ops} \times (137/1836)\,. \quad (4a)$$

Similarly,

$$\alpha\beta^2 = (t/t^P)^2(\hbar c/e^2)(m_e/m_p) = \text{No. ops} \times (137/1836)\,. \quad (4b)$$

That is, the number of ops differs from the Eddington-Dirac large number cubed by a factor of the fine structure constant times the proton-electron mass ratio. Since the number of ops is $\approx 10^{120}$, as shown above, and the fine-structure constant times the proton-electron mass ratio is ≈ 10, the number of ops is a factor of 10 larger than the Eddington-Dirac large number cubed. In other words, whether or not the approximate equality embodied by the Eddington-Dirac large number is a coincidence, the fact that the number of operations that can have been performed by the Universe is related to this large number is not.

The above sections calculated how many elementary logical operations that can have been performed on how many bits during various phases of the history of the Universe. As noted above, there are three distinct interpretations of the numbers calculated. The first interpretation simply states that the number of ops and number of bits given here are upper bounds on the amount of computation that can have been performed since the Universe began. This interpretation should be uncontroversial: existing computers have clearly performed far fewer ops on far fewer bits [21].

The second interpretation notes that the numbers calculated give a lower bound on the number of bits and the number of operations that must be performed by a quantum computer that performs a direct simulation of the Universe. This interpretation should also be uncontroversial: quantum computers can accurately simulate any physical system that evolves according to local interactions, using the same amounts of energy and Hilbert space volume as the system itself [8–10]. It is an open question as to how to simulate quantum gravity, but string theory and M theory provide potential theories of quantum gravity [22], and these theories should also be accessible to efficient simulation on a quantum computer. If so, then quantum computation might provide an alternative formulation for a "theory of everything."

The third interpretation—that the numbers of bits and ops calculated here represent the actual memory capacity and number of elementary quantum logic operations performed by the Universe—is more controversial. That the Universe registers an amount of information equal to the logarithm of its number of accessible states seems reasonable. And virtually all physical interactions can operate as quantum logic gates. But whether or not it makes sense to identify an elementary quantum logic operation with the local evolution of information-carrying degrees of freedom by an average angle of $\pi/2$ is a question whose answer must await further developments in the relationship between physics and computation.

The amount of information the Universe can register and the amount of information processing it can perform can be calculated using the physics of information processing. To date, the Universe can have performed 10^{120} ops on 10^{90} bits (10^{120} bits if quantum gravity is taken into account), enough to factor a million-bit number using the classical number field sieve algorithm, and enough to factor a 10^{60} bit number using Shor's quantum algorithm. Is the Universe a computer? It is certainly *not* a digital computer running Linux or Windows. But the Universe certainly does represent and process quantifiable amounts of information in a systematic fashion.

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