Photon Propagator, Monopoles, and the Thermal Phase Transition in Three Dimensional Compact QED

M. N. Chernodub
*ITEP, B. Cheremushkinskaja 25, Moscow, 117259, Russia
and Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan*

E.-M. Ilgenfritz
Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan

A. Schiller
Institut für Theoretische Physik and NTZ, Universität Leipzig, D-04109 Leipzig, Germany
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We investigate the gauge boson propagator in the three dimensional compact Abelian gauge model in the Landau gauge at finite temperature. The presence of the monopole plasma in the confinement phase leads to the appearance of an anomalous dimension in the momentum dependence of the propagator. The anomalous dimension as well as an appropriate ratio of photon wave function renormalization constants with and without monopoles is observed to be an order parameter for the deconfinement phase transition. We discuss the relation between our results and the confining properties of the gluon propagator in non-Abelian gauge theories.

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Three dimensional compact electrodynamics ($cQED_3$) shares two outstanding features of QCD, confinement [1] and chiral symmetry breaking [2]. With some care, it might be helpful for the understanding of certain nonperturbative aspects of QCD to study them within $cQED_3$. The nonperturbative properties of $cQED_3$ deserve interest by themselves because this model was shown to describe some features of Josephson junctions [3] and high-T_c superconductors [4].

Here, we want to elaborate on $cQED_3$ as a toy model of *confinement*. Indeed, this has been the first nontrivial case in which confinement of electrically charged particles was understood analytically [1]. Confinement is caused here by a plasma of monopoles which emerge due to the compactness of the gauge field. Other common features of the two theories are the existence of a mass gap and of a confinement-deconfinement phase transition at some nonzero temperature. According to universality arguments [5] the phase transition of $cQED_3$ is expected to be of the Kosterlitz-Thouless type [6].

In QCD_4, the deconfinement phase transition is widely believed to be caused by loss of monopole *condensation* (for a review, see Ref. [7]) within the effective dual superconductor approach [8]. Studying the dynamics of the monopole current inside gluodynamics, monopole decondensation at the critical temperature is appearing as *depercolation,* i.e., the decay of the infrared, percolating monopole cluster into short monopole loops [9]. This change of vacuum structure has a *dimensionally reduced* analog in the 3D monopole-antimonopole pair binding which has been observed in $cQED_3$ [10,11].

At present, the gluon propagator in QCD_4 is under intensive study. The analogies mentioned before encouraged us to study the similarities between the gauge boson propagators in both theories. In order to fix the role of the monopole plasma in $cQED_3$, not just for confinement of external charges but also for the nonperturbative modification of the gauge boson propagator, we consider it in the confinement and the deconfined phases. On the other hand, on the lattice at any temperature we are able to separate the monopole contribution to the propagator by means of Eq. (2) below.

We have chosen the Landau gauge since it has been adopted in most of the investigations of the gauge boson propagators in QCD [12,13] and QED [14,15]. In order to avoid the problem of Gribov copies [16], the alternative Laplacian gauge has been used recently [17]. The Coulomb gauge, augmented by a suitable global gauge in each time slice (minimal Coulomb gauge) has been advocated both analytically [18] and numerically [19].

The numerical lattice results for gluodynamics show that the propagator for all these gauges in momentum space is less singular than p^{-2} in the immediate vicinity of $p^2 = 0$. Moreover, the results for the propagator at zero momentum are ranging from a finite [17] (Laplacian gauge) to a strictly vanishing [16,18,19] (Coulomb gauge) value. Recent investigations in the Landau gauge show that, besides the suppression at $p \to 0$, the propagator is enhanced at intermediate momenta which can be characterized by an anomalous dimension [12] (see the last reference in [12] for a comparison of different model functions).

In the present Letter we demonstrate that the momentum behavior of the photon propagator in QED_3 is also described by a Debye mass and by an anomalous dimension which both vanish at the deconfinement transition. This mechanism can be clearly attributed to magnetic monopoles. The plasma contribution is relatively easy to exhibit by explicit calculation and can be eliminated by *monopole subtraction* on the level of the gauge fields. The results of a study of the propagator in SU(2) gluodynamics have been interpreted [13] in a similar spirit, where P vortices appearing in the maximal center gauge were shown to be essential for the enhancement of the Landau gauge propagator at intermediate momenta.

For our lattice study we have adopted the Wilson action, $S[\theta] = \beta \sum_p (1 - \cos\theta_p)$, where θ_p is the U(1) field strength tensor represented by the plaquette curl of the compact link field θ_l, and β is the lattice coupling constant related to the lattice spacing a and the continuum coupling constant g_3 of the 3D theory, $\beta = 1/(ag_3^2)$. We focus here on the difference between the confined and the deconfined phase. All results presented have been obtained on lattices of size $32^2 \times 8$. The finite temperature phase transition is known to take place [11,20] at $\beta_c \approx 2.35$.

The Landau gauge fixing is defined by maximizing the functional $\sum_l \cos\theta_l^G$ over all gauge transformations G. For details of the Monte Carlo algorithm, we refer to [11]. A more complete presentation of our studies, including also a thorough analysis of the propagator in the zero temperature case, is in preparation [21]. Details on the implementation of Landau gauge fixing, including the elimination of zero momentum modes and the careful control of double Dirac strings, can be found in Refs. [15,21].

We study the gauge boson propagator, $\langle \theta_\mu(x)\theta_\nu(0)\rangle$, in the momentum space. The propagator is a function of the lattice momentum, $p_\mu = 2\sin(\pi k_\mu/L_\mu)$, where $k_\mu = 0, \ldots, L_\mu/2$ is an integer. We discuss here the finite temperature case and focus on the temporal component of the propagator,

$$D_{33}(\mathbf{p}^2, 0) = \frac{1}{L_x L_y L_z} \langle \theta_3(\mathbf{p}, 0)\theta_3(-\mathbf{p}, 0)\rangle, \qquad (1)$$

as a function of the spatial momentum, $\mathbf{p}^2 = \sum_{\mu=1}^{2} p_\mu^2$. We recall that at finite temperature the confining properties of static electrically charged particles are encoded in the temporal component of the gauge boson field, θ_3.

In order to pin down the effect of monopoles we have divided the gauge field θ_l into a regular (photon) and a singular (monopole) part which can be done following Ref. [22]. In the notation of lattice forms this is written

$$\theta = \theta^{\mathrm{phot}} + \theta^{\mathrm{mon}}, \qquad \theta^{\mathrm{mon}} = 2\pi\Delta^{-1}\delta p[j], \qquad (2)$$

where Δ^{-1} is the inverse lattice Laplacian and the 0-form ${}^*j \in \mathbb{Z}$ is nonvanishing on the sites of the dual lattice occupied by the monopoles. The 1-form ${}^*p[j]$ corresponds to the Dirac strings (living on the links of the dual lattice) which connect monopoles with antimonopoles, $\delta^* p[j] = {}^*j$. For a Monte Carlo configuration, we have fixed the gauge, then located the Dirac strings, $p[j] \neq 0$, and constructed the monopole part θ^{mon} of the gauge field according to the last equation in (2). The photon field is just the complement to the monopole part according to the first equation of (2).

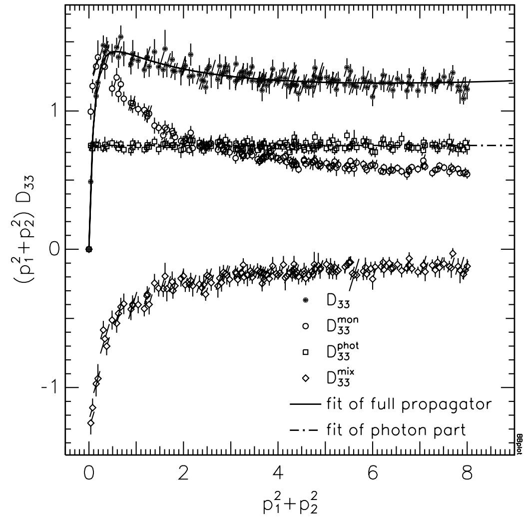

FIG. 1. Different contributions to the full D_{33} propagator (multiplied by $\mathbf{p}^2$) vs spatial lattice momentum squared and fits as described in the text for $\beta = 1.8$ on a $32^2 \times 8$ lattice.

The photon and monopole parts of the gauge field contribute to the propagator, $D = D^{\mathrm{phot}} + D^{\mathrm{mon}} + D^{\mathrm{mix}}$, where D^{mix} represents the mixed contribution from regular *and* singular fields. We show the propagator for $p = (\mathbf{p}, 0)$ together with the separate contributions, multiplied by $\mathbf{p}^2$ and averaged over the same $\mathbf{p}^2$ values, in Fig. 1 for coupling constant $\beta = 1.8$.

The regular part of the propagator has perfectly the free field form

$$D_{33}^{\mathrm{phot}} = \frac{1}{\beta} \frac{Z^{\mathrm{phot}}}{\mathbf{p}^2}, \qquad (3)$$

at all available β. The perturbative propagator defined in terms of θ_l is obviously proportional to g_3^2, which is taken into account by the factor $1/\beta$ in Eq. (3). The fits of the photon part of the propagator by the above expression give the parameter Z^{phot} as a function of lattice coupling (dash-dotted line in Fig. 1 for $\beta = 1.8$).

The singular contribution to the gauge boson propagator shows a maximum in $\mathbf{p}^2 D_{33}^{\mathrm{mon}}$ at some momentum (Fig. 1), moving with increasing β nearer to $|\mathbf{p}|a = 0$. The mixed

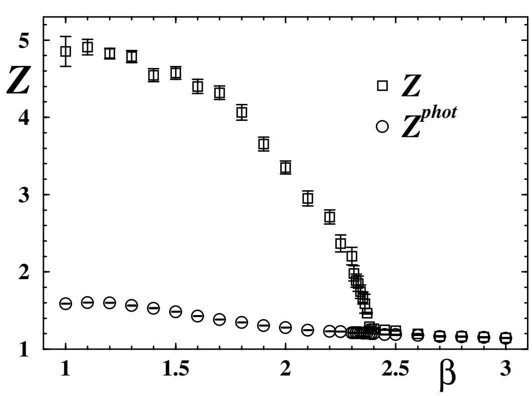

FIG. 2. Coefficients Z of fit (4) for full propagator and Z^{phot} for photon contribution (3) vs β.

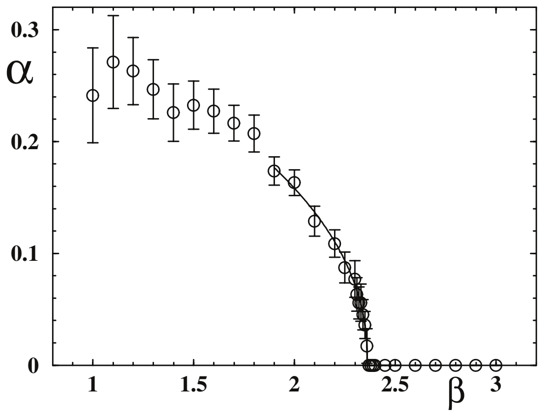

FIG. 3. Anomalous dimension α vs β and its best fit near β_c using function (6).

component gives a negative contribution to $\mathbf{p}^2 D_{33}^{\mathrm{mix}}$, growing with decreasing momentum. The central point of our Letter is that all these contributions together do *not* sum up to a simple massive Yukawa propagator. To quantify the difference between a Yukawa-type propagator and the actual behavior we use the the following four-parameter model function for $D_{33}(\mathbf{p}^2, 0)$:

$$D_{33}(\mathbf{p}^2, 0) = \frac{Z}{\beta} \frac{m^{2\alpha}}{\mathbf{p}^{2(1+\alpha)} + m^{2(1+\alpha)}} + C\,, \qquad (4)$$

where Z, α, m, and C are the fitting parameters. This model is similar to some of Refs. [12,23] where the propagator in gluodynamics has been studied.

The first part of the function (4) implies that the photon acquires a Debye mass m (due to screening [1]) together with the anomalous dimension α. The (squared) photon wave function renormalization constant Z describes the renormalization of the photon wave function due to quantum corrections. The second part of (4) represents a δ-function-like interaction in coordinate space.

Before fitting we average the propagator over all lattice momenta at the same $\mathbf{p}^2$ to improve rotational invariance. Thus the errors entering the fits include both the variance among the averages for individual momenta and the individual errors. The fits were performed using standard MATHEMATICA packages combined with a search for the global minimum in χ^2/d.o.f. To check the stability of the fits, we studied several possibilities of averaging and thinning out the data sets, a procedure which will be discussed elsewhere [21].

The model function (4) works perfectly for all $\mathbf{p}^2$ and couplings β. For $\beta \geq 2.37$ the best fit for mass parameter m and anomalous dimension α are both consistent with zero. Therefore we set $m = 0$ and $\alpha = 0$ for these values of β to improve the quality of the fit of Z and C.

It turns out that the inclusion of a constant term, C, in the model function (4) is crucial for obtaining good fits in the confinement phase, despite the fact that it is very small [as a function of β the parameter C decreases from $C(1.0) = 0.18(4)$ to $C(2.2) = 0.009(2)$; it rapidly vanishes in the deconfined phase]. Similarly to m and

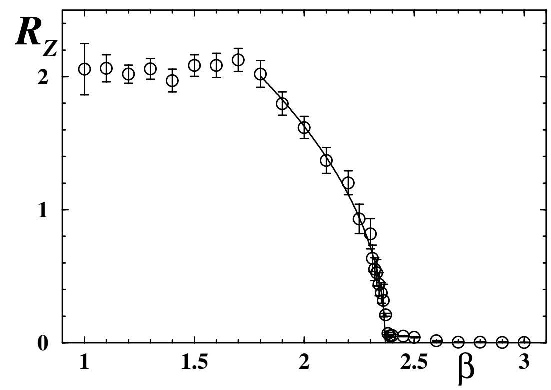

FIG. 4. Same as in Fig. 3 for ratio R_Z, Eq. (5).

α parameters we set C to zero for $\beta \geq 2.45$, where C becomes smaller than 10^{-4}.

An example of the best fit of the full propagator for $\beta = 1.8$ is shown in Fig. 1 by the solid line [with $C = 0.033(5)$]. The parameter Z distinguishes clearly between the two phases (Fig. 2). It coincides with the photon part Z^{phot} (defined without monopoles) in the deconfined phase while it is much larger in the confined phase. This indicates that the photon wave function gets strongly renormalized by the monopole plasma. In contrast, the factor Z^{phot} smoothly changes crossing the deconfinement transition at $\beta_c \approx 2.35$.

The anomalous dimension α also distinguishes the two phases (Fig. 3): it is equal to zero in the deconfinement phase (perturbative behavior) while in the confinement phase the monopole plasma causes the anomalous dimension growing to $\alpha \approx 0.25-0.3$.

To characterize the properties of Z and α approaching the phase transition we fit the excess of the ratio of Z's over unity,

$$R_Z(\beta) = \frac{Z(\beta)}{Z^{\mathrm{phot}}(\beta)} - 1\,, \qquad (5)$$

and the anomalous dimension α in the following form:

$$f_i(\beta) = h_i(\beta_c^{(i)} - \beta)^{\gamma_i}, \qquad \beta < \beta_c^{(i)}, \qquad (i = \alpha, Z), \qquad (6)$$

where $i = Z, \alpha$. The $\beta_c^{(\alpha,Z)}$ are the pseudocritical couplings which might differ on finite lattices.

The best fits f_α and f_Z are shown in Figs. 3 and 4, respectively. The solid lines in both plots extend over the fitting region. The corresponding parameters are presented in Table I. The pseudocritical couplings $\beta_c^{(\alpha)}$ and $\beta_c^{(Z)}$ are in agreement with previous numerical studies [11,20] giving $\beta_c = 2.346(2)$. Note that the critical exponents γ_i are close to $1/2$, both for the anomalous dimension α and for R_Z expressing the ratio of photon field renormalization constants.

TABLE I. Best parameters for the fits (6).

i	h_i	$\beta_c^{(i)}$	γ_i
α	0.250(9)	2.363(3)	0.50(2)
Z	2.63(7)	2.368(5)	0.48(3)

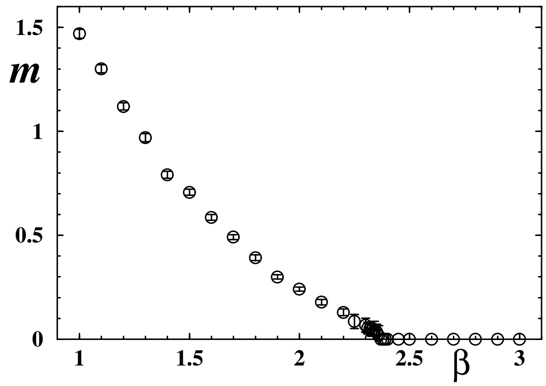

FIG. 5. The mass m vs β.

Finally, the β dependence of the mass parameter, m, is presented in Fig. 5. As expected, the mass scale generated is nonvanishing in the confinement phase due to the presence of the monopole plasma [1]. It vanishes at the deconfinement transition point when the very dilute remaining monopoles and antimonopoles form dipoles [11].

Summarizing, we have shown that the presence of the monopole plasma leads to the appearance of a nonvanishing anomalous dimension $\alpha > 0$ in the boson propagator of $cQED_3$ in the confinement phase. We hope that our observation stimulates an analytical explanation.

At this stage of studying $cQED_3$ as a model of confinement we conjecture that in the case of QCD the Abelian monopoles defined within the Abelian projection may be responsible for the anomalous dimension of the gluon propagator observed in Refs. [12,23]. If true, a monopole subtraction procedure analogous to that employed here would be able to demonstrate this. We found that the anomalous dimension α and the ratio of the photon wave function renormalization constants with and without monopoles, R_Z (5), represent alternative, also nonlocal order parameters characterizing the confinement phase.

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