Noise Activated Nonlinear Dynamic Sensors

L. Gammaitoni*

Dipartimento di Fisica, Universitá di Perugia, Perugia, Italy
and Instituto Nazionale di Fisica Nucleare, Sezione di Perugia, Perugia, Italy
and Istituto Nazionale di Fisica della Materia, Sezione di Perugia, I-06100 Perugia, Italy

A. R. Bulsara†

Space and Naval Warfare Systems Center, Code D-363, 49590 Lassing Road, San Diego, California 92152-6147
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We introduce a novel dynamical description for a wide class of nonlinear physical sensors operating in a noisy environment. The presence of unknown physical signals is assessed via the monitoring of the residence times in the metastable attractors of the system. We show that the presence of ambient noise, far from degrading the sensor operation, can actually improve its sensitivity and provide a greatly simplified readout scheme, as well as significantly reduce processing procedures for this new class of devices that we propose to call noise activated nonlinear dynamic sensors. Such devices can also show interesting dynamical features such as the *resonant trapping effect*.

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A large class of dynamic sensors have nonlinear input-output characteristics, often corresponding to a bistable potential energy function that underpins the sensor dynamics. Examples include magnetic field sensors, e.g., the simple fluxgate sensor [1,2], ferroelectric sensors [3], and mechanical sensors [4], e.g., acoustic transducers, made with piezoelectric materials. In many cases, the detection of a small target signal (dc or low frequency) is based on a spectral technique [1]wherein a known periodic bias signal is applied to the sensor to saturate it and drive it very rapidly between its two (locally) stable attractors (corresponding to the minima of the potential energy function, when the attractors are fixed points). Often, the amplitude of the bias signal is taken to be quite large in order to render the response largely independent of the noise. The effect of a target dc signal is, then, to skew the potential, resulting in the appearance of features at even harmonics of the bias frequency ω [5] in the system response. The spectral amplitude at 2ω is, then, proportional to the bias frequency and the square of the target signal amplitude; hence, the spectral amplitude can be used to yield the target signal. In practice, a feedback mechanism is frequently utilized for reading out the asymmetry-producing target signal via a nulling technique [1].

The above readout scheme has some drawbacks. Chief among them is the requirement of large onboard power to provide a high-amplitude, high-frequency bias signal. The feedback electronics can also be cumbersome and introduce their own noise floor into the measurement, and, finally, a high-amplitude, high-frequency bias signal often increases the noise floor in the system. We propose here a description of the system dynamics which makes possible the use of a new measurement technique based on the system output residence times in its steady states. The residence time statistics in a bistable system were proposed for the first time in [6] as a quantifier for the *stochastic resonance* [7] phenomenon. Here we propose to use such statistics in order to gain information on the presence of small-unknown target signals. We start by stressing that, without any external target signal, the two residence times (we will assume a two-state system for the remainder of this Letter) will be, on average, identical: The presence of even a small amount of noise leads to a residence times distribution (RTD) about a mean value, due to uncertainties in the switching time. The presence of an external target signal usually renders the potential asymmetric with a concomitant difference in the mean residence times which, to first order, is proportional to the asymmetry-producing target signal itself. We propose to monitor the difference between the mean residence times of the two states of the system and to use this observable as a quantifier for detecting the presence of the target signal. This procedure has some advantages compared to the standard procedure: It can be implemented experimentally without complicated feedback electronics, with or without the bias signals. In fact, the difference in residence times is quantifiable even in the *absence* of the periodic bias signal, with only noise driving the sensor between its steady states, although practical considerations, e.g., observation times that depend on the relative magnitude of the noise standard deviation and the barrier height, may limit the applicability of this procedure in some practical cases. The residence-times based technique works without the knowledge of the computationally demanding power spectral amplitude of the system output (in most cases a simple averaging procedure on the system output works just fine) and, finally, it performs well in the presence of noise.

In realistic scenarios, the nonlinear sensors are operated in a noisy environment with just a few dynamical quantities that can be adjusted to improve their performance. In order to examine the optimal operating condition(s), we consider the case in which ambient noise characteristics (correlation time and standard deviation) and target signal characteristics *cannot* be adjusted at will, while the periodic bias amplitude A (and frequency) and barrier height b can be chosen with a certain freedom. We present calculations on a simple two-state system (the Schmitt trigger, ST) that serves as a good prototype for more complex bistable sensors and permits the computation of the residence-times statistics, whence some of the ideas outlined above can be quantified. Simulation results using a double well potential system, instead of a Schmitt trigger, are presented in [8,9]. We start with the simplest model of a driven ST with static thresholds located at $\pm b$, an applied controllable sinusoidal bias signal $A(t) = A \sin\omega t$, and a dc target signal ϵ ($\epsilon \ll b$). We assume the background noise $\zeta(t)$ to be Gaussian bandlimited (exponentially correlated) having zero mean, and correlation time τ. The noise $\zeta(t)$ stems from a white-noise driven Ornstein-Uhlenbeck process: $\dot{\zeta} = -\lambda\zeta + \sigma F(t)$, where $\lambda \equiv \tau^{-1}$, and $F(t)$ is white noise having zero mean and unit intensity σ^2. Then, $\zeta(t)$ has correlation function $\langle\zeta(t) \times \zeta(s)\rangle = \sigma_\zeta^2 \exp[-|t - s|/\tau]$ with variance $\sigma_\zeta^2 = \sigma^2\tau/2$.

In Fig. 1 we show digitally simulated data from the ST for the quantity of interest $\langle\Delta T\rangle = |\langle T_+\rangle - \langle T_-\rangle|$ as a function of A and b, $\langle T_+\rangle$ and $\langle T_-\rangle$ being the mean residence times in the upper and the lower state of the ST, respectively. It is interesting to note that $\langle\Delta T\rangle$ increases monotonically with b for a chosen A and becomes larger when A becomes smaller, for a fixed noise intensity. Such a behavior suggests a novel way to operate nonlinear sensors in a noisy environment. We call NANDS (noise activated nonlinear dynamic sensors) the class of bistable sensors that can be operated as dc or low frequency signal detectors. For NANDS, the dynamical observable is the difference between the (mean) residence time in each stable state, when the switches between the states are activated solely by the ambient noise (i.e., $A = 0$). NANDS devices have some clear advantages over usual sensors, e.g., reduced bias amplitude, or even the complete absence of the external periodic bias, and consequent reduction in power requirements. On the other hand, the performances of such devices are conditioned by the statistical properties of the noise. Specifically, in order to obtain a large enough signal-to-noise ratio on $\langle\Delta T\rangle$, it is necessary to consider the statistical distribution of the residence times $\langle T_+\rangle$ and $\langle T_-\rangle$. In the presence of large mean or variance of the residence time observable (low noise level and/or large barrier height), it might be necessary to acquire a long time series before being able to estimate appropriately the observable $\langle\Delta T\rangle$.

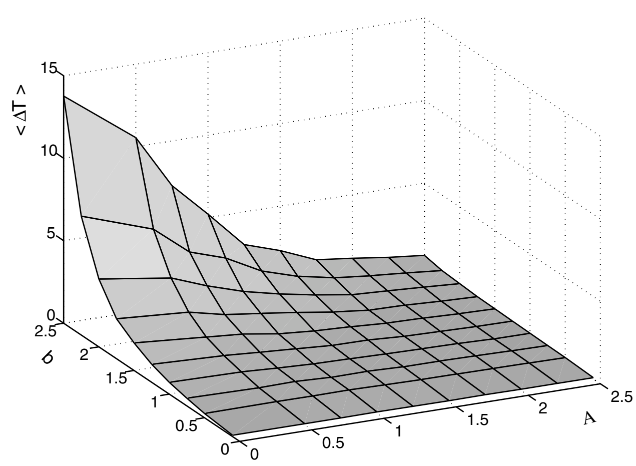

FIG. 1. $\langle\Delta T\rangle$ in arbitrary time units (atu) as a function of A and b for the Schmitt trigger. Other parameter values are $\sigma_\zeta = 1.5$, $\tau = 1$ atu, $\omega = 1$ atu^{-1}, and $\epsilon = 0.1$.

In order to reach a quantitative understanding of the behavior of $\langle\Delta T\rangle$, we start by considering a purely deterministic crossing event ($A > b + \epsilon$ and $\sigma_\zeta = 0$). One readily obtains $t_1 = \omega^{-1}\sin^{-1}(\frac{b-\epsilon}{A})$ and $t_2 = \omega^{-1}[\sin^{-1}(\frac{b+\epsilon}{A}) + \pi]$, for the times of first crossing of the upper and lower thresholds, respectively (assuming that we start at the origin). The residence times in the two states (assuming instantaneous switches) are $T_+ = t_2 - t_1$ and $T_- = t_1 + \frac{2\pi}{\omega} - t_2$ whence we obtain for the difference in residence times $\Delta T = T_+ - T_-$:

$$\Delta T = 2\omega^{-1}\left[\sin^{-1}\left(\frac{b+\epsilon}{A}\right) - \sin^{-1}\left(\frac{b-\epsilon}{A}\right)\right]. \quad (1)$$

Defining a "sensitivity" via $S(\epsilon) = \frac{d\Delta T}{d\epsilon}$, we obtain

$$S = \frac{2}{\omega A}\left\{\left[1 - \left(\frac{b+\epsilon}{A}\right)^2\right]^{-1/2} + \left[1 - \left(\frac{b-\epsilon}{A}\right)^2\right]^{-1/2}\right\}, \quad (2)$$

which clearly increases with ϵ, saturating at $\bar{\epsilon} = A - b$. One may show that other (nonsinusoidal) bias waveforms lead to enhanced sensitivity. For instance, a waveform comprising a superposition of a square and a triangular wave has been studied. The resultant (deterministic) sensitivity is independent of ϵ, and far better than the sinusoidal bias case [10].

To understand the role of the noise, we studied the normalized RTD by measuring the residence times in both states of the ST (i.e., we compute the distribution of residence times regardless of the residence state). In Fig. 2, the RTD thus obtained for various noise intensities ($A = 1.5b$ and a dc target signal $\epsilon = 0.2b$) is presented. We note the following.

1. For small noise ($\sigma_\zeta \ll A - b$), the RTD presents two well-separated almost-symmetric peaks centered about the mean values $\langle T_{+,-}\rangle$.

2. As long as the noise stays small ($\sigma_\zeta < A - b$), the mean values $\langle T_{+,-}\rangle$ are roughly the same as the deterministic values (1) computed above (the larger A, the less they depart from the computed values, see also Fig. 3).

3. In the presence of increasing amounts of noise ($\sigma_\zeta > A - b$), the two peaks of the RTD tend to merge as a consequence of an increasing number of purely noise activated switches between the stable states; simultaneously, the RTDs develop noise-dependent tails.

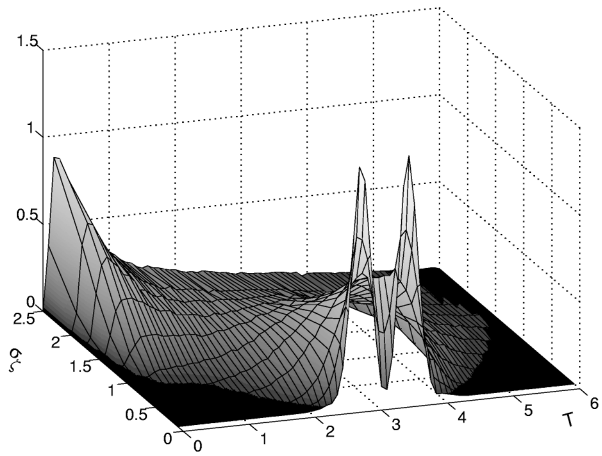

FIG. 2. Residence times distribution for the Schmitt trigger stable states measured for increasing values of the noise standard deviation σ_ζ. Each RTD (for a fixed value of the noise standard deviation) is normalized. Time is measured in atu. Other parameter values are $b = 1$, $A = 1.5$, $\tau = 1$ atu, $\omega = 1$ atu^{-1}, and $\epsilon = 0.2$.

4. For large noise ($\sigma_\zeta \gg A - b$), the switching mechanism is completely dominated by noise. $\langle \Delta T \rangle$ decreases and eventually goes to zero when $\sigma_\zeta \to \infty$.

In Fig. 3 we show in detail the dependence of $\langle \Delta T \rangle$ on σ_ζ for three different values of the bias amplitude A. It is interesting to note that the departure from the deterministic case (zero noise) is, in general, nonmonotonic. We observe

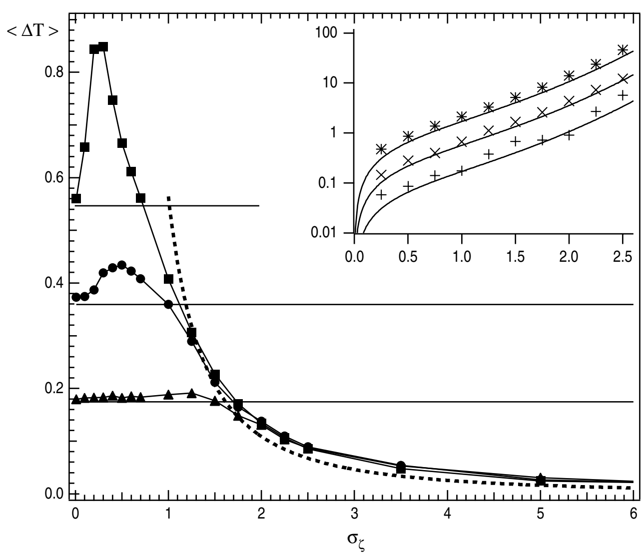

FIG. 3. $\langle \Delta T \rangle$ versus σ_ζ for three different values of the amplitude of the bias signal: $A = 1.25$ (squares), $A = 1.5$ (circles), $A = 2.5$ (triangles). Other parameter values are $b = 1$, $\tau = 1$ atu, $\omega = 1$ atu^{-1}, and $\epsilon = 0.1$. The straight horizontal lines indicate the respective deterministic ΔT values as computed via (1). The dashed curve shows the prediction from (5) for $A = 0$. Inset: $\langle \Delta T \rangle$ *versus* b for three different values of the target signal amplitude: $\epsilon = 0.03$ (+), $\epsilon = 0.1$ ($\times$), $\epsilon = 0.3$ (*), with zero bias signal ($A = 0$) and $\sigma_\zeta = 4$. Continuous lines represent the estimate obtained from (5). Other parameter values as in the main figure.

that a small amount of noise, instead of decreasing $\langle \Delta T \rangle$, makes it rise to a maximum, at values significantly larger than the predicted one for the purely deterministic switching. The amplitude of this effect and the position of the maximum are clearly a function of A. In order to explain such behavior, we recall the notion of *resonant trapping* introduced recently [11]: In a configuration in which the bias signal amplitude A is taken larger than the threshold b, the switch event is ordinarily controlled by the bias signal itself. It can happen, however, that when $A(t)$ approaches the upper level (lower level) the noise $\zeta(t)$ is smaller than $-(A - b)$ [larger than $(A - b)$]. In such cases the switch event gets frustrated, unless $\zeta(t)$ recrosses the boundary $-(A - b)$ [$(A - b)$] prior to the sign reversal of $A(t)$. Such a failure mechanism was first observed [11] in a double well system: There a frustrated switch is attributable to the fact that the escape time out of the unstable well increases with noise intensity and reaches a maximum for an optimal value of σ_ζ, whence the term *resonant trapping.* A detailed description of this effect is given in [11] where dependencies arising from characteristic time scales in the system (signal period, intrawell relaxation time, and the noise correlation time) are discussed. For our purposes, we observe that the positions of the maxima in Fig. 3 are well described by the condition $\sigma_\zeta = A - b$ in agreement with the behavior of the maximum of the trapping probability, as reported in [11]. The maximum in $\langle \Delta T \rangle$ is due to an increase of $\langle T_+ \rangle$ (for $\epsilon > 0$) caused by a failure in the down switch mechanism. The switch is thus postponed for a time $2\pi/\omega$ and the RTD for T_+ develops peaks at $\frac{2n+1}{2}\frac{2\pi}{\omega}$ [10]. It is interesting to observe that failures in the switch mechanism [both in the up ($- \to +$)] and in the down ($+ \to -$) direction] cooperate to increase $\langle \Delta T \rangle$, thus increasing the detectability (see also [12]) for small ϵ, i.e., *the right amount of noise improves detectability.* A detailed description of the beneficial effects of *resonant trapping* to our detection method will be discussed in a forthcoming paper, where we discuss also the effect of the shape of the forcing signal and the interpretation of this effect in terms of *noise-enhanced* sensitivity due to the specific shape of the function $\langle \Delta T \rangle$

The behavior of $\langle \Delta T \rangle$ for large noise intensity is almost independent of A (Fig. 3); in this regime, the response tends to purely noise driven behavior. Using the above dynamics for the state-point $x(t)$, for $A = 0$, we may write down the standard expression [13] for the mean first passage time $\langle T_- \rangle$ to make the transition from the starting point (the reflecting boundary) at the lower (shifted) threshold $-b + \epsilon$ to the absorbing barrier located at the upper threshold $b + \epsilon$:

$$\langle T_- \rangle = \frac{2}{\sigma^2} \int_{-b+\epsilon}^{b+\epsilon} dx\, e^{x^2/\sigma^2\tau} \int_{-\infty}^{x} dz\, e^{-z^2/\sigma^2\tau}$$
$$= \tau\sqrt{\pi} \int_{u_2}^{u_1} e^{u^2}[1 + \mathrm{erf}(u)]\, du\,, \qquad (3)$$

where $u_1 = (b + \epsilon)/(\sigma\sqrt{\tau})$, and $u_2 = (-b + \epsilon)/(\sigma\sqrt{\tau})$. The reverse mean passage time is easily written down as

$$\langle T_+ \rangle = \tau\sqrt{\pi} \int_{u_2}^{u_1} e^{u^2}[1 - \text{erf}(u)]\, du. \qquad (4)$$

Then, we may write $\langle \Delta T \rangle$ as the difference in these two expressions. Expanded to first order in ϵ, we obtain

$$\langle \Delta T \rangle = 4\epsilon\sqrt{\pi\tau/\sigma^2}\exp[b^2/2\sigma_\zeta^2]\text{erf}\left(\frac{b}{\sqrt{2\sigma_\zeta^2}}\right) + O(\epsilon^2), \qquad (5)$$

This expression is compared with numerically obtained separations $\langle \Delta T \rangle$ in the small ϵ limit (typically, this is the limit in which most experiments operate) in Fig. 3 (inset).

For *subthreshold* bias signals, the crossing events are noise controlled and the RTD multimodal [14] (with a noise-dependent tail) in general. The "stochastic resonance" scenario [15] may be exploited to yield better signal processing. This situation has received considerable attention in the literature (see, e.g., [15]); we do not address it here.

Experiments with a ring core fluxgate magnetometer are currently underway at FOI–Stockholm, aimed at demonstrating these ideas. Preliminary results indicate that the observable $\langle \Delta T \rangle$ can, in fact, be readily computed without recourse to a feedback circuit, and with bias signal amplitudes ranging from somewhat suprathreshold to vanishing, using the ideas discussed above. In these experiments, the sensor noise is usually not Gaussian [16] due to incoherent domain wall dynamics. However, new magnetic materials as well as sophisticated techniques that allow the construction of micromagnetic sensors or sensors having at most a few magnetic domains [17] have led to a significant lowering of the non-Gaussian noise so that the main source of noise is Gaussian correlated, as assumed in this paper. Additional experiments involving ferroelectric thin films are also underway at SSC, San Diego.

The results of this paper show that it is possible to operate a nonlinear dynamic sensor as a "neural" device in which a crossing or "firing" rate to a threshold is the observable of interest. The background noise may be constructively utilized to switch the sensor between its stable steady states, with the effects of a small target dc signal manifested as an inequality in the switch rates to the thresholds. In the new class of NANDS devices, the sensor need not be biased with large periodic drives or operated with a feedback loop. Elaborate detection schemes based on a more extended use of the statistical properties of the RTD other than the sample mean can be devised in order to improve detection performances. A detailed discussion on this point will be presented elsewhere [10].

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*Email address: luca.gammaitoni@pg.infn.it
†Email address: bulsara@spawar.navy.mil

[1] See, e.g., W. Bornhofft and G. Trenkler, in *Sensors, A Comprehensive Survey,* edited by W. Gopel, J. Hesse, and J. Zemel (VCH, New York, 1989), Vol. 5; P. Ripka, Sens. Actuators **A33**, 129 (1992).

[2] M. Rao, H. Krishnamurthy, and R. Pandit, Phys. Rev. B **42**, 856 (1990); F. Liorzou, B. Phelps, and D. Atherton, IEEE Trans. Magn. **36**, 418 (2000).

[3] See, e.g., I. Bunget and M. Prohescu, *Physics of Solid Dielectrics* (Elsevier, New York, 1984); M. Rao and R. Pandit, Phys. Rev. B **43**, 3373 (1991).

[4] See, e.g., J. Fraden, *Handbook of Modern Sensors* (AIP Press, New York, 1997), and references therein.

[5] R. Bartussek, P. Hanggi, and P. Jung, Phys. Rev. E **49**, 3930 (1994); A. R. Bulsara, M. E. Inchiosa, and L. Gammaitoni, Phys. Rev. Lett. **77**, 2162 (1996).

[6] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta, and S. Santucci, Phys. Rev. Lett. **62**, 349 (1989).

[7] For a review, see A. R. Bulsara and L. Gammaitoni, Phys. Today **49**, No. 3, 39 (1996); L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 225 (1998).

[8] M. Karlsson, J. Robinson, A. Bulsara, and L. Gammaitoni, in Proceedings of the Meeting on Marine Electromagnetics (MARELEC), Stockholm, 2001.

[9] A. Bulsara, M. Karlsson, J. Robinson, and L. Gammaitoni, in Proceedings of the Meeting on Marine Electromagnetics (MARELEC), Stockholm, 2001.

[10] A. Bulsara, C. Seberino, L. Gammaitoni, M. Karlsson, B. Lundquist, and J. Robinson (to be published).

[11] F. Apostolico, L. Gammaitoni, F. Marchesoni, and S. Santucci, Phys. Rev. E **55**, 36 (1997); F. Marchesoni, F. Apostolico, L. Gammaitoni, and S. Santucci, Phys. Rev. E **58**, 7079 (1998).

[12] J. M. G. Vilar and J. M. Ruby, Phys. Rev. Lett. **86**, 950 (2001).

[13] See, e.g., C. Gardiner, *Handbook of Stochastic Processes* (Springer-Verlag, Berlin, 1983).

[14] L. Gammaitoni, F. Marchesoni, E. Menichella-Saetta, and S. Santucci, Phys. Rev. Lett. **62**, 349 (1989); P. Jung, Phys. Rep. **234**, 176 (1993); A. Longtin, A. Bulsara, D. Person, and F. Moss, Biol. Cybernet. **70**, 569 (1994).

[15] For reviews, see A. Bulsara and L. Gammaitoni, Phys. Today **49**, No. 3, 39 (1996); L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998).

[16] G. Durin, in *Proceedings of the 14th International Conference on Noise in Physical Systems and* $1/f$ *Fluctuations* (World Scientific, Singapore, 1997); G. Bertotto, *Hysteresis in Magnetism* (Academic Press, New York, 1998).

[17] J. Deak, A. Miklich, J. Slonczewski, and R. Koch, Appl. Phys. Lett. **69**, 1157 (1996); R. Koch and J. Rozen, Appl. Phys. Lett. **78**, 1897 (2001).