Decoherence and $1/f$ Noise in Josephson Qubits

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We propose and study a model of dephasing due to an environment of bistable fluctuators. We apply our analysis to the decoherence of Josephson qubits, induced by background charges present in the substrate, which are also responsible for the $1/f$ noise. The discrete nature of the environment leads to a number of new features which are mostly pronounced for slowly moving charges. Far away from the degeneracy this model for the dephasing is solved exactly.

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Any quantum system during its evolution gets entangled with the surrounding environment. This effect is known as *decoherence* [1,2]. Besides the understanding of its role in many fundamental questions, decoherence is studied because it will ultimately limit the performance of a quantum computer [3]. Solid state nanodevices seem the natural arena to fulfill the requirements of large scale integrability and flexibility in the design, though, due to the presence of many types of low energy excitations in the environment, decoherence represents a serious limitation. Proposals to implement a quantum computer using superconducting nanocircuits are proving to be very promising [4,5] and several experiments have already highlighted the quantum properties of these devices [6].

In superconducting nanocircuits various sources of decoherence are present [4,7], such as fluctuations originating from the surrounding circuit, quasiparticle tunneling, fluctuating background charges (BC), and flux noise. In this Letter we introduce and study a model for decoherence due to a discrete environment, which describes what is considered the most serious limitation for Josephson qubits in the charge regime, i.e., the decoherence originating from fluctuating charged impurities. The common belief is that this effect originates from random traps for single electrons in dielectric materials surrounding the island. These fluctuations cause the $1/f$ noise directly observed in single electron tunneling devices [8,9].

The system under consideration is a Cooper pair box [4]. Under appropriate conditions (charging energy E_C much larger than the Josephson coupling E_J and temperatures $k_BT \ll E_J$) only two charge states are important, and the Hamiltonian of the qubit $\mathcal{H}_Q$ reads

$$\mathcal{H}_Q = \frac{\delta E_c}{2}\,\sigma_z - \frac{E_J}{2}\,\sigma_x\,,$$

where the charge basis $\{|0\rangle, |1\rangle\}$ is expressed using the Pauli matrices, and the bias $\delta E_c \equiv E_C(1 - C_x V_x/e)$ can be tuned by varying the applied gate voltage V_x. The environment is a set of BCs electrostatically coupled to the qubit. The resulting total Hamiltonian is

$$\mathcal{H} = \mathcal{H}_Q - \frac{1}{2}\,\sigma_z \sum_i v_i b_i^\dagger b_i + \sum_i \mathcal{H}_i\,,$$
$$\mathcal{H}_i = \varepsilon_{ci} b_i^\dagger b_i + \sum_k [T_{ki} c_{ki}^\dagger b_i + \text{H.c.}] + \sum_k \varepsilon_{ki} c_{ki}^\dagger c_{ki}\,. \tag{1}$$

Here $\mathcal{H}_i$ describes an isolated BC: the operators b_i ($b_i^\dagger$) destroy (create) an electron in the localized level ε_{ci}. This electron may tunnel, with amplitude T_{ki}, to a band described by the operators c_{ki} and $c_{ki}^\dagger$ and the energies ε_{ki}. For simplicity we assume that each localized level is connected to a distinct band. An important scale is the total switching rate $\gamma_i = 2\pi \mathcal{N}(\epsilon_{ci})|T_i|^2$ ($\mathcal{N}$ is the density of states of the electronic band, $|T_{ki}|^2 \approx |T_i|^2$), which characterizes the classical relaxation regime of each BC. Finally the coupling with the qubit is such that each BC produces a bistable extra bias v_i. The same model for the BCs has been used in Ref. [10] and explains experiments on charge trapping in systems of small tunnel junctions.

Our aim is to investigate the effect of the BC environment on the dynamics of the qubit. In this Letter we focus on decoherence *during* the quantum time evolution, due to an environment which produces $1/f$ noise. Thus we are concerned with BCs with a distribution of switching rates $\sim 1/\gamma$, in the range $[\gamma_m, \gamma_M]$ [11].

We used several techniques as second order perturbation theory in the couplings v_i, or in the BC band couplings T_{ki}, Heisenberg equations of motion, and a real-time path-integral analysis [12–14]. The picture which emerges is that the decoherence produced by each BC depends qualitatively on the ratio $g_i = v_i/\gamma_i$, so it is convenient to distinguish between *weakly coupled* BCs ($g_i \ll 1$) and *strongly coupled* BCs (in the other regimes). We stress that a *strongly coupled* BC does not necessarily have large coupling v_i with the qubit. Indeed in the physical situation we discuss in this Letter the energy scale associated with the total extra bias produced by the set of BCs is much smaller than the level splitting of the qubit, $\sqrt{\delta E_c^2 + E_J^2}$. To summarize our results, we find that, as far as decoherence is

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concerned, a single weakly coupled BC behaves as a source of Gaussian noise, whose effect is fully characterized by the power spectrum of the unperturbed equilibrium fluctuations of the extra bias operator $v_i b_i^\dagger b_i$, given by $s_i(\omega) = v_i^2/[2\cosh^2(\beta\varepsilon_{ci}/2)]\gamma_i/(\gamma_i^2 + \omega^2)$. Each weakly coupled BC contributes independently to decoherence. Instead our quantum mechanical treatment points out that decoherence due to a strongly coupled BC shows pronounced features of its discrete character, as saturation effects when $g_i \gg 1$, and dependence on initial conditions.

We consider, for the sake of clarity, two special operation points for the qubit: (i) charge degeneracy, $\delta E_c = 0$, and (ii) the case where tunneling can be neglected, $E_J = 0$. For this latter case, where pure dephasing occurs without relaxation, we find an exact solution.

(i) Charge degeneracy ($\delta E_c = 0$).—Relaxation (Γ_r) and dephasing (Γ_ϕ) rates of the qubit given by the golden rule (GR) [12,13]

$$\Gamma_\phi^{\mathrm{GR}} = \frac{1}{2}\Gamma_r^{\mathrm{GR}} = \frac{1}{4}\mathrm{S}(E_J) \qquad (2)$$

depend only on the power spectrum $S(\omega) = \sum_i s_i(\omega)$ at $\omega = E_J$. This simple result would readily allow an estimate of the rates $\Gamma_{r,\phi}$ [15] from independent measurements of $S(\omega)$ [8,9,16]. Being second order in v_i, the golden rule is appropriate only for weakly coupled BCs. We study the general problem by using the Heisenberg equations of motion. For the average values of the qubit observables $\langle\sigma_\alpha\rangle$ (sum over $\alpha, \beta = x, y, z$ is implicit), we obtain ($\hbar = 1$)

$$\langle\dot{\sigma}_\alpha\rangle = E_J \epsilon_{\alpha\beta}\langle\sigma_\beta\rangle + \eta_{\alpha\beta}\sum_i^N v_i\langle b_i^\dagger b_i \sigma_\beta\rangle,$$

where N is the total number of BCs, and $\epsilon_{yz} = -\epsilon_{zy} = 1$, and $\eta_{xy} = -\eta_{yx} = 1$ are the only nonzero entries. On the right-hand side, averages of new operators which also involve the localized levels and the bands are generated. New equations have to be considered and the iteration of this procedure yields an infinite chain. A closed set of $3 + 3N$ equations is obtained by factorization of high order averages: we ignore the cumulants $\langle b_i^\dagger b_i b_j^\dagger b_j\rangle_c$ and $\langle b_i^\dagger b_i b_j^\dagger b_j \sigma_\alpha\rangle_c$ for $i \neq j$ and insert the relaxation dynamics for the BCs in the approximated terms. This method gives accurate results for general values of g_i even if v_i/E_J is not very small, as we checked by comparing with numerical evaluation of the reduced density matrix of the qubit with few BCs. The results are presented in Figs. 1 and 2, where the time Fourier transform of $\langle\sigma_z(t)\rangle$, proportional to the average charge on the island, is shown. We assumed factorized initial conditions for the qubit and the BCs. We first consider a set of weakly coupled BCs in the range $[10^{-2}, 10]\ E_J$ which determines $1/f$ noise in a frequency interval around the operating frequency. The coupling strengths v_i have been generated uniformly with approximately zero average and magnitudes chosen in order to yield the amplitude of typical measured spectra [8,9,16] (extrapolated at GHz frequencies). These BCs are weakly coupled, and determine a dephasing rate which reproduces the prediction of the golden rule equation (2) (Fig. 1, dotted line, $\Gamma_\phi^{\mathrm{GR}}/E_J \approx 1.65 \times 10^{-4}$) . Now we add a slower (and strongly coupled $g_0 = v_0/\gamma_0 = 8.3$) BC, in order to extend the $1/f$ spectrum to lower frequencies. The added BC gives negligible contribution to the power spectrum at E_J so according to Eq. (2) it should not modify Γ_ϕ. Instead, as shown in Fig. 1, we find that the strongly coupled BC alone determines a dephasing rate comparable to that of the weakly coupled BCs. The overall Γ_ϕ is more than twice the prediction of the golden rule. If we further slow down the added BC we find that Γ_ϕ increases toward values $\sim\gamma_0$. This indicates that the effect of strongly coupled BCs on decoherence saturates (we

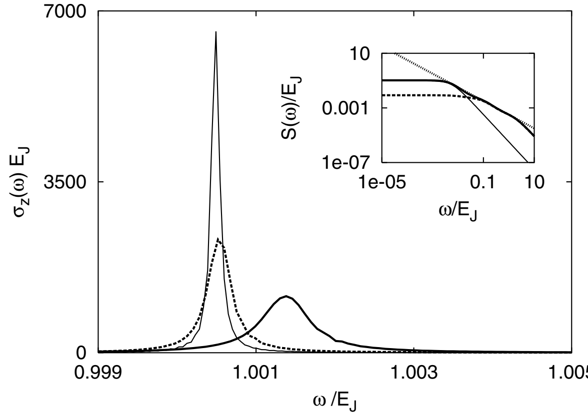

FIG. 1. The Fourier transform $\sigma_z(\omega)$ for a set of weakly coupled BCs plus a single strongly coupled BC (thick line). The separate effect of the coupled BC ($g_0 = 8.3$, thin line) and the set of weakly coupled BCs (dotted line) is shown for comparison. Inset: corresponding power spectra. In all cases the noise level at E_J is fixed to the value $\mathrm{S}(E_J)/E_J \approx 3.18 \times 10^{-4}$.

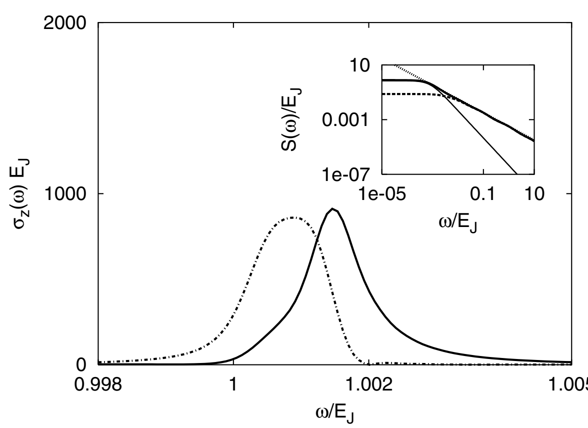

FIG. 2. The Fourier transform $\sigma_z(\omega)$ for a set of weakly coupled BCs plus a strongly coupled BC ($v_0/\gamma_0 = 61.25$) prepared in the ground (dotted line) or in the excited state (thick line). Inset: corresponding power spectra (the thin line corresponds to the extra BC alone).

discuss later similar results for the case of pure dephasing, where this conclusion can be made sharp).

In this regime we also observe effects related to the initial preparation of the strongly coupled BC (see Fig. 2). Finally we checked different sets of BCs with the same power spectrum. They yield larger decoherence if BCs with $g \gtrsim 1$ are present. The golden rule result [Eq. (2)] *underestimates* the effect of these strongly coupled BCs. All the features presented above are a direct consequence of the discrete nature of the environment.

Pure dephasing ($E_J = 0$).—In the absence of the tunneling term Eq. (1) is a model for pure dephasing. The charge on the qubit island is conserved, $[\mathcal{H}, \sigma_z] = 0$, but superpositions of charge states dephase. Hence the off-diagonal elements of the reduced density matrix of the qubit in the charge basis, or equivalently the averages of the raising and lowering operators $\sigma_\pm$, decay in time.

It is possible to show by direct calculation that, for a product initial condition, $\langle\sigma_\pm\rangle$ factorizes exactly in independent contributions of each BC, $\langle\sigma_-(t)\rangle = \langle\sigma_-(0)\rangle \prod_j \exp\{-i(\delta E_c - v_j/2)t\} f_j(t)$. Using a real-time path-integral technique [13], the general form of $f_j(t)$ in Laplace space is obtained:

$$f_j(\lambda) = \frac{\lambda + K_{1,j}(\lambda) - i v_j/2 \delta p_j^0}{\lambda^2 + (v_j/2)^2 + \lambda K_{1,j}(\lambda) + v_j/2 K_{2,j}(\lambda)}, \tag{3}$$

where $\delta p_j^0 = 1 - 2\langle b_j^\dagger b_j\rangle_{t=0}$. The kernels $K_{1,j}(\lambda)$ and $K_{2,j}(\lambda)$ are given by formal series expressions. An explicit form is obtained in the noninteracting blip approximation (NIBA) [13,14], $K_{1,j}(\lambda) = \gamma_j$, $K_{2,j}(\lambda) = -\gamma_j/\pi[\psi(1/2 + \beta/2\pi(\lambda - i\epsilon_{cj})) - \psi(1/2 + \beta/2\pi(\lambda + i\epsilon_{cj}))]$. In order to appreciate the validity of the NIBA result, we notice that it is also obtained by the Heisenberg equations with the only assumption being that the band is in equilibrium.

We now discuss the results in the physically relevant limit, where the BCs have an incoherent dynamics. In this case an analytic form for $\langle\sigma_-(t)\rangle$ is found,

$$\begin{aligned}\frac{\langle\sigma_-(t)\rangle}{\langle\sigma_-(0)\rangle} &= e^{-i\delta E_c t} \prod_{j=1}^{N} e^{i v_j t/2} \\ &\quad \times \{A_j e^{-\gamma_j/2(1-\alpha_j)t} + (1 - A_j) e^{-\gamma_j/2(1+\alpha_j)t}\} \\ &\equiv \exp\{-i\delta E_c t\} \exp\{-\Gamma(t) + iE(t)\},\end{aligned} \tag{4}$$

where we defined $\alpha_j = \sqrt{1 - g_j^2 - 2ig_j \tanh(\beta\varepsilon_{cj}/2)}$ and $A_j = \frac{1}{2\alpha_j}(1 + \alpha_j - i\delta p_j^0 g_j)$. This result can be obtained both by approximating the kernel $K_{2,j}(\lambda) \sim K_{2,j}(0)$, valid in the limit $\varepsilon_{ci}, v_i, \gamma_i \ll K_B T$, and by the exact result of a semiclassical analysis. In this last case the coupling operator $\sum_i v_i b_i^\dagger b_i$ is substituted by a classical stochastic process $\mathcal{E}(t)$, the sum of random telegraph processes, yielding

$$\frac{\langle\sigma_-(t)\rangle}{\langle\sigma_-(0)\rangle} = e^{-i\delta E_c t}\left\langle\left\langle \exp\left[-i\int_0^t dt'\, \mathcal{E}(t')\right]\right\rangle\right\rangle, \tag{5}$$

where $\langle\langle\cdots\rangle\rangle$ is the average over the possible realizations of $\mathcal{E}(t)$ with given initial conditions δp_j^0.

The form of Eq. (4) elucidates the different roles of weakly and strongly coupled BCs in the decoherence process. Dephasing due to each BC comes from the sum of two exponential terms. If $g_j \ll 1$ only the first term is important, and the corresponding rate is $\approx 1/[4\cosh^2(\beta\varepsilon_{cj}/2)] v_j^2/\gamma_j$, the golden rule result. If $g_j \gg 1$ the two terms are of the same order, and the decay rate is $\sim\gamma_j$, the switching rate of the BC. The main effect of strong coupling with the qubit is a static energy shift. That is, for the slower BCs ($g_j \gg 1$) the contribution to the decoherence rate saturates to $\sim\gamma_j$. At short times $t \ll 1/\gamma_j$ the initial conditions, which may take the values $\delta p_j^0 = \pm 1$, determine the transient behavior.

We now apply this result to sets of BCs which produce $1/f$ noise. We stress that, while saturation of dephasing due to a *single* BC is physically intuitive, it is not *a priori* clear whether this holds also for the $1/f$ case, where a *large* number ($\sim 1/\gamma$) of slow fluctuators (strongly coupled BCs) is involved. In Fig. 3 we show the results for a sample with a number of BCs per decade $n_d = 1000$ and with v_i distributed with small dispersion around $\langle v\rangle = 9.2 \times 10^7$ Hz. Initial conditions $\delta p_j^0 = \pm 1$ are distributed according to $\langle\delta p_j^0\rangle = \delta p_{\rm eq}$, the equilibrium value. To illustrate the different role played by BCs with $g_j \ll 1$ and $g_j \gg 1$, we consider sets with $\gamma_M = 10^{12}$ Hz and different γ_m. In this case the dephasing is given by BCs with $\gamma_j > 10^7$ Hz $\approx \langle v\rangle/10$. The main contribution comes from three decades at frequencies around $\langle v\rangle$. The overall effect of the strongly coupled BCs ($\gamma_j < \langle v\rangle/10$) is minimal, despite their large number.

Finally we compare our results with the Gaussian approximation. It amounts to estimating the average

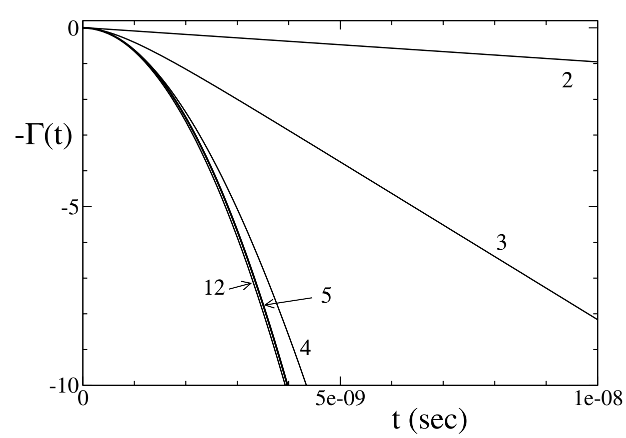

FIG. 3. Saturation effect of slow BCs for a $1/f$ spectrum. Relevant parameters ($\langle v\rangle = 9.2 \times 10^7$ Hz, $n_d = 1000$) give typical experimental measured noise levels and reproduce the observed decay of the echo signal [9] in charge Josephson qubits. Couplings v_i are distributed with dispersion $\langle\Delta v\rangle/\langle v\rangle = 0.2$. $\Gamma(t)$ is almost unaffected by strongly coupled charges (the labels are the number of decades included).

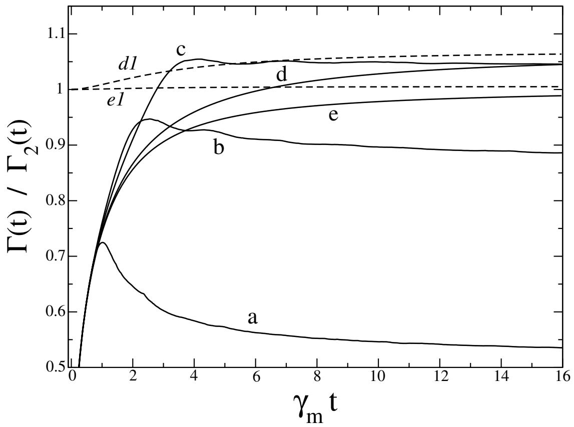

FIG. 4. Ratio $\Gamma(t)/\Gamma_2(t)$ for a $1/f$ spectrum between $\gamma_m = 2 \times 10^7$ and $\gamma_M = 2 \times 10^9$ with different numbers of BCs per decade: (*a*) $n_d = 10^3$, (*b*) $n_d = 4 \times 10^3$, (*c*) $n_d = 8 \times 10^3$, (*d*) and (*d1*) $n_d = 4 \times 10^4$, (*e*) and (*e1*) $n_d = 4 \times 10^5$. The solid lines correspond to $\delta p_j^0 = \pm 1$, and the dashed lines correspond to equilibrium initial conditions for the BCs.

in Eq. (5) by its second cumulant and taking $\delta p_j^0 = \delta p_{\text{eq}}$ [17],

$$\Gamma_2(t) = \frac{1}{\pi} \int_0^\infty d\omega \, S(\omega) \frac{1 - \cos(\omega t)}{\omega^2}. \qquad (6)$$

This formula fails to describe BCs with $g_j \gg 1$. For instance, $\Gamma_2(t)$ at a fixed t scales with the number of decades and does not show saturation. The Gaussian approximation should become correct if the environment has a very large number of extremely weakly coupled BCs. We check this limit by comparing $\Gamma_2(t)$ with Eq. (4). The power spectrum $S(\omega)$ is identical for all the curves in Fig. 4 but is obtained by sets of charges with different n_d and $\langle v \rangle$. The Gaussian behavior is recovered for $t \gg 1/\gamma_m$ if n_d is large (all the BCs are weakly coupled). If in addition we take $\delta p_j^0 = \delta p_{\text{eq}}$ in Eq. (4), $\Gamma(t)$ approaches $\Gamma_2(t)$ also at short times. Hence decoherence depends separately on n_d and $\langle v \rangle$, whereas in the Gaussian approximation only the combination $n_d \langle v^2 \rangle$, which enters $S(\omega)$, matters. In other words, the characterization of the effect of slow sources of $1/f$ noise requires knowledge of moments of the bias fluctuations higher than $S(\omega)$.

In conclusion, we studied decoherence due to an environment of bistable charges. We found that the average coupling between individual BCs and the qubit is an important scale of the problem: BCs such that $\gamma_i \ll v_i$ show pronounced features of their discrete dynamics, as saturation and transient behavior. The physical picture we obtain for the decoherence effects due to BCs is not sensitive to the details of the model Hamiltonian (1), but is mainly determined by the discrete character of the environment. Thus this approach can also be applied to different physical systems as phase qubits in the presence of flux noise [7].

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