Pairing of Cooper Pairs in a Fully Frustrated Josephson-Junction Chain

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(Received 7 February 2002; published 17 May 2002)

We study a one-dimensional Josephson-junction chain embedded in a magnetic field. We show that, when the magnetic flux per elementary loop equals half the superconducting flux quantum $\phi_0 = h/2e$, a local $\mathbb{Z}_2$ symmetry arises. This symmetry is responsible for a nematic Luttinger liquid state associated with bound states of Cooper pairs. We analyze the phase diagram and discuss some experimental possibilities to observe this exotic phase.

DOI: 10.1103/PhysRevLett.88.227005 PACS numbers: 74.50.+r, 71.10.Fd, 71.23.An, 73.20.Jc

During the past twenty years, Josephson-junction arrays have proved to be very good tools to investigate classical and quantum phase transitions [1]. Recently, much attention has been paid to systems which display highly degenerate classical ground states [2] due to the presence of Aharonov-Bohm cages [3]. Interestingly, a glassy vortex phase without disorder has been predicted for such two-dimensional (2D) structures [4] in agreement with experimental observations [5]. In this Letter, we investigate the influence of quantum fluctuations on such systems in a 1D model introduced in [6]. In this remarkably simple example, the huge classical degeneracy is a direct consequence of a local $\mathbb{Z}_2$ symmetry which is unbroken in the presence of quantum fluctuations. We show that this may stabilize an unusual nematic Luttinger liquid (LL) phase in which charge $4e$ bound states of Cooper pairs are the elementary objects.

We consider the chain of loops shown in Fig. 1 embedded in a uniform magnetic field. We denote by ϕ the magnetic flux per elementary plaquette and we set $\gamma = 2\pi\phi/\phi_0$, where $\phi_0 = h/2e$ is the superconducting flux quantum. Each site of this lattice is supposed to be occupied by a superconducting island. A convenient description of the low-energy Hilbert space of this system involves local boson operators $a_n^\dagger,\ b_n^\dagger,\ c_n^\dagger$ $(a_n,\ b_n,\ c_n)$ that create (destroy) Cooper pairs on the three types of islands of the lattice, respectively represented by black, grey, and white circles in Fig. 1. The system is described by the following Josephson coupling Hamiltonian:

$$H_J = -t_J \sum_n a_n^\dagger (b_n + c_n + b_{n-1} + e^{-i\gamma} c_{n-1}) + \text{H.c.} \tag{1}$$

We first focus on the special value $\gamma = \pi$ (half a flux quantum per loop). As shown in [6], this Hamiltonian has, in this case, a single-particle spectrum composed of three highly degenerate flat bands $\varepsilon_0 = 0$, $\varepsilon_\pm = \pm 2t_J$. The corresponding eigenstates can be chosen as strictly localized (cage states) around each fourfold coordinated site (see Fig. 1). This leads naturally to the notion of Aharonov-Bohm cages discussed in [3].

Let us introduce the set of boson operators $A_{\alpha,n}^\dagger$ $(A_{\alpha,n})$ that creates (destroys) one Cooper pair with energy ε_α $(\alpha = 0, \pm)$ in a cage state localized around the nth fourfold site. These operators can be simply expressed as a linear combination of the operators $a_n^\dagger, b_n^\dagger, c_n^\dagger, b_{n-1}^\dagger, c_{n-1}^\dagger$ only, whose coefficients are given in Fig. 1 so that we get

$$H_J = \sum_{n,\alpha} \varepsilon_\alpha A_{\alpha,n}^\dagger A_{\alpha,n} . \tag{2}$$

In this present form, H_J clearly exhibits a local U(1) symmetry. We shall now study the effect of boson-boson interaction on this symmetry. Therefore, we consider a real valued function $n \mapsto s_n$, and we construct a unitary operator U_s defined by $U_s A_{\alpha,n} U_s^{-1} = e^{-is_n} A_{\alpha,n}$ which commutes with H_J. Using the precise form of the cage states, we easily obtain

$$U_s a_n^\dagger a_n U_s^{-1} = a_n^\dagger a_n , \tag{3}$$

$$U_s b_n^\dagger b_n U_s^{-1} = \cos^2(\Delta_n) b_n^\dagger b_n + \sin^2(\Delta_n) c_n^\dagger c_n + z_n , \tag{4}$$

$$U_s c_n^\dagger c_n U_s^{-1} = \sin^2(\Delta_n) b_n^\dagger b_n + \cos^2(\Delta_n) c_n^\dagger c_n - z_n , \tag{5}$$

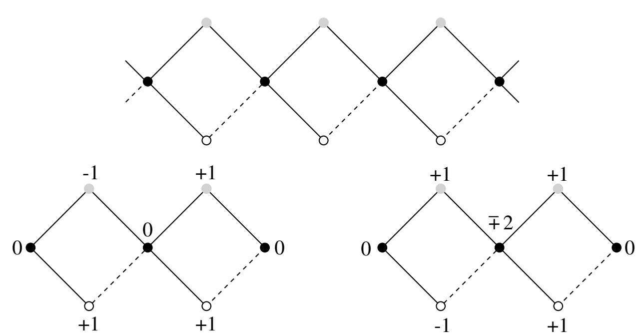

FIG. 1. The chain of loops and the three (non-normalized) cage eigenstates corresponding to ε_0 (left) and $\varepsilon_\pm$ (right). The dashed lines symbolize the hopping term $-t_J e^{-i\gamma}$.

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where we have set $z_n = i\sin(2\Delta_n)(b_n^\dagger c_n - c_n^\dagger b_n)/2$ and $\Delta_n = (s_{n+1} - s_n)/2$. From these transformation laws, we can readily see that any interaction term involving the bilinear operators $a_m^\dagger a_m$, $b_n^\dagger b_n + c_n^\dagger c_n$ preserves the local U(1) symmetry. Physically, this symmetry implies that the total number of bosons in each cage is separately conserved and the system remains an insulator. However, this symmetry is fragile since it is easy to find two-body interactions which break it. For instance, a Hubbard-like interaction term, $\sum_n (a_n^\dagger a_n)^2 + (b_n^\dagger b_n)^2 + (c_n^\dagger c_n)^2$, has this effect which is manifested by the appearance of the delocalized two-particle bound states discussed in [6]. An exciting feature of this system is that this type of interaction still preserves a subgroup of the full U(1) corresponding to a local $\mathbb{Z}_2$ symmetry. This subgroup corresponds to $s_n = 0[\pi]$ for all n. With this restriction, it is easy to check that the operator $b_m^\dagger b_m b_n^\dagger b_n + c_m^\dagger c_m c_n^\dagger c_n$ commutes with U_s for all (m, n). This local $\mathbb{Z}_2$ symmetry has an important physical consequence since it means that the parity of the total number of bosons in each cage is separately conserved. Therefore, if two-particle interactions lead to coherent transport through the chain for a many-boson system, quasi off-diagonal long-range order may occur only for composite objects built with an even number of original bosons, i.e., here, of Cooper pairs. In other words, a superconducting Josephson-junction chain with this geometry and half a flux quantum per loop may realize a quasi-Bose condensate (in fact a LL) of charge $4e$ composite bosons.

To discuss in more detail the physics of this system, it is useful to analyze these symmetry considerations for quantum rotor models. These offer the advantage of an intuitively simple classical limit defined from the phase of a superconducting order parameter. Formally, we introduce three phase fields, $\theta_n, \varphi_n, \chi_n$, and their canonically conjugate fields, $\Pi_{\theta,n}, \Pi_{\varphi,n}, \Pi_{\chi,n}$, which are related to the local Bose operators by $a_n^\dagger = \Pi_{\theta,n}^{1/2} e^{i\theta_n}$, $b_n^\dagger = \Pi_{\varphi,n}^{1/2} e^{i\varphi_n}$, $c_n^\dagger = \Pi_{\chi,n}^{1/2} e^{i\chi_n}$. Assuming that the local particle number fluctuations are small, we obtain the quantum phase Hamiltonian [1]

$$H = \frac{E_C}{2} \sum_n \Pi_{\theta,n}^2 + \Pi_{\varphi,n}^2 + \Pi_{\chi,n}^2 - E_J \sum_n \cos(\theta_n - \varphi_n) + \cos(\theta_n - \chi_n) + \cos(\theta_n - \varphi_{n-1}) + \cos(\theta_n - \chi_{n-1} - \gamma), \qquad (6)$$

where E_C is the charging energy and E_J is the Josephson coupling between islands. Note that the present modeling of capacitive effects is not meant to be very realistic since, for the sake of simplicity, we have not taken into account off-diagonal elements of the capacitance matrix. This choice corresponds to a local Hubbard-like interaction term between Cooper pairs. For convenience, we set $\sqrt{E_C E_J} = 1$.

The classical ground state of H is easily obtained for any γ. Indeed, if we set $x_n = \theta_{n+1} - \theta_n - \gamma/2$, and eliminate φ_n and χ_n, minimizing H is equivalent to minimizing $F(x_n) = -|\cos(x_n/2 + \gamma/4)| - |\cos(x_n/2 - \gamma/4)|$ for all n. As shown in Fig. 2, F has two local minima in $x_n = 0$ and $x_n = \pi$. For $0 < \gamma < \pi$, one has $F(0) < F(\pi)$, and the classical ground state is unique (up to a global U(1) degeneracy). By contrast, for $\gamma = \pi$, one has $F(0) = F(\pi)$ so that, for a given plaquette, we obtain two degenerate ground states (up to a global translation of the phase variables) which are illustrated in Fig. 3. These states differ only in the sign of the superconducting currents which circulate around the plaquette. For a chain made up of N loops, we thus obtain 2^N degenerate classical ground states up to a global translation of the phase variables.

This huge degeneracy is a direct consequence of a local $\mathbb{Z}_2$ symmetry of H. Note that H is not invariant under the full local U(1) group related to the Aharonov-Bohm cages. This occurs since the Josephson term in H may be written as a strongly nonlinear expression of the basic local Bose operators. For the $\mathbb{Z}_2$ transformations, it is an easy task to translate the U_s operators in the language of phase variables. An interesting local $\mathbb{Z}_2$ transformation is provided by a kink in the s_m's. Let U_n be the transformation defined by $s_m = 0$ for $m \leq n$ and $s_m = \pi$ for $m > n$. This transformation does not modify the phase variables for $m \leq n$, whereas it shifts them by π if $m > n$ and it permutes φ_n and χ_n. Thus, its main physical effect is to change the currents flowing around the plaquette located between n and $n + 1$ into their opposite value. From this description, we deduce that, by starting from a given classical ground state, we may generate any other ground state by applying a finite sequence of such U_n operators. We also see that, in the classical limit considered here, the local $\mathbb{Z}_2$ symmetry is spontaneously broken, yielding ground states with well-defined local circulating supercurrents. Note that U_n also leaves most conjugate variables unchanged except $\Pi_{\varphi,n}$ and $\Pi_{\chi,n}$ which are exchanged. As a result, we may add to H any term involving these conjugate fields without breaking the local $\mathbb{Z}_2$ symmetry, provided the spatial symmetry between the φ_n and χ_n degrees of freedom is respected. Experimentally, this would require tight control of offset charges since these may seriously alter an otherwise excellent geometrical symmetry of the chain.

For real systems, it may become important to take into account quantum fluctuations of the phase variables, especially when the superconducting islands are so small that their charging energy E_C can no longer be neglected in comparison to the Josephson coupling energy E_J. For a single loop and $\gamma = \pi$, these quantum fluctuations have been shown, theoretically [7] and experimentally

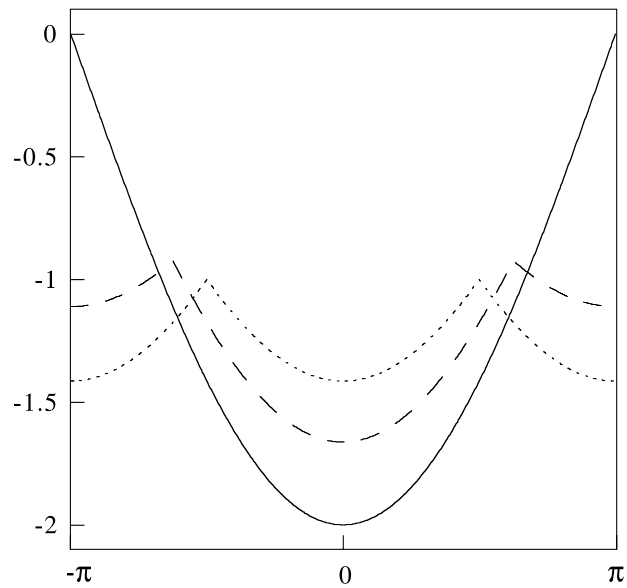

FIG. 2. Behavior of $F(x_n)$ for $\gamma = 0$ (solid line), $\gamma = 3\pi/4$ (dashed line), and $\gamma = \pi$ (dotted line).

[8,9], to induce tunneling between the two degenerate classical ground states shown in Fig. 3. The true quantum mechanical ground state is therefore a macroscopic linear superposition of these two classical states and provides a simple example of a "Schrödinger cat." For a system with N loops, eigenstates are classified according to the various irreducible representations of the local $\mathbb{Z}_2$ group which mix all the 2^N classical ground states. One of our next goals is to describe how quantum fluctuations lift the degeneracy among these representations, which is an artifact of the classical limit.

At small $g = \sqrt{E_C/E_J}$, the properties of the system are actually very similar to those of a quantum XY model. For small γ, we thus expect the infinite chain to be in a LL phase for $g < g^*(\gamma)$ and in a gapful insulating (I) phase for $g > g^*(\gamma)$. The transition at $g^*(\gamma)$ is of a Berezinskii-Kosterlitz-Thouless (BKT) type [10]. Simple spin wave calculations using the harmonic approximation of H around its classical ground state predict $g^*(\gamma) = \sqrt{\cos(\gamma/4)}\, g^*(0)$ with $g^*(0) = \pi\sqrt{3}/2$. The main effect of the magnetic field, in this simple approximation, is thus to replace g by an effective $g_{\mathrm{eff}} = g/\sqrt{\cos(\gamma/4)}$ which controls all the correlation function exponents in the LL phase.

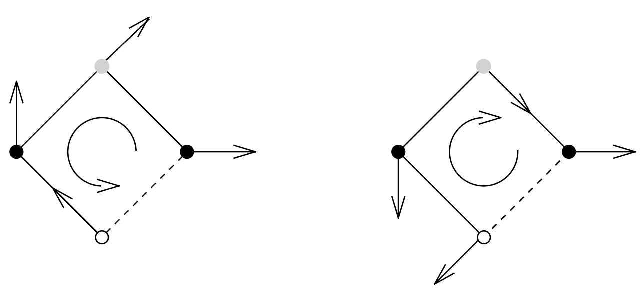

FIG. 3. Two possible classical ground states of H with different chirality.

To analyze the effect of the quantum fluctuations in the vicinity of $\gamma = \pi$ where the additional local $\mathbb{Z}_2$ symmetry emerges, it is convenient to eliminate the twofold coordinated islands to get a simple description of the low-energy physics of this system. Therefore, instead of H, we now consider the following Hamiltonian:

$$H_{XY} = \sum_n \frac{g'}{2} \Pi_{\theta,n}^2 - \frac{1}{g'} \{\cos[p(\theta_n - \theta_{n+1} - \gamma/2)] + \epsilon \cos(\theta_n - \theta_{n+1} - \gamma/2)\}. \tag{7}$$

The parameter g' is provided by fitting the exponent of the correlation function $\langle e^{i(\theta_m - \theta_n)} \rangle$ in the semiclassical regime with its value obtained with H in the harmonic approximation. This choice leads to $g' = 2^{5/4}3^{-1/2}g$. The parameter $\epsilon = 4|\gamma - \pi|$ is determined from the energy splitting between the two local minima of the single loop potential energy. Finally, in our case, we have $p = 2$ but we discuss hereafter the properties of H_{XY} for an arbitrary p.

The Hamiltonian H_{XY} has a local $\mathbb{Z}_p$ symmetry at $\epsilon = 0$, corresponding to the local transformations T_a: $\theta_j \mapsto \theta_j + 2\pi a_j/p$, where a_j is an integer. The irreducible representations of this group are easily obtained in a basis which diagonalizes simultaneously the $\Pi_{\theta,j}$'s. For a state $|\psi\rangle$ such that $\Pi_{\theta,j}|\psi\rangle = l_j|\psi\rangle$, where l_j is an integer, we have $T_a|\psi\rangle = \exp(i\sum_j \frac{2\pi}{p} l_j a_j)|\psi\rangle$. Writing $l_j = m_j + pn_j$ with m_j and n_j integers and $0 \leq m_j \leq p - 1$, we find that the set of m_j's completely specifies the irreducible representation of the local $\mathbb{Z}_p$ group. For each such representation, the action of the corresponding projector on the approximate Gaussian ground state of H_{XY} produces a natural trial wave function, at least when $g' \ll 1$. We have computed the expectation value of H_{XY} on these states. Doing so, we noticed that $2\pi/p$ tunnel processes occurring on different lattice sites are mostly uncorrelated. Neglecting completely these correlations we obtain

$$\langle H_{XY} \rangle = \frac{g'L^2}{2N} - \frac{Ce^{-f}}{g'} \sum_j \cos\left[\frac{2\pi}{p}(m_j - L/N)\right], \tag{8}$$

up to a constant energy independent of the representation and to factors of order e^{-2f}. In (8), $L = \sum_j l_j$ is the total angular momentum, C is a number close to $2\pi^2 - 8$ at small g', and $f \simeq 4\pi/pg'$. The ground state is therefore obtained by choosing the identity representation of the local $\mathbb{Z}_p$ group ($m_j = 0$).

Next, we see that the term proportional to ϵ couples different irreducible representations of the local $\mathbb{Z}_p$ group. When $p = 2$ the action of this perturbation on the 2^N low-energy trial states just discussed is well described by a quantum Ising model in a transverse magnetic field:

$$H_{\mathrm{I}} = -\frac{1}{g'}\left(Ce^{-f}\sum_n \sigma_n^X + D\epsilon \sum_n \sigma_n^Z \sigma_{n+1}^Z\right), \tag{9}$$

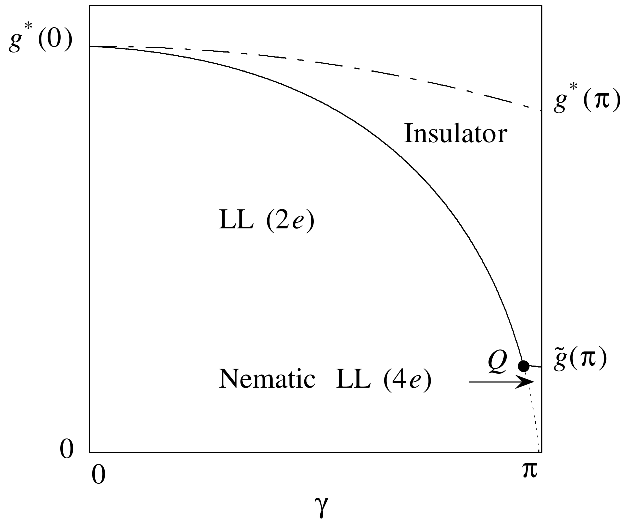

FIG. 4. Schematic phase diagram as a function of $g = \sqrt{E_C/E_J}$, and $\gamma = 2\pi\phi/\phi_0$. LL stands for Luttinger liquid.

where D is close to 1 for small g'. In terms of these Ising variables, the local $\mathbb{Z}_2$ symmetry corresponds to interchanging the $|+\rangle$ and $|-\rangle$ states on any given subset of sites. It is therefore implemented by the σ_n^X operators. This model has a continuous phase transition (in the universality class of the 2D Ising model) at its self-dual point which corresponds to $\epsilon = (C/D)e^{-2\pi/g'}$. Furthermore, it is easy to see that H_{XY} exhibits a BKT transition at $\gamma = \pi$ (so $\epsilon = 0$) corresponding to the loss of quasi-long-range order for the nematic order parameter $e^{2i\theta}$. In the harmonic approximation, this occurs at $\tilde{g}(\pi) = g^*(\pi)/4$.

Gathering this information obtained in various limits, we get the phase diagram drawn in Fig. 4. Besides the two familiar phases, namely, the (I) phase at large g and the LL phase characterized by an algebraic order of the $e^{i\theta}$ order parameter, the most interesting result is the presence of a remarkable nematic Luttinger liquid (NLL) phase which may be viewed as a quasiordered condensate of pairs of Cooper pairs associated with the order parameter $e^{2i\theta}$.

The dotted line (Fig. 4) of a 2D Ising-type separates the two Luttinger phases. The Ising order parameter is vanishing in the NLL phase and builds up in the conventional LL phase where its square is proportional to the algebraically decaying part of the $e^{i\theta}$ autocorrelation function. The physical picture presented here has already been uncovered by Lee and Grinstein [11] in the framework of a 2D classical XY model with squared cosine interaction. In their language, the BKT transition from the $2e$ LL phase to the I phase involves 2π vortices of the θ field, in contrast to π vortices between the NLL phase and the I phase. The nature of the multicritical point Q remains mysterious to us. Note that the coupling between XY and Ising-like degrees of freedom has already been extensively studied [12–17] in the context of fully frustrated Josephson-junction arrays. Nevertheless, in these works, the Ising order parameter has a very different nature since it describes the possible melting of the vortex lattice. It implies that the XY transition has to appear at lower temperatures than the Ising transition [12–14]. In our case and in the vicinity of $\gamma = \pi$, the XY transition occurs for larger g than the Ising one. So, the effect of the Ising domain walls on the phase stiffness is very different in the two situations.

To conclude, we have established the possibility of macroscopic condensation (in the sense of a LL) of charge $4e$ objects in a Josephson-junction chain for $\gamma \simeq \pi$. Experimentally, it may be possible to detect this binding of Cooper pairs by connecting such a network to superconducting leads. Indeed, we expect here a phenomenon analogous to Andreev's reflection: an ordinary Cooper pair of charge $2e$ entering the chain at low energy (compared to the gap between different representations of the local $\mathbb{Z}_2$ group) will leave behind a pair of charge $-2e$ so that a charge $4e$ composite object may propagate along the chain. Another possibility is to close the chain into a large ring. In this geometry, we expect quantum oscillations of the global current with respect to the magnetic flux across the ring with an elementary period $\phi_0/2$ as long as $\gamma \simeq \pi$.

We thank P. Lecheminant, D. Mouhanna, B. Pannetier, and E. Serret for fruitful and stimulating discussions.

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