Statistics of Magnetic Noise in Neutron Star Crusts

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The neutron star crust magnetodynamics is demonstrated to exhibit erratic jumps at the fields corresponding to a sharp change of nuclide magnetic moments induced by quantization effects. Such a noise originates from magnetic avalanches and shows intensity and statistical properties which are favorably compared to the burst activity of soft gamma repeaters.

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The discovery of a superintense gamma-ray outburst (giant flare) from SGR 0526-66 on 5 March 1979 [1,2] is related to pioneering observations of soft gamma repeaters (SGRs). Almost a copy of such a superhigh luminosity, $L_G \sim 10^{44.5}$ ergs, flare was seen on 27 August 1998 from SGR 1900 + 14 [3]. Many properties of both events, such as a sharp change in the persistent x-ray flux during the giant flare [4] and a multipeaked pulse profile [5,6], indicate very large magnetic fields of essentially multipolar geometry with dipole components $B_{\text{dipole}} \sim (4\text{–}8) \times 10^{14}$ G, revealed from the magnetic-braking spin-down mechanism [7]. Estimates based on applications of the Newtonian scalar virial theorem [8] in conjunction with numerical calculations (see, e.g., [9,10]) assert the possibility of significantly stronger stellar magnetic fields suggesting thereby that multipole components in inner star regions (e.g., crust) can be larger by about 2–3 orders of magnitude than on the surface, in the same manner as solar magnetic fields [11].

The observations intensified during the past decade (see, e.g., [3–7,12–19] and references therein) reveal that SGRs commonly emit short ($\sim$0.1 s) outbursts with $L_X \sim 10^3\text{–}10^4 L_{\text{Edd}}$ far above the Eddington limit L_{Edd} [20], while sub-Eddington persistent x-ray luminosities $L_P \sim 10^{27.5}\text{–}10^{29}$ W are similar to anomalous x-ray pulsars (AXPs). During short (typically, weeks to months) intervals the intensive burst activity displays signals of self-organized criticality, e.g., power law dependence of the burst number on the intensity, log normal distribution of waiting times between the bursts [13–16]. Such active phases are separated by relatively long (years) and quasiregular quiescent periods [7].

Many of SGR's features are well explained within the concept of ultramagnetized neutron stars ("magnetars") [5,6,9,21]. SGR bursts have been proposed [21] to originate from solid crust fractures induced by magnetic fields. However, some observations, such as the quasiperiodicity of active phases and rather stable, without noticeable spin-up glitches, spinning down, controvert such a starquake triggering mechanism and stimulate the search for alternative models, e.g., collisions of a strange star with asteroids [22], effects of boson condensation in the superconducting core [23].

We argue in this paper that such properties can also be well understood within the "magnetar" concept by exploring triggering mechanisms associated with the release of magnetic energy stored in neutron star crusts [24]. The intervals of intense activity are related to a sharp, steplike change of the magnetization because of inhomogeneous crust structure [24–26]. At such conditions the demagnetization proceeds as erratic jumps associated with magnetic avalanches and sharp energy injection to the magnetosphere. Such processes are similar to those known for the Barkhausen effect (see, e.g., [27]) but arise in strongly magnetized media at the energy scale larger by about 30 orders of magnitude.

This paper focuses on an example of outer crusts which consists of nearly spherical nuclei of a mass number A with an average density of bound nucleons $\mathcal{D}_{\text{Nb}} = A/V_{\text{WS}}$, related to the Wigner-Seitz volume V_{WS} and estimated as tenths of normal nuclear density $\mathcal{D}_0$ (see, e.g., [28] and references therein). The nucleons populate discrete energy levels in such strongly inhomogeneous matter. At field strengths $b_n \sim 10^{16}\text{–}10^{17}$ G corresponding to crossings of nuclear levels the structure of nuclei changes dramatically [24–26] leading, in particular, to an abrupt stepwise field dependence of a nucleus magnetic moment $m = \mu_N \sum_n \nu_n \theta(b - b_n)$ with the nucleon magneton μ_N and step function $\theta(x < 0) = 0$, $\theta(x \geq 0) = 1$. It is worthwhile to notice similar quantum fluctuations of the magnetization for rod and plate configurations of inner crusts [28], while analogous jump anomalies for delocalized electrons are known as the de Haas–van Alphen effect [27].

We consider the outer crusts as polycrystalline structures with nuclei arranged in a close packed (plausibly bcc [28]) lattice and assume the dipolar interaction between magnetic moments. Since such a system shows ferromagnetic ordering (cf., e.g., [29]), the crusts can be viewed as a hypercubic lattice of Π domains. The magnetization of the ith domain,

$$P_i = \frac{m_i}{V_{\text{WS}}} = \frac{m_i}{A\mu_N} p, \qquad p \approx \frac{\mathcal{D}_{\text{Nb}}}{\mathcal{D}_0} 10^{15} \text{ Oe}, \quad (1)$$

is determined by the local field

$$b_i = H(t) + J \sum_{j \in \mathrm{NN}} P_j - \eta \mathcal{P} + f_i . \qquad (2)$$

Here the adiabatically changing (see below), with time t, magnetic field H originates plausibly from the star core, the term containing an overall magnetization $\mathcal{P} = \sum_{i=1}^{\Pi} P_i/\Pi$, and parameter η accounts for the demagnetizing effect in a global form [27], while the nearest-neighbor (NN) domains contribute with the coupling strength J. The random fields $f_i = f_i^{st} + f_i^{d}$ implement an uncorrelated disorder as well as fluctuations, and are assumed to satisfy the Gaussian distribution $W(f) = \exp\{-f^2/2R^2\}/\sqrt{2\pi}\, R$, as one expects from the central limit theorem. The static random component f_i^{st} simulates irregular crystalline anisotropies and arbitrary varying interaction strengths caused by inhomogeneities and disorder in the form of defects, grain boundaries, and impurities in the crystalline structures. The dynamical component f_i^{d} accounts for interaction and correlation effects beyond the NN coupling and, therefore, it is determined by a certain configuration of moments $\{m_i\}$. We refer to such a model as a randomly jumping interacting moments (RJIM) model.

The conditions $b_{n+1} - b_i \gg R$ and $b_i - b_n \gg R$ correspond to a quiescent phase of star evolution when almost all the moments equal to $\mathcal{M}_n = \mu_{\mathrm{N}} \sum_{k=1}^{n} \nu_k$. For magnetic fields of considered strengths the estimates based on nonlinear terms in Ohm's law [30] due to ambipolar diffusion [31] yield a relatively short diffusion time of about $10^{1.5}$ yr. Such high rates might be detected by indirect measurements (see, e.g., [32] and references therein) during a couple of years. The recent analysis [33] of field evolution indicates that period clustering of magnetars is consistent with an assumption of magnetic field confined to the crust with its decay induced by Hall cascade.

As the magnetic field decreases the moment domains progressively jump to $\mathcal{M}_{n-1}$, $\mathcal{M}_{n-2}, \ldots$, when the difference between local effective fields b_i [Eq. (2)] and the respective quantities b_n, $b_{n-1}, \ldots$ changes sign. Because of the nearest-neighbor interaction, the jumping moment can result in the jump of a neighbor domain, which in turn might lead to the reducing moment of another neighbor, and so on, generating thereby an avalanche of moment jumps. The linear speed c_{m} of the avalanche propagation is determined by the ratio of the lattice constant $a \approx 10$ fm and the relaxation time $\tau_{\mathrm{N}} \approx 10^{-20.5}$ s for nuclear reconfiguration associated with magnetic response, $c_{\mathrm{m}} \approx a/\tau_{\mathrm{N}} \approx 10^8$ cm/s (for more details see [24]). Then for outer crusts of a linear size, $l_{\mathrm{crust}} \sim 100$ m, the estimate of the avalanche spanning time, $\tau_{\mathrm{av}} \approx l_{\mathrm{crust}}/c_{\mathrm{m}} \sim 0.1$ ms, is consistent with the rising time of SGR bursts [1–3,6,12]. The field $H(t)$ remains almost constant on the time scale τ_{av}, while the magnetization reduces sharply on a value proportional to the avalanche size. The corresponding excess of magnetic energy is released in the magnetosphere. Since the velocity of magnetoplasma waves (i.e., Alfvén waves) is close to the speed of light c, the linear size of the strongly excited magnetosphere region exceeds the value $R \approx l_{\mathrm{crust}} c/c_{\mathrm{m}}$, comparable to the neutron star radius. Subsequent cooling of photon-electron-positron plasma via gamma-ray emission from this region generates a SGR-burst event [21].

By assuming the field strength $H \sim 10^{16.5}$ G and employing an estimate [Eq. (1)], the upper limit of emitted energy is evaluated as

$$E_{\max}^{\mathrm{SGR}} \approx H \mathcal{P} l_{\mathrm{crust}}^3 \sim 10^{42} \text{ ergs} \qquad (3)$$

and is found to be in a good agreement with SGR-burst observations. When demagnetization jumps involve the inner crust as well, the avalanche linear size is an order of magnitude larger and the energy release extends up to $10^{44.5}$ ergs, a value corresponding to giant flare events.

Employing the mean-field approximation the contribution of NN domains $J \sum_{\mathrm{NN}} P_j$ to the local field Eq. (2) is replaced by an interaction with the overall magnetization $\mathcal{P}$ of the system. Then random fields can be viewed as mean-field fluctuations (cf., e.g., [34]). For simplicity we consider hereafter a single jump in stepwise m_i at the field strength b_1 with $\nu_1 = 1$. The average number of reducing moments $\bar{n}_{\mathrm{ind}}$ caused by a jump of one domain is given by the probability of finding a random field within an interval $[h, h + J_{\mathrm{eff}} p/\Pi]$ weighted with the domain number Π. Here $h = b_1 - J_{\mathrm{eff}} \bar{\mathcal{P}} - H$, and the coupling constant $J_{\mathrm{eff}} = J - \eta$ is effectively reduced because of the demagnetizing effect. At $J_{\mathrm{eff}} p/\Pi \ll R$ we evaluate $\bar{n}_{\mathrm{ind}} \approx J_{\mathrm{eff}} p W(h)$.

Calculating the ensemble average magnetization $\bar{\mathcal{P}}$ gives the mean-field magnetic equation of state (MEOS)

$$\bar{\mathcal{P}} = p \int W(f) m_i \, df = \frac{p}{2}\left(1 + \mathrm{erf}\left[\frac{-h}{\sqrt{2}R}\right]\right) \qquad (4)$$

with the error function $\mathrm{erf}[x]$.

The negatively defined magnetic susceptibility $\chi = d\bar{\mathcal{P}}/dH = p/[W(h)^{-1} - J_{\mathrm{eff}} p]$ corresponds to $\bar{n}_{\mathrm{ind}} > 1$, and yields the adiabatic spinodal region located between the singularity lines (i.e., critical fields) $H_{\mathrm{c}}^{\pm}(R) = b_1 - (J_{\mathrm{eff}} p/2)\big(1 \pm \pi^{3/2} r \sqrt{-\ln(r)} \mp \mathrm{erf}[\sqrt{-\ln(r)}]\big)$ which meet at the critical point $\{H_{\mathrm{c}}, R_{\mathrm{c}}\} = \{b_1 - (J_{\mathrm{eff}} p/2), J_{\mathrm{eff}} p/\sqrt{2\pi}\}$. Here $r = \sqrt{2\pi} R/J_{\mathrm{eff}} p$, and $+$ $(-)$ indicates the upper (lower) spinodal line. With increasing demagnetization energy (i.e., parameter η) the spinodal region narrows, indicating thereby dynamical tuning to critical fields. At such fields H_{C} the magnetization curve $\bar{\mathcal{P}}(H)$ shows diverging slope, while $\bar{n}_{\mathrm{ind}} = 1$. In the vicinity of such a region on the $\{H, R\}$ plane the system exhibits the widest distribution of avalanche sizes with a power law behavior.

Thus when the magnetic field approaches critical values the star enters the phase of intensified burst activity. The expectation time T_{s} for an inactive evolution is then determined by the ratio of the spacing β_{jmp} in magnetic moment jump anomalies and the change rate $\dot{B}$ of the overall field

[24] $T_s \approx -\beta_{\rm jmp}/\dot{B}$. Since the magnetic energy $F_B \sim B^2$ dominates and powers the emission we find $\dot{B} \sim -L_{\rm p}/B$. Using the familiar [20] relation for fields on star magnetic poles, $B_{\rm p} \sim \sqrt{L_{\rm p}}$, and assuming the proportionality with the crust field, $B \sim B_{\rm p}$, we obtain $T_{\rm s}^2 L_{\rm p} = \text{const}$. As illustrated in Fig. 1, SGR observables are well reproduced by this expression. Such systematics predict an expectation time of 3–4 years for the periodicity of intensive burst sets for SGR 1627-41 [19], suggesting thereby the next probable active phase in the fall of 2001 or 2002.

The cumulative avalanche size distribution in the vicinity of the critical point is compared in Fig. 2 with the cumulative fluence distribution, i.e., the burst number with a fluence exceeding the certain value. The observations by various missions are in good agreement with simulations, when accounting for the scale of the energy release [Eq. (3)], remoteness $\sim$10 kpc (e.g., [6,12]), and isotropic emission of the sources. Such an event number dependence is well fitted by the power law with an exponent 0.67 which corresponds to the value 1.67 for the differential distribution and provides a signal of self-organized criticality in the burst statistics.

As seen in Fig. 3 for different SGRs the waiting time distributions as a function of the reduced time, i.e., the time divided by the time at the maximum, display a universal function. The data are well reproduced by simulations and fitted in the vicinity of a maximum by the log normal function. Such a property points out the single time scale for SGR-burst triggering processes. Within RJIM such a time scale is determined by the ratio of the disorder parameter R and the field change rate: $\tau = R/\dot{B}$. Therefore, the scaling with respective time leads to a universal function.

In summary, the magnetodynamics of neutron star crusts have been considered within the RJIM model accounting in a phenomenological manner for quantum fluctuations due to the discrete level structure, internuclide coupling, disorder, and demagnetization energy. The comparison of model predictions with observational data allows one, therefore, to quantify crust properties in terms of the respective set of parameters, introduced here hypothetically. As demonstrated, at magnetic fields corresponding to jump anomalies of nuclide magnetic moments (caused by, e.g., level crossings), the demagnetization proceeds as sharp steps due to avalanche propagations. As a consequence, sudden energy releases to the magnetosphere lead to SGR bursts. The crust seismic activity is not implied within such a triggering mechanism, corroborative with a lack of spin-up glitches in the rotation of such pulsars. The quasiperiodic (with the field) magnetic moment jumps are consistent with some regularities in the SGR-burst emissions. As shown the scaling properties predicted by the RJIM model for, e.g., the burst intensity and waiting time distributions, are in good agreement with SGR observations supporting thereby the credibility of the RJIM model. As implied within considered treatment the specific classifying feature of SGRs is plausibly represented by the crust ultrastrong multipolar magnetic field components matching the strength region of important quantization effects in nuclide magnetization. For outer crusts such fields exceed 10^{16} G, while weaker fields are expected for neutron-rich nuclides of inner crusts [24–26]. Further implications of the proposed magnetic emission mechanism in the analysis of SGR activity can provide better understanding of neutron star crust, in particular, strengths and evolution of magnetic fields.

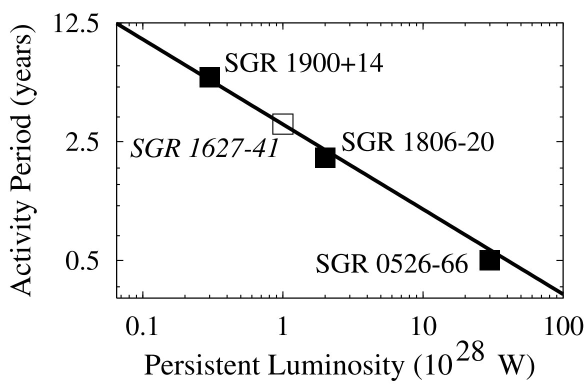

FIG. 1. Period of SGR's active phases versus the persistent luminosity. The solid squares represent observational data from [1,2,15–17] for SGR 1900 + 14, from [18] for SGR 1806-20, from [35] for SGR 0526-66, as also discussed in [22]. The solid line indicates the systematics with const $\approx 10^{43.5}$ erg $\cdot$ s (see text). The open square shows an expected position for SGR 1627-41 according to the persistent luminosity $L_{\rm p} \approx 10^{28}$ W [19].

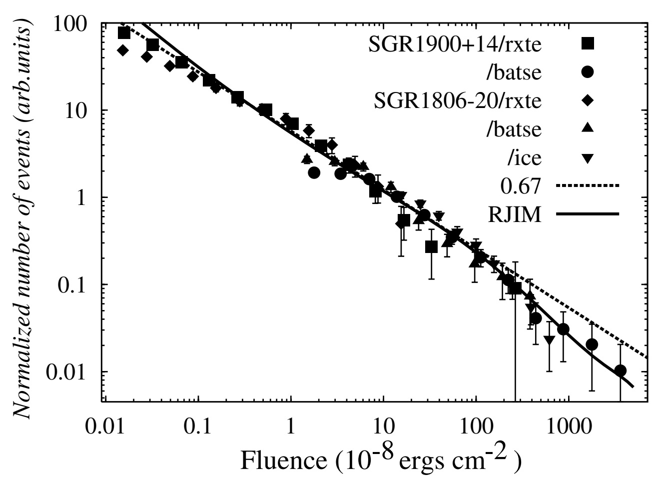

FIG. 2. Normalized cumulative fluence distribution in arbitrary (arb.) units of SGR bursts. The results of the RXTE and BATSE observations for SGR 1900 + 14 from [15] are shown by squares and circles, respectively. RXTE (diamonds), BATSE (up-triangles), and ICE (down-triangles) data for SGR 1806-20 are from [16]. The solid line represents the avalanche size distribution from RJIM for the cubic lattice of a size $(150)^3$. The dashed line denotes the power law distribution.

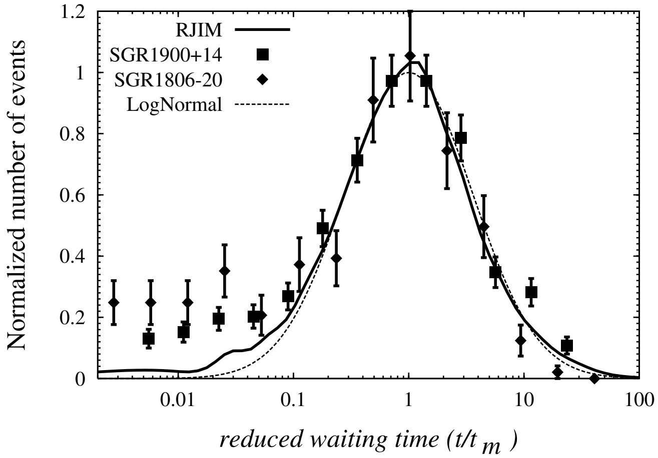

FIG. 3. The reduced waiting time distribution between the successive RXTE/PCA bursts from SGR 1900 + 14 (squares) 15] and SGR 1806-20 (diamonds) [16] is compared with the waiting time distribution between avalanches (solid curve). The dashed line represents the fit to the log normal distribution of the width 3.6.

Finally, we note that arrays of atomic clusters and/or quantum dots (see, e.g., [36]) can display similar noisy magnetodynamics at conditions far from magnetization reversal. Such an effect might be employed as a tool to analyze the roughness and disorder in an array.

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