Reconnection and the Ideal Evolution of Magnetic Fields

Allen H. Boozer
Columbia University, New York, New York 10027
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A magnetic evolution is ideal if it is consistent with the field being embedded in a perfectly conducting fluid. Faraday's law implies the evolution is ideal when the parallel component of the electric field is the derivative of a scalar potential, a condition that generically holds in any local region of space. Reconnection requires the non-existence of such a potential. In systems with two periodic directions, non-existence focuses reconnection onto the surfaces in which the magnetic field lines close on themselves, the rational surfaces. This rational surface effect does not arise in astrophysics but does appear in periodic simulation codes. Effects that could give astrophysical reconnection are discussed.

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I. Introduction.—The evolution of a magnetic field $\vec{B}(\vec{x},t)$ is called ideal if $\partial\vec{B}/\partial t = \vec{\nabla}\times(\vec{u}\times\vec{B})$. This is the evolution equation for a magnetic field embedded in a perfectly conducting fluid moving with velocity $\vec{u}(\vec{x},t)$. We will show that a generic magnetic evolution always appears ideal in a local region of space. The velocity $\vec{u}$ has the interpretation of being the velocity of the magnetic field lines, Sec. III.

Faraday's law, $\partial\vec{B}/\partial t = -\vec{\nabla}\times\vec{E}$, provides one relation between an evolving magnetic field and an electric field. The properties of the medium in which the magnetic field is embedded provide a second. The second relation is often far more complicated than the conventional Ohm's law, $\vec{E}+\vec{v}\times\vec{B} = \frac{\eta}{\mu_0}\vec{\nabla}\times\vec{B}$ with $\vec{v}$ the velocity of the medium and η its resistivity. The velocity the field lines $\vec{u}$ is in general distinct from the velocity of the medium $\vec{v}$, Sec. IV. Regardless of the complexities of the actual medium in which a magnetic field is embedded, the evolution of that magnetic field is consistent with its being embedded in an ideal fluid, a fluid with $\eta = 0$?

A given magnetic evolution determines through Faraday's law the electric field up to an arbitrary gradient. From this we will show that the evolution of a magnetic field appears ideal in any region of space for which a function $\Phi(\vec{x},t)$ exists that satisfies the magnetic differential equation [1] $\vec{B}\cdot\vec{\nabla}\Phi = -\vec{E}\cdot\vec{B}$. The electric field is determined by the physics of the medium in which the magnetic field is embedded. The most important solvability constraint [2,3] on the equation for $\Phi(\vec{x},t)$ is that the loop integral of the electric field along any closed magnetic field line vanishes, $\oint\vec{E}\cdot d\vec{\ell} = 0$. If this constraint is not satisfied, $\Phi(\vec{x},t)$ becomes multivalued, which means it is not a proper function of position. However, we will show that for a generic magnetic field, a solution Φ always exists in a sufficiently local region of space.

Fast reconnection [4] is a nonideal evolution of a magnetic field in which the nonideal behavior is concentrated in thin layers. Fast reconnection (usually called just reconnection) is considered an extremely important phenomenon in laboratory and space plasmas. Fast reconnection can easily be observed in a computer code with two periodic directions [5–7] or in a geometric torus, such as a toroidal device for magnetic fusion. In either case, the solvability constraint $\oint\vec{E}\cdot d\vec{\ell} = 0$ focuses reconnection on the surfaces on which the magnetic field lines close on themselves when periodically extended, Sec. III. In toroidal geometry these surfaces are called rational magnetic surfaces.

In space and astrophysics, magnetic field lines rarely close on themselves, so the localization of nonideal magnetic behavior by the failure of the solvability constraint $\oint\vec{E}\cdot d\vec{\ell} = 0$ on closed field lines is an unlikely explanation for fast reconnection. The reconnection phenomena observed in computer codes that have periodic boundary conditions in two directions apparently have little to do with the reconnection observed space and astrophysical systems. In Sec. V phenomena that can give reconnectionlike behavior in space or astrophysical plasmas are discussed.

The model of the medium in which the magnetic field is embedded can fundamentally change the properties of nonideal magnetic evolution, like fast reconnection [5]. Nevertheless, a number of properties of magnetic evolution can be obtained from Faraday's law alone. These properties, which are the focus of this paper, are model independent and are, therefore, of particular importance.

II. Demonstration.—Faraday's law, $\partial\vec{B}/\partial t = -\vec{\nabla}\times\vec{E}$, determines the electric field associated with an evolving magnetic field $\vec{B}(\vec{x},t)$ to within an additive gradient, $\vec{\nabla}\Phi(\vec{x},t)$. The Ohm's law for a perfectly conducting fluid flowing with a velocity $\vec{u}$ is usually given as $\vec{E}+\vec{u}\times\vec{B} = 0$. However, because of the arbitrary additive gradient, the evolution is consistent with the magnetic field being embedded in a perfectly conducting fluid if

$$\vec{E}+\vec{u}\times\vec{B} = -\vec{\nabla}\Phi\,. \qquad (1)$$

Equation (1) is a much weaker condition for an ideal evolution than $\vec{E}+\vec{u}\times\vec{B} = 0$ that was used, for example, in the seminal work [8] of Lau and Finn on reconnection and magnetic nulls. They considered an electric

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potential but assumed that it had to satisfy the constraint $\vec{B} \cdot \vec{\nabla}\Phi = 0$.

The paradox of a generic magnetic evolution appearing ideal is demonstrated by showing that an arbitrary vector field $\vec{E}(\vec{x}, t)$ can be locally written in the form of Eq. (1). If the magnetic field has no nulls, which means $|\vec{B}|$ is nonzero, the potential Φ can be chosen so the component of Eq. (1) along the magnetic field is locally satisfied for an arbitrary $\vec{E}(\vec{x}, t)$. The required equation for the potential is

$$\vec{B} \cdot \vec{\nabla}\Phi = -\vec{E} \cdot \vec{B}. \tag{2}$$

This differential equation for Φ is always locally solvable if $|\vec{B}|$ is nonzero. If $|\vec{B}|$ is nonzero, the flow velocity $\vec{u}$ can balance the components of $\vec{E}(\vec{x}, t)$ perpendicular to the magnetic field in Eq. (1). The required flow perpendicular to the magnetic field is

$$\vec{u}_\perp = \frac{(\vec{E} + \vec{\nabla}\Phi) \times \vec{B}}{B^2}. \tag{3}$$

The parallel flow is arbitrary.

At a magnetic null Eqs. (2) and (3) appear singular. However, near a generic null of a magnetic field, these equations are nonsingular, and the evolution can be viewed as ideal. The evolution near a magnetic null is locally ideal if the flow $\vec{u}$ can be chosen so $\vec{\nabla} \times (\vec{E} + \vec{u} \times \vec{B}) = 0$ with $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$. An identity of vector calculus is

$$\vec{\nabla} \times (\vec{u} \times \vec{B}) = \vec{u}\vec{\nabla} \cdot \vec{B} - \vec{B}\vec{\nabla} \cdot \vec{u} + \vec{B} \cdot \vec{\nabla}\vec{u} - \vec{u} \cdot \vec{\nabla}\vec{B}. \tag{4}$$

Consequently, near a magnetic null at $\vec{x} = \vec{x}_0(t)$, the evolution is ideal if a flow $\vec{u}$ can be found such that

$$\vec{u} \cdot \vec{\nabla}\vec{B} = -\left(\frac{\partial\vec{B}}{\partial t}\right)_{\vec{x}_0}. \tag{5}$$

Near the null the magnetic field has the form $\vec{B} = \overleftrightarrow{\mathcal{B}} \cdot [\vec{x} - \vec{x}_0(t)]$. The tensor $\overleftrightarrow{\mathcal{B}}$ has two properties. (i) $\overleftrightarrow{\mathcal{B}}$ has no trace, because the divergence of the magnetic field is zero. (ii) $\overleftrightarrow{\mathcal{B}}$ is symmetric or not depending on whether the magnetic field is curl-free. However, for a generic null the tensor $\overleftrightarrow{\mathcal{B}}$ can satisfy no other conditions such as having a zero determinant, $|\overleftrightarrow{\mathcal{B}}|$. The reason is that a zero of the magnetic field requires $B_x(x, y, z) = 0$, $B_y(x, y, z) = 0$, and $B_z(x, y, z) = 0$, which is three equations with three unknowns (x, y, z). An additional condition, such as $|\overleftrightarrow{\mathcal{B}}| = 0$, would be four conditions on three unknowns. At a magnetic null, Eq. (5) is $\overleftrightarrow{\mathcal{B}} \cdot \vec{u} = -\partial\vec{B}/\partial t$. This matrix equation can be solved for $\vec{u}$ when the tensor $\overleftrightarrow{\mathcal{B}}$ has a nonzero determinant, and the solution is $\vec{u} = d\vec{x}_0/dt$. If the determinant $|\overleftrightarrow{\mathcal{B}}|$ is nonzero, one can show that the field is zero only at a point, not along a curve, and that a small perturbation can move, but cannot remove, the null of the magnetic field.

For a point null, $|\overleftrightarrow{\mathcal{B}}|$ nonzero, Eq. (2) for Φ and Eq. (3) for $\vec{u}_\perp$ are nonsingular at the null. Equation (2) requires $(\vec{\nabla}\Phi)_{\vec{x}_0} = -\vec{E}(\vec{x}_0)$, which implies $\vec{E} + \vec{\nabla}\Phi = \overleftrightarrow{\mathcal{E}} \cdot (\vec{x} - \vec{x}_0)$ near the null. Inserting this form into Eq. (3), one finds that both the numerator and denominator of the equation for $\vec{u}_\perp$ depend quadratically on the distance from the null. Since B^2 is nonzero in all directions away from a point null, the expression for $\vec{u}_\perp$ is nonsingular in the limit as $\vec{x} \to \vec{x}_0$.

In some theoretical examples of reconnection, the magnetic field is orthogonal to a symmetry direction. In such cases, a magnetic null is a null along a line in the symmetry direction. However, an arbitrarily small perturbation can change such a magnetic field into a field that is zero only at separated points. Consequently, the case of a null along a line is not generic and presumably has little physical relevance. It should be noted that in the four coordinates of space-time, (x, y, z, t), nulls can occur at isolated points of all three components of the magnetic field plus the determinant $|\overleftrightarrow{\mathcal{B}}|$. These isolated space-time points are the places where magnetic nulls can appear or disappear.

III. Interpretation of $\vec{u}$.—As we have seen, the generic evolution of a magnetic field is at least locally consistent with the magnetic field being embedded in a perfectly conducting fluid flowing with a velocity $\vec{u}$. What is the interpretation of the flow velocity $\vec{u}$ when the magnetic field is embedded in a resistive or insulating medium? The velocity $\vec{u}$ can be interpreted as the flow of the field lines. More precisely, it is the velocity of a set of coordinates that is defined by the magnetic field.

Let θ and φ be any pair of coordinates; then an arbitrary vector can be written as $\vec{A} = \psi_t\vec{\nabla}\theta - \psi_p\vec{\nabla}\varphi + \vec{\nabla}g$, which implies any magnetic field can be written in the form [3]

$$\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}\theta + \vec{\nabla}\varphi \times \vec{\nabla}\psi_p. \tag{6}$$

If $\vec{B} \cdot \vec{\nabla}\varphi = (\vec{\nabla}\psi_t \times \vec{\nabla}\theta) \cdot \vec{\nabla}\varphi \neq 0$, the quantities $(\psi_t, \theta, \varphi)$ have a finite Jacobian and can be used as coordinates. This coordinate system is defined by $\vec{x}(\psi_t, \theta, \varphi, t)$, which gives positions in space as a function of $(\psi_t, \theta, \varphi)$ and time.

The magnetic field line trajectories in $(\psi_t, \theta, \varphi)$ coordinates are given by Hamilton's equations of motion with $\psi_p(\psi_t, \theta, \varphi, t)$ the Hamiltonian [3],

$$\frac{d\theta}{d\varphi} = \frac{\partial\psi_p}{\partial\psi_t} \quad \text{and} \quad \frac{d\psi_t}{d\varphi} = -\frac{\partial\psi_p}{\partial\theta}. \tag{7}$$

The velocity of the $(\psi_t, \theta, \varphi)$ coordinates is

$$\vec{u} \equiv \frac{\partial\vec{x}(\psi_t, \theta, \varphi, t)}{\partial t}. \tag{8}$$

The electric field is related to the vector and scalar potentials by $\vec{E} = -\partial\vec{A}/\partial t - \vec{\nabla}\Phi_{\vec{x}}$. This equation can be rewritten in $(\psi_t, \theta, \varphi)$ coordinates in the form [3,9]

$$\vec{E} + \vec{u} \times \vec{B} = \left(\frac{\partial\psi_p}{\partial t}\right)_{\psi_t,\theta,\varphi} \vec{\nabla}\varphi - \vec{\nabla}\Phi. \tag{9}$$

If $\partial\psi_p/\partial t = 0$, the magnetic field lines can distort, but no topology change can occur. This is equivalent to saying that the motion of $(\psi_t, \theta, \varphi)$ coordinates can absorb the evolution of the field if a $\vec{u} = \partial\vec{x}/\partial t$ and a Φ can be found with $\vec{E} + \vec{u} \times \vec{B} = -\vec{\nabla}\Phi$. This equation has the same form as Eq. (1) but a subtly different interpretation.

Nonideal magnetic behavior, or reconnection, requires that $\partial\psi_p/\partial t$ be nonzero. If a solution for Φ exists to $\vec{B} \cdot \vec{\nabla}\Phi = -\vec{E} \cdot \vec{B}$ with a nonsingular field line flow $\vec{u}$, then there is no reconnection.

A doubly periodic system, $\vec{x}(\psi_t, \theta + 1, \varphi) = \vec{x}(\psi_t, \theta, \varphi + 1) = \vec{x}(\psi_t, \theta, \varphi)$, in which the magnetic field lines locally lie on constant ψ_t surfaces, $\vec{B} \cdot \vec{\nabla}\psi_t = 0$, clarifies what is meant by nonideal behavior and reconnection. In such a system a θ coordinate exists that has the property that ψ_p depends on only ψ_t and t. These $(\psi_t, \theta, \varphi)$ coordinates are the action-angle variables of Hamiltonian mechanics, with φ the canonical time, or the magnetic coordinates of plasma physics [3,10]. In these coordinates, the equation $\vec{B} \cdot \vec{\nabla}\Phi = -\vec{E} \cdot \vec{B}$ can be solved essentially analytically for Φ. To do this, Fourier expand the parallel electric field,

$$\frac{\vec{E} \cdot \vec{B}}{\vec{B} \cdot \vec{\nabla}\varphi} = \sum_{m,n} V_{mn}(\psi_t) e^{i2\pi(n\varphi - m\theta)}, \tag{10}$$

then the Fourier coefficients of Φ are

$$\Phi_{mn}(\psi_t) = \frac{i}{2\pi} \frac{V_{mn}(\psi_t)}{n - \iota(\psi_t)m}. \tag{11}$$

The rotational transform $\iota(\psi_t) \equiv d\psi_p/d\psi_t$. Resonant Fourier components of Φ have a singular form on rotational surfaces on which the rotational transform is the ratio of two integers, $\iota = N/M$. On such surfaces, magnetic field lines close on themselves after being extended M periods of the φ coordinate and N periods of the θ coordinate. If the electric and magnetic fields are well-behaved functions of position, Φ is also well behaved except on the surfaces in which the field lines close on themselves, the rational surfaces. Near the rational surfaces, the singularities of Φ cause a singular flow for the magnetic field lines. Ignoring the spatial variation of the magnetic field, $\vec{B} \cdot \vec{\nabla} \times \vec{u} \approx -\nabla_\perp^2\Phi$ with the perpendicular sign meaning perpendicular to the field lines. This singularity of the flow of the field lines is what causes the failure of the ideal evolution of a magnetic field near rational magnetic surfaces. The existence of rational surfaces naturally leads to a breaking of the ideal evolution of a magnetic field in thin layers and to fast reconnection.

In systems in which the magnetic field lines do not lie in doubly periodic surfaces, which presumably includes all astrophysical systems, singularities in the field line flow cannot be caused by rational surfaces. Other ways exist for a singular flow of the magnetic field lines to arise, Sec. V, and must be the cause of reconnection in astrophysical systems. In addition to fast reconnection, which is focused on rational surfaces, doubly periodic systems can also have a slow reconnection phenomenon. This arises when the zero-zero Fourier term of the V_{mn} series is nonzero. This zero-zero term is called the loop voltage

$$V_{00}(\psi_t, t) \equiv \frac{\partial}{\partial\psi_t} \int \vec{E} \cdot \vec{B}\, d^3x. \tag{12}$$

However, in this evolution the magnetic field lines remain on nested toroidal surfaces. In other words, the topology of the surfaces on which field lines lie remains fixed. Indeed, in a region in which the loop voltage is a spatial constant, the magnetic field is independent of time. In tokamak literature, this is an Ohmic steady state in which a constant magnetic field is maintained against slow resistive decay by a transformer induced loop voltage.

Boundary conditions on rigid perfectly conducting surfaces can cause reconnection. A rigid perfectly conducting surface is defined by making the electric field zero, $\vec{E} = 0$, and the flow velocity of the field lines zero, $\vec{u} = 0$, on the surface, which implies, Eq. (1), the potential Φ is constant on the surface. On field lines that leave and reenter a rigid perfect conductor, the potential Φ must satisfy boundary conditions at both ends, which are generally inconsistent with the existence of a global solution to $\vec{B} \cdot \vec{\nabla}\Phi = -\vec{E} \cdot \vec{B}$. The reconnection implied by this failure of global solvability is often of the slow variety, which means without singular layers. Effects that can lead to singular layers and fast reconnection are discussed in Sec. V.

IV. Consistency with Ohm's law.—If an evolving magnetic field is embedded in a resistive fluid, how can the evolution of the field appear ideal? The most familiar model of a resistive fluid uses the Ohm's law $\vec{E} + \vec{v} \times \vec{B} = \eta\vec{j}$. The most general expression for the electric field that is consistent with an ideal evolution for the magnetic field is $\vec{E} + \vec{u} \times \vec{B} = -\vec{\nabla}\Phi$. These two expressions for the electric field imply a magnetic evolution is ideal if and only if $(\vec{v} - \vec{u}) \times \vec{B} = \eta\vec{j} - \vec{\nabla}\Phi$. This condition is satisfied if a function $\Phi(\vec{x}, t)$ exists with $\vec{B} \cdot \vec{\nabla}\Phi = \eta\vec{j} \cdot \vec{B}$. The argument given in Sec. III implies this is true for any generic magnetic field in a sufficiently local region of space. The velocity $\vec{v} - \vec{u}$ is the velocity of the resistive fluid relative to the magnetic field lines. In other words, the evolution of a magnetic field may be ideal even when the evolution of the fluid in which it is embedded is dissipative. An obvious example is a plasma that is maintained in a steady state by sources of particle and energy as the plasma diffuses across the toroidal magnetic surfaces of a stellarator with no transformer. In this case the magnetic field is stationary, $\vec{u} = 0$ and $\partial\psi_p/\partial t = 0$. If a magnetic field is embedded in a conducting fluid, then the ideal evolution of the field is a much easier condition to satisfy than is the ideal evolution of the fluid.

The equation for the electric field in a plasma is far more complicated than in a simple resistive fluid. However, the basic condition for an ideal evolution of an embedded magnetic field is similar. The force balance equation for each species of charged particles in a plasma can be written in the form

$$\vec{E} + \vec{v}_s \times \vec{B} = \vec{R}_s. \tag{13}$$

The velocity of the species is $\vec{v}_s$, the sum of the forces on the species due to inertia, pressure, and dissipation is $q_s n_s \vec{R}_s$, and the charge density is $q_s n_s$. A corollary to Eq. (13) is $\vec{R}_s \cdot \vec{B} = \vec{E} \cdot \vec{B}$, which implies $\vec{R}_s \cdot \vec{B}$ is the same for all species. If $\vec{R}_s \cdot \vec{B} = \vec{B} \cdot \vec{\nabla} w(\vec{x})$ for any species, the magnetic evolution is ideal.

Even when the magnetic field evolution is ideal, the various species can cross the magnetic field lines with each species having its own velocity. Equations (1), (13), and $\vec{R}_s \cdot \vec{B} = \vec{B} \cdot \vec{\nabla} w(\vec{x})$ imply

$$\vec{R}_s = (\vec{v}_s - \vec{u}) \times \vec{B} + \vec{\nabla} w, \tag{14}$$

which implies species s flows across the magnetic field lines at the velocity

$$(\vec{v}_s - \vec{u})_\perp = \frac{(\vec{R}_s - \vec{\nabla} w) \times \vec{B}}{B^2}. \tag{15}$$

In other words, the tying of the magnetic field to any plasma species is a much easier constraint to break than is the constraint of an ideal evolution for the magnetic field itself. The breaking of the tying of the ions to the magnetic field by ideal inertial effects allows reconnection to proceed more rapidly in a plasma than in a simple resistive fluid. This has been the subject of much recent work and the topic of a recent tutorial paper [5].

V. Conclusion.—The generic evolution of a magnetic field is locally an ideal evolution, which means the flow is consistent with the field being embedded in a perfectly conducting fluid flowing with a velocity $\vec{u}$. If the magnetic field is actually embedded in a nonideal or insulating medium, then the velocity $\vec{u}$ is the flow of a coordinate system in which the magnetic field lines preserve their topology provided a well-behaved function, $\Phi(\vec{x}, t)$, exists that satisfies the differential equation $\vec{B} \cdot \vec{\nabla}\Phi = -\vec{E} \cdot \vec{B}$. The fundamental requirement for reconnection of magnetic field lines is that there is no such function. In systems with two periodicity directions, this requirement focuses the reconnection into thin layers around the rational surfaces where the field lines close on themselves when periodically extended. The applicability of computer codes that have two periodic directions to space and astrophysical plasmas is at best obscure. Since a generic magnetic field can have only isolated nulls, models that have a null of the magnetic field at each point along a curve, such as a symmetric reconnection without a guide field, must have little relevance to the evolution of naturally occurring magnetic fields.

In space and astrophysical plasmas reconnectionlike phenomena can occur for at least four reasons that have nothing to do with closed magnetic field lines. Two of the four types of phenomena involve nonideal magnetic effects. (i) Each field line that enters and leaves a volume of interest has length L that is parametrized by its entry point. Discontinuities in $\Phi(\vec{x}, t)$ can be produced if the lengths of field lines in an open system have discontinuities and may be responsible for focusing reconnection into thin layers. (ii) Given an arbitrary magnetic field $\vec{B}(\vec{x})$, the separation δ of neighboring field lines ($\delta \to 0$) will typically increase exponentially, $\delta \approx \delta_0 \exp(K\ell)$ with $1/K$ called Lyapunov length and ℓ the distance along a field line. [Typical Hamiltonians have an exponential separation of trajectories, and magnetic field lines are the trajectories of a Hamiltonian, Eq. (7).] The exponential separation of field lines coupled with the equation $\partial\Phi/\partial\ell = -\vec{E} \cdot \vec{B}/B$ for the potential implies $|\vec{\nabla}\Phi| \propto \exp(K\ell)$. The exponentially large currents required to drive the flows associated with $\vec{\nabla}\Phi \times \vec{B}$ can break the constraint of an ideal evolution when $K\ell \gg 1$. Two other phenomena may look like reconnection, but the magnetic evolution can be ideal. (iii) Loss of equilibrium and instabilities driven by the currents that are required to maintain the magnetic field are well known phenomena in an ideal plasma model. The energy that can be released by these phenomena can be a significant fraction of the energy in the field produced by the plasma current. Typically the energy goes into the kinetic energy of the plasma, which means accelerating the plasma to speeds that are a fraction of the Alfvén speed. Because of their mass, most of this kinetic energy is in the ions. (iv) An ideal magnetic evolution often leads to a high current density along the magnetic field lines. If the current density is greater than a critical value, which depends on the electron temperature and number density $j_D \approx en\sqrt{T_e/m_e}$, the electric field will exceed the Dreicer electric field [11]. The electron distribution will then form a high energy, or runaway tail, which could produce phenomena that resemble the electron heating expected from reconnection.

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