

## Reynolds Number Dependence of Streamwise Velocity Spectra in Turbulent Pipe Flow

J. F. Morrison, W. Jiang, B. J. McKeon, and A. J. Smits

*Department of Mechanical & Aerospace Engineering, Princeton University, Princeton, New Jersey 08544*  
(Received 11 September 2001; published 13 May 2002)

Spectra of the streamwise velocity component in fully developed turbulent pipe flow are presented for Reynolds numbers up to  $5.7 \times 10^6$ . Even at the highest Reynolds number, streamwise velocity spectra exhibit incomplete similarity only: while spectra collapse with both classical inner and outer scaling for limited ranges of wave number, these ranges do not overlap. Thus similarity may not be described as complete, and a region varying with the inverse of the streamwise wave number,  $k_1$ , is not expected, and any apparent  $k_1^{-1}$  range does not attract any special significance and does not involve a universal constant. Reasons for this are suggested.

DOI: 10.1103/PhysRevLett.88.214501

PACS numbers: 47.27.Jv, 02.50.-r, 05.45.-a, 47.27.Nz

Reynolds number similarity is an essential concept in describing the fundamental properties of turbulent wall-bounded flow. Unlike the drag coefficient for bluff bodies, that for a turbulent boundary layer continues to decrease with increasing Reynolds number because the small-scale motion near the surface is directly affected by viscosity at any Reynolds number. Therefore Reynolds number similarity is very important in design and is a vital tool for the engineer, who, plied with information from either direct numerical simulations or wind-tunnel tests (or both), may well have to extrapolate over several orders of magnitude in order to estimate quantities such as drag at engineering or even meteorological Reynolds numbers. Perhaps the most well-known example of Reynolds number similarity is the region of log velocity variation (the log law) found in wall-bounded flows which, at sufficiently high Reynolds numbers, exists regardless of the nature of the surface boundary condition or the form of the outer imposed length scale. The log law may be derived by an overlap argument, where the overlap is said to occur between a near-wall region described by “wall” variables, that is, a velocity scale  $u_\tau$  and a length scale  $\nu/u_\tau$  (the superscript “+” denotes nondimensionalization with wall variables), and an outer layer that depends on “outer” variables, that is, a velocity scale  $u_\tau$  and a length scale that is, for pipe flow, the radius  $R$ . Here,  $u_\tau = \sqrt{\tau_w/\rho}$ ,  $\tau_w$  is the wall shear stress,  $\rho$  is the density, and  $\nu$  is the kinematic viscosity [1].

For turbulence at sufficiently large distances from the wall, Kolmogorov’s famous theories express the most important way in which Reynolds number similarity is used [2]. Yet, of increasing importance is whether or not the large scales also exhibit Reynolds number similarity when suitably scaled. In the context of wall-bounded turbulent flow, there is growing interest in using these ideas to develop subgrid models and boundary conditions for large-eddy simulations (LES). In LES, only the large scales are resolved so that in the near-wall region where all the eddies are “small,” there is a need to model not only the energy drain from the resolved scales but also to provide an “off-the-surface” condition for the simulation. In

this context, self-similarity could be very useful. However, turbulence at any point consists of a range of scales so that simple similarity may exist only under a restrictive range of conditions. The situation is complicated by the fact that wall turbulence is highly anisotropic, owing to the different effects of the viscous constraint (the “no-slip” condition) and the impermeability constraint. The latter leads to a reduction in the wall-normal ( $v$ ) component at a streamwise wave number,  $k_1$ , that is roughly inversely proportional to the distance from the wall,  $y$ . The effect of this “blocking” or “splatting” is to increase the wall-parallel ( $u$  and  $w$ ) components down to the viscous dominated sublayer. This leads to the expectation that the spectrum for the  $v$  component at low wave numbers depends only on  $y$ , while spectra for the  $u$  and  $w$  components depend on both  $y$  and an outer length scale such as  $R$ . Here we concentrate on the behavior of the one-dimensional velocity spectrum for the streamwise velocity component,  $\phi_{11}(k_1)$ .

A particularly useful concept that explains much of the foregoing is Townsend’s theory concerning “active” and “inactive” motion [3]. The active motion comprises the shear-stress ( $\rho\overline{uv}$ ) bearing motion which therefore scales on  $y$  and  $u_\tau$  only. On the other hand, the inactive motion is of large scale, does not bear any significant shear stress and, to first order, resides only in the  $u$  and  $w$  components. Townsend [4,5] described the inactive motion as a “meandering or swirling” motion that contributes “to the Reynolds [shear] stress much further from the wall than the point of observation,” but not, as noted in [6] “at the point of observation.” Intrinsic to the concept of active motion are “attached wall eddies,” which are “in some sense are attached to the wall” and are therefore governed by  $u_\tau$ . Moreover, they “have diameters proportional to distance of their ‘centres’ from the wall” and therefore have properties determined by the length scale  $y$ . If it is supposed that the population of wall eddies is dominated by such structures, then simple similarity may exist.

The purpose of this Letter is to report important new measurements over a very large Reynolds number range in fully developed pipe flow. The data at very high Reynolds numbers are of special interest for scaling arguments at

low wave numbers since the viscous length scale may be assumed to be insignificant. Of particular note is that the measurements do not provide convincing support of the widely held notion that there exists a range of stream-wise wave numbers,  $k_1$ , in which  $\phi_{11}$  scales as  $k_1^{-1}$ , so exhibiting self-similarity. Yet evidence to the contrary abounds in laboratory data (with the usual restrictions in Reynolds number—a limitation to which the present data are not subject), both for fully developed pipe flow [7,8] and boundary layers [9,10]. Literature reporting observations in the atmospheric surface layer, a field to which the present results are particularly relevant, yields similar conclusions [11]. Here, we suggest that this result is actually *not* at variance with Townsend's later theory [5] which, in this respect, is contrary to his earlier work [4]. Moreover, we suggest reasons why this might be so.

Given the uniqueness of our results (in terms of high Reynolds number) and the prominence given to previous work confirming  $\phi_{11} \propto k_1^{-1}$ , it is clear that a careful reappraisal of the conditions under which a self-similar  $k_1^{-1}$  range may exist is required. Its oft-stated appearance (so much so, that the existence of a  $k_1^{-1}$  range is often taken for granted [12]) might plausibly be the result of interpretation alone: a prescribed slope over some region of wave number can usually be found in turbulence spectra on log-log axes. However, more sceptical views are also available [13]. There are several derivations, but here we restrict ourselves to a reappraisal of the dimensional analysis of Perry and co-workers [7–10], in the limit of infinite Reynolds number.

Scalings for “large” scales (in which the direct effects of viscosity may be neglected) that contribute to the stream-wise velocity component may be expressed functionally as

$$\phi_{11} = f(y, u_\tau, R, k_1). \quad (1)$$

Outer scaling suggests that the wall-normal distance,  $y$ , is not important and that dimensional analysis therefore yields

$$\frac{\phi_{11}(k_1)}{Ru_\tau^2} = \frac{\phi_{11}(k_1R)}{u_\tau^2} = g_1(k_1R), \quad (2)$$

while, alternatively, “inner” scaling suggests the exclusion of  $R$  as a relevant length scale so that, at higher wave numbers,

$$\frac{\phi_{11}(k_1)}{yu_\tau^2} = \frac{\phi_{11}(k_1y)}{u_\tau^2} = g_2(k_1y). \quad (3)$$

The veracity of these scalings is usually judged by the degree of collapse of the spectra at wave numbers lower than that at which spectral transfer becomes important. In the range of wave numbers  $R^{-1} < k_1 < y^{-1}$  over which both Eqs. (2) and (3) are valid (that is, where collapse is evident with *both* scalings, as required by asymptotic matching), it follows that

$$\phi_{11}(k_1) = Ru_\tau^2 g_1(k_1R) = yu_\tau^2 g_2(k_1y). \quad (4)$$

Dimensional arguments and direct proportionality between  $g_1$  and  $g_2$  therefore imply

$$\frac{\phi_{11}(k_1R)}{u_\tau^2} = \frac{A_1}{k_1R} = g_1(k_1R), \quad (5)$$

and

$$\frac{\phi_{11}(k_1y)}{u_\tau^2} = \frac{A_1}{k_1y} = g_2(k_1y), \quad (6)$$

where  $A_1$  is a universal constant. Collapse with both length scales therefore suggests a self-similar structure such that  $\phi_{11}(k_1) \propto u_\tau^2 k_1^{-1}$ . One could therefore call this situation “complete similarity.” However, it is possible that, for example, while  $y$  and  $u_\tau$  might form a complete parameter set to define the motion in the range of wave numbers over which collapse is apparent [Eq. (3)], these wave numbers might in fact be too high for collapse to be possible using  $R$  and  $u_\tau$  [Eq. (2)]. Thus simultaneous collapse is not possible. We shall refer to this situation as “incomplete similarity,” in which case the constant  $A_1$  in Eqs. (5) and (6) cannot be universal.

This analysis is predicated on two principal assumptions. The first is that the viscosity,  $\nu$ , does not enter the problem. This requires that  $k_1 \ll u_\tau/\nu$ . In turn, this requires the Reynolds number to be sufficiently high, or equivalently that  $y$  is sufficiently large such that the energy-containing scales are not affected directly by viscosity. The second assumption is that  $u_\tau$  is the correct velocity scale for both inner and outer scaling. In particular, in conformity with Townsend's theory, it supposes that inactive motion arises primarily through the influence of attached eddies and that therefore  $u_\tau$  is the appropriate velocity scale.

Below, spectra are presented in premultiplied form on linear-log axes since a linear ordinate enables a closer scrutiny of scalings than that afforded by a logarithmic one. In particular, any  $k_1^{-1}$  dependence will show as a horizontal line. In addition, the use of nondimensional axes ensures that not only the ordinate but also the area under the spectra is directly proportional to energy. Integration of the spectra, therefore, yields  $u^2/u_\tau^2$ . Spectra are given in the form

$$\frac{k_1R\phi_{11}(k_1R)}{u_\tau^2} = h_1(k_1R), \quad (7)$$

for outer scaling, and for inner scaling, in the form

$$\frac{k_1y\phi_{11}(k_1y)}{u_\tau^2} = h_2(k_1y). \quad (8)$$

In the context of the present experiment, it is useful to clarify precisely what the foregoing analysis indicates. Strictly, as long as  $\nu/u_\tau \ll y < R$  (the Reynolds number is “high”), Eqs. (5) and (6) should both show a  $k_1^{-1}$  range for  $R^{-1} \ll k_1 \ll y^{-1}$ . In order to remove the ambiguity concerning the relative values of  $y$  and  $R$ , we have chosen to fix alternately  $y$  in Eq. (5) and then  $R$  in Eq. (6). Then,

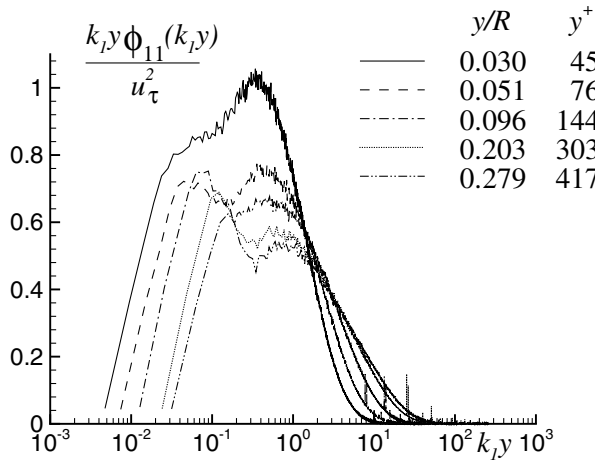


FIG. 1. Inner scaling,  $Re_D = 5.50 \times 10^4$ ,  $R^+ = 1.50 \times 10^3$ .

Eq. (7) invites us to retain only  $R$  and  $u_\tau$ . Thus while  $y$  is fixed,  $u_\tau$  is varied by changing the pressure drop along the pipe. In practice, this involves a change of Reynolds number (strictly Kármán number,  $Ru_\tau/\nu$ ) since changing  $R$  is more difficult, but changing the Reynolds number does not pose a problem as long as it is sufficiently high such that the wave number range of interest is not directly affected by viscosity. Alternatively, Eq. (8) invites the use of  $y$  and  $u_\tau$  only for any fixed  $R$ . In this case, we have merely varied  $y$  (subject to  $\nu/u_\tau \ll y < R$ ) at a fixed Reynolds number, although, as long as  $\nu$  can be neglected, a value of  $y$  at any Reynolds number might be chosen.

Details of the pressurized pipe and results from extensive Pitot-tube measurements of the mean velocity are provided in [1]. Here, fluctuating velocity measurements are made using standard hot-wire techniques using wires with length-to-diameter ratios  $l/d = 200$  for low Reynolds numbers, and  $l/d = 100$  for  $Re_D \geq 10^6$ . Typically, the resolution is such that  $k_1 \eta \approx O(1)$ , where  $\eta$  is the Kolmogorov length scale. Further details are given in [14].

Using inner scaling, Fig. 1 shows  $\phi_{11}(k_1 y)$  in the form given by Eq. (8) for  $Re_D = 5.50 \times 10^4$  over the range in  $y$  for which collapse might be expected. Figure 2 shows data for  $Re_D = 5.7 \times 10^6$  plotted in the same form. In Fig. 1, it is evident that the Reynolds number is simply too low for collapse to be possible. Note that  $R^+ = 1500$  only. At the highest Reynolds number (Fig. 2), there is some collapse for  $0.05 < k_1 y < 1$  approximately, the range increasing with Reynolds number. However, the collapse is not along a horizontal line, suggesting incomplete similarity. Figure 3 shows the same data as in Fig. 2, but plotted using outer scaling. At the highest Reynolds number (Fig. 3) peaks around  $k_1 R \sim 1$  for  $0.03 \leq y/R \leq 0.05$  suggest collapse may be possible, but inspection of Fig. 2 in the region of  $k_1 y \sim 0.03$  shows that the same data clearly do not collapse using inner scaling. Instead, spectra at all positions show discrete separate peaks. This also suggests incomplete similarity. Figure 4 shows spectra scaled

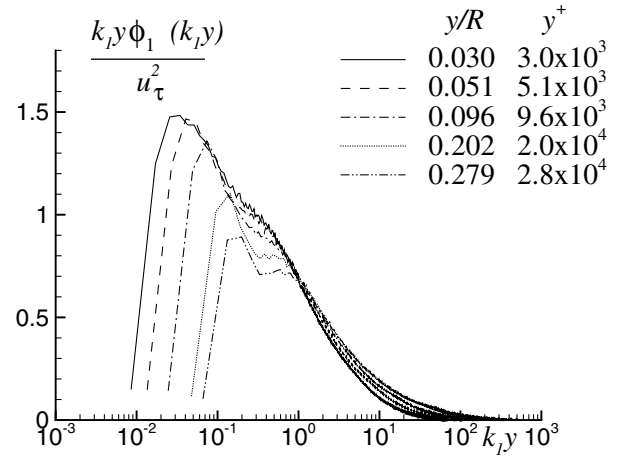


FIG. 2. Inner scaling,  $Re_D = 5.7 \times 10^6$ ,  $R^+ = 10^5$ .

using outer scaling for data at all three Reynolds numbers with fixed  $y/R = 0.1$ . At the two higher Reynolds numbers, there is some collapse at low wave numbers, and as in Fig. 3 for  $k_1 R \sim 1$ , apparently occurring on a horizontal line over about half a decade in  $k_1 R$ . However, as the collapse is not apparent with inner scaling (Fig. 2), this again suggests only incomplete similarity. Note also that values of the ordinate in regions where collapse might be seen to be possible for Figs. 1–4 varies from one figure to another. This behavior is obviously inconsistent with a universal value of  $A_1$ .

On balance, it would appear that while collapse of the velocity spectra may be possible with either inner or outer scaling (incomplete similarity), it is unlikely that simultaneous collapse with both scalings over the same wave number range is possible, at least up to the maximum Reynolds number attained here. That is, complete similarity is not observed. As suggested in [15], the behavior of  $\overline{u^2}^+$  is consistent with the concept of inactive motion which (a) increases with Reynolds number and (b) increases as  $y/R$  decreases. On the basis of (a) and (b) alone, complete

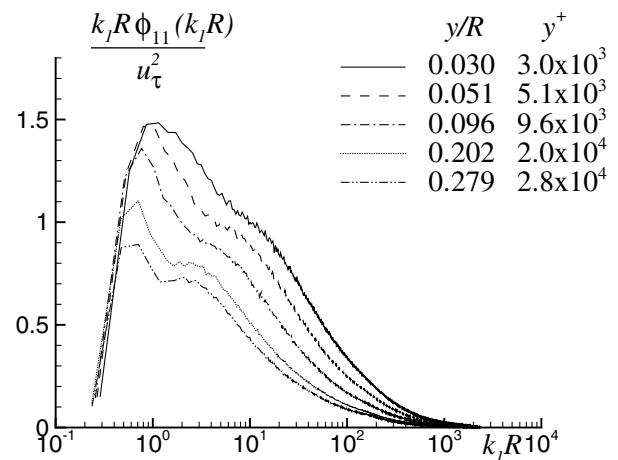
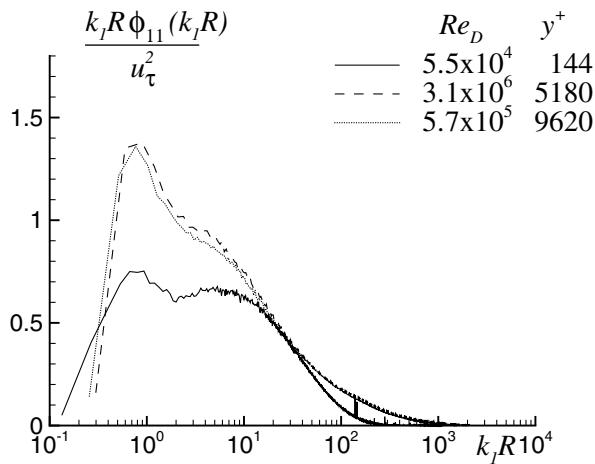


FIG. 3. Outer scaling,  $Re_D = 5.7 \times 10^6$ ,  $R^+ = 10^5$ .

FIG. 4. Outer scaling,  $y/R = 0.1$ .

similarity at practical Reynolds numbers is unlikely. While the active motion scales on  $y$  and  $u_\tau$  only (in the limit of infinite Reynolds number) as Townsend proposed, the inactive component always requires three scales, namely,  $y$ ,  $R$ , and a velocity scale, in compliance with (a) and (b) above. This shows that active and inactive components interact, as exemplified by the Reynolds number dependence of the peak in  $\overline{u^2}^+$  at the position of maximum energy production in the sublayer [15]. Different velocity scales for the inner and outer scalings of Eqs. (5) and (6) are an even more serious proposition: self-similarity would not then be possible under any circumstance. The present theory does not distinguish the  $u$ - and  $w$ -velocity components, and it suggests the presence of complete similarity in both directions. Yet this is inconsistent with the mean shear in the  $x$  direction being responsible for quasi-stream-wise vortices which have no spanwise equivalent. Although the present data do not support the existence of a self-similar  $k_1^{-1}$  range, Townsend suggested as early as 1976 [5] that “It now appears that simple similarity of the motion is not possible with attached eddies . . . .” Moreover, negative results are often not reported so that evidence supporting a self-similar range may not be as overwhelming as might

appear at first sight. Thus, even though at high-enough Reynolds numbers a region exhibiting complete similarity is found for the mean velocity, no such similarity is apparent for the higher moments. Given the considerations outlined here, such a result should not be unexpected.

The support of ONR under Grants No. N00014-98-1-0525 and No. N00014-99-1-0340 is gratefully acknowledged. J.F.M. is indebted to both the Engineering and Physical Sciences Research Council (GR/M64536/01 and GR/M84855/01) and the Royal Academy of Engineering (United Kingdom) for additional financial support.

- [1] M. V. Zagarola and A. J. Smits, *J. Fluid Mech.* **373**, 33 (1998).
- [2] S. G. Saddoughi and S. V. Veeravalli, *J. Fluid Mech.* **268**, 333 (1994).
- [3] A. A. Townsend, *J. Fluid Mech.* **11**, 97 (1961).
- [4] A. A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1976), 1st ed.
- [5] A. A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1976), 2nd ed.
- [6] P. Bradshaw, *J. Fluid Mech.* **30**, 241 (1967).
- [7] A. E. Perry and C. J. Abell, *J. Fluid Mech.* **79**, 785 (1977).
- [8] A. E. Perry, S. Henbest, and M. S. Chong, *J. Fluid Mech.* **165**, 163 (1986).
- [9] I. Marušić and A. E. Perry, *J. Fluid Mech.* **298**, 389 (1995).
- [10] I. Marušić, A. K. M. Uddin, and A. E. Perry, *Phys. Fluids* **9**, 3718 (1997).
- [11] B. A. Kader and A. M. Yaglom, in *Turbulence and Coherent Structures*, edited by O. Métais and M. Lesieur (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991), pp. 387–412.
- [12] V. Nikora, *Phys. Rev. Lett.* **83**, 734 (1999).
- [13] A. M. Yaglom, in *New Approaches and Concepts in Turbulence*, edited by T. Dracos and A. Tsinober (Birkhäuser, Basel, Boston, 1993), pp. 7–27.
- [14] J. F. Morrison, W. Jiang, B. J. McKeon, and A. J. Smits (to be published).
- [15] J. F. Morrison, W. Jiang, B. J. McKeon, and A. J. Smits (to be published).