Correlated Perturbations from Inflation and the Cosmic Microwave Background

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(Received 5 July 2001; published 13 May 2002)

We compare the latest cosmic microwave background data with theoretical predictions including correlated adiabatic and cold dark matter (CDM) isocurvature perturbations with a simple power-law dependence. We find that there is a degeneracy between the amplitude of correlated isocurvature perturbations and the spectral tilt. A negative (red) tilt is found to be compatible with a larger isocurvature contribution. Estimates of the baryon and CDM densities are found to be almost independent of the isocurvature amplitude. The main result is that current microwave background data do not exclude a dominant contribution from CDM isocurvature fluctuations on large scales.

DOI: 10.1103/PhysRevLett.88.211302 PACS numbers: 98.70.Vc, 98.80.Cq

Increasingly accurate measurements of temperature anisotropies in the cosmic microwave background sky offer the prospect of precise determinations of both cosmological parameters and the nature of the primordial perturbation spectra. The recent Boomerang [1], DASI [2], and Maxima [3] data have shown evidence for three peaks in the cosmic microwave background (CMB) temperature anisotropy power spectrum as expected in inflationary scenarios. In this context the CMB data support the current "concordance" model based on a spatially flat Friedmann-Robertson-Walker universe dominated by cold dark matter and a cosmological constant [4]. In addition, the CMB data no longer show any signs of being in conflict with the big bang nucleosynthesis data [5].

In the studies which have estimated the cosmological and primordial parameters with these new data sets, only the case of purely adiabatic perturbations has been considered so far. That is, the perturbation in the relative number densities of different particle species is taken to be zero. Although this assumption is justified for perturbations originating from single-field inflationary models, it does not necessarily follow when there is more than one field present during inflation [6–10]. Other possible primordial modes are isocurvature [11,12] (also referred to as "entropy") modes in which the particle ratios are perturbed but the total energy density is unperturbed in the comoving gauge.

Most previous studies have examined the extent to which a statistically independent isocurvature contribution to the primordial perturbations may be constrained by CMB and large-scale structure data [13,14]. It has recently been shown that multifield inflationary models in general produce correlated adiabatic and isocurvature perturbations [7–10]. These correlations can dramatically change the observational effect of adding isocurvature perturbations [12,15]. Up until now, only the case of scale-invariant correlated adiabatic and entropy perturbations has been considered. Trotta *et al.* [16] found (with an earlier CMB data set) that in this case the cold dark matter (CDM) isocurvature mode was likely to be very small if not entirely absent, though they did find that a neutrino isocurvature mode contribution [12] was not ruled out. In this Letter we examine to what extent a correlated CDM isocurvature mode is consistent with the recent CMB data when a tilted power law spectrum is allowed.

Nonadiabatic perturbations are produced during a period of slow-roll inflation in the presence of two or more light scalar fields, whose effective masses are less than the Hubble rate. On subhorizon scales, fluctuations remain in their vacuum state so that when fluctuations reach the horizon scale their amplitude is given by $\widehat{\delta\phi}_{i*} \simeq (H_*/2\pi)\hat{a}_i$, where the subscript $*$ denotes horizon crossing and $\hat{a}_i$ are independent normalized Gaussian random variables, obeying $\langle \hat{a}_i \hat{a}_j \rangle = \delta_{ij}$. The total comoving curvature and entropy perturbation at any time during two-field inflation can quite generally be given in terms of the field perturbations, along and orthogonal to the background trajectory, as [8]

$$\hat{\mathcal{R}} \propto \cos\theta\,\widehat{\delta\phi}_1 + \sin\theta\,\widehat{\delta\phi}_2, \tag{1}$$

$$\hat{S} \propto -\sin\theta\,\widehat{\delta\phi}_1 + \cos\theta\,\widehat{\delta\phi}_2, \tag{2}$$

where θ is the angle of the inflaton trajectory in field space. Although the curvature and entropy perturbations are uncorrelated at horizon crossing, any change in the angle of the trajectory, θ, will begin to introduce correlations [8]. Further correlations may be introduced by the model dependent dynamics when inflation ends and the fields' energy is transformed into radiation and/or dark matter. The comoving curvature perturbation, $\mathcal{R}_{\rm rad}$, on large scales during the radiation-dominated era is related to the conformal Newtonian metric perturbation, Φ, by $\mathcal{R}_{\rm rad} = 3\Phi/2$. The isocurvature perturbation is $S_{\rm rad} = \delta\rho_{\rm CDM}/\rho_{\rm CDM} - (3/4)\delta\rho_\gamma/\rho_\gamma$ and remains constant on large scales until it reenters the horizon. On large scales the CMB temperature perturbation can be expressed in terms of the primordial perturbations [7]

$$\frac{\hat{\delta T}}{T} \approx \frac{1}{5}(\hat{\mathcal{R}}_{\rm rad} - 2\hat{S}_{\rm rad}). \quad (3)$$

The general transformation of linear curvature and entropy perturbations from horizon crossing during inflation to the beginning of the radiation era will be of the form

$$\begin{pmatrix} \hat{\mathcal{R}}_{\rm rad} \\ \hat{S}_{\rm rad} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{R}S} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} \hat{\mathcal{R}}_* \\ \hat{S}_* \end{pmatrix}. \quad (4)$$

Two of the matrix coefficients, $T_{\mathcal{R}\mathcal{R}} = 1$ and $T_{S\mathcal{R}} = 0$, are determined by the physical requirement that the curvature perturbation is conserved for purely adiabatic perturbations and that adiabatic perturbations cannot source entropy perturbations on large scales [17]. The remaining terms will be model dependent. If the fields and their decay products completely thermalize after inflation then $T_{SS} = 0$ and there can be no entropy perturbation if all species are in thermal equilibrium characterized by a single temperature, T. This means that it is unlikely that a neutrino isocurvature perturbation could be produced by inflation unless the reheat temperature is close to that at neutrino decoupling shortly before primordial nucleosynthesis takes place. On the other hand, a cold dark matter species could remain decoupled at temperatures close to, or above, the supersymmetry breaking scale yielding T_{SS}. The simplest assumption being that one of the fields can itself be identified with the cold dark matter [7].

The slow evolution (relative to the Hubble rate) of light fields after horizon crossing translates into a weak scale dependence of both the initial amplitude of the perturbations at horizon crossing and the transfer coefficients $T_{\mathcal{R}S}$ and T_{SS}. Parametrizing each of these by simple power laws over the scales of interest, requires three power laws to describe the scale dependence in the most general adiabatic and isocurvature perturbations,

$$\hat{\mathcal{R}}_{\rm rad} = A_r k^{n_1} \hat{a}_r + A_s k^{n_3} \hat{a}_s, \quad (5)$$

$$\hat{S}_{\rm rad} = B k^{n_2} \hat{a}_s. \quad (6)$$

The generic power law spectrum of adiabatic perturbations from single-field inflation can be described by two parameters, the amplitude and tilt, A and n. Uncorrelated isocurvature perturbations require a further two parameters, whereas we now have in general six parameters. The dimensionless cross correlation

$$\cos\Delta = \frac{\langle \mathcal{R}_{\rm rad} S_{\rm rad} \rangle}{(\langle \mathcal{R}^2_{\rm rad} \rangle \langle S^2_{\rm rad} \rangle)^{1/2}} = \frac{{\rm sgn}(B) A_s k^{n_3}}{\sqrt{A_r^2 k^{2n_1} + A_s^2 k^{2n_3}}} \quad (7)$$

is in general scale dependent.

We will investigate in this Letter the restricted case where all the spectra share the same spectral index and hence Δ is scale independent. This might naturally arise in the case of almost massless fields, where the scale dependence of the field perturbations is primarily due to the decrease of the Hubble rate during inflation, which is common to both perturbations and yields $n_i < 0$. In the following analysis we also allow $n_i > 0$, but we shall see that blue power spectra of this type are not favored by the data.

We then have four parameters, $A = \sqrt{A_r^2 + A_s^2}$, B, Δ, and n describing the effect of correlated perturbations, where $n = 1 + 2n_i$ is defined to coincide with the standard definition of the spectral index for adiabatic perturbations. We leave an investigation of the full six parameters for future work.

By defining the entropy-to-adiabatic ratio $B^* = B/A$, the parameter A becomes an overall amplitude that can be marginalized analytically (see below). In the following, to simplify notation, we write $A = 1$ and drop the asterisk from B^*. We limit the analysis to $B > 0$ and $0 < \Delta < \pi$, since there is complete symmetry under $\Delta \to -\Delta$ and under $(B \to -B, \Delta \to \pi - \Delta)$. Further, we allow three background cosmological parameters to vary, $\omega_b \equiv \Omega_b h^2$, $\omega_c \equiv \Omega_{\rm CDM} h^2$, and Ω_Λ, where $\Omega_{b,{\rm CDM},\Lambda}$ is the density parameter for baryons, CDM, and the cosmological constant, respectively. Since we assume spatial flatness, the Hubble constant is $h^2 = \frac{\omega_c + \omega_b}{1 - \Omega_\Lambda}$. Our aim is therefore to constrain the six parameters,

$$\alpha_i \equiv \{B, \Delta, n, \omega_b, \omega_c, \Omega_\Lambda\},$$

by comparison with CMB observations. We consider the COBE data analyzed in [18], and the recent high-resolution Boomerang [1], Maxima [3], and DASI data [2]. In order to concentrate on the role of the primordial spectra (and limit the numerical computation required) we will fix the neutrino masses (zero) and spatial curvature (zero). We will also neglect any contribution from tensor (gravitational wave) perturbations.

We use a CMBFAST code [19] modified in order to allow correlated perturbations to calculate the expected CMB angular power spectrum, C_l, for all parameter values. [Our C_l is defined as $C_l = l(l+1)C_l^*/(2\pi)$, where C_l^* is the square of the multipole amplitude.] The computations required can be considerably reduced by expressing the spectrum for a generic value of B and Δ as a function of the spectra for other values. Let us denote the purely adiabatic and isocurvature spectra when $B = 1$ as $[C_l]_{\rm ad}$ and $[C_l]_{\rm iso}$, respectively, and the correlation term for totally correlated perturbations $B = 1, \Delta = 0$ as $[C_l]_{\rm corr}$. Then we can write the generic spectrum for arbitrary B and Δ as

$$C_l = [C_l]_{\rm ad} + B^2 [C_l]_{\rm iso} + 2B\cos\Delta [C_l]_{\rm corr}. \quad (8)$$

We can obtain $[C_l]_{\rm corr}$ from Eq. (8) and using any $B\cos\Delta \neq 0$. The library spectra $[C_l]_{\rm ad}$ and $[C_l]_{\rm iso}$ and $[C_l]_{\rm corr}$ can then be used to evaluate C_l for any B and Δ. A different set of library spectra will be needed for each set of cosmological parameters. When $n_1 \neq n_3$ then Δ is not generally scale independent and so it would be necessary to evaluate the shape of the cross-correlation spectra $[C_l]_{\rm corr}$ for each form of $\Delta(k)$, but one can always perform the scaling with respect to B analytically.

The remaining input parameters requested by the CMBFAST code are set as follows: $T_{\rm CMB} = 2.726K$,

$Y_{\rm He} = 0.24$, $N_\nu = 3.04$. All our likelihood functions below are obtained by marginalizing over τ_c, the optical depth to Thomson scattering, in the range $(0, 0.2)$ (larger τ_c have a very small likelihood). We did not include the cross correlation between band powers because it is not available, but it should be less than 10% according to [1]. An offset log-normal approximation to the band-power likelihood has been advocated by [18] and adopted by [1,3], but the quantities necessary for its evaluation are not available. Since the offset log normal reduces to a log normal in the limit of small noise, we evaluated the log-normal likelihood

$$-2\log L(\alpha_j) = \sum_i [Z_{\ell,t}(\ell_i;\alpha_j) - Z_{\ell,d}(\ell_i)]^2 \sigma_\ell^{-2}, \quad (9)$$

where $Z_\ell \equiv \log \hat{C}_\ell$, the subscripts t and d refer to the theoretical quantity and to the real data, respectively, $\hat{C}_\ell$ are the spectra binned over some interval of multipoles centered on ℓ_i, σ_ℓ are the experimental errors on $Z_{\ell,d}$, and the parameters are denoted collectively as α_j.

The overall amplitude parameter A can be integrated out analytically using a logarithmic measure $d\log A$ in the likelihood. Analogously, we can marginalize over the relative calibration uncertainty of the Boomerang, Maxima, and DASI data (see [1,3]), by an analytic integration to obtain the final likelihood function that we discuss in the following. We neglected beam and pointing errors, but we checked that the results do not change significantly even where increasing the calibration errors by 50%. We assume a linear integration measure for all the other parameters.

In order to compare with the Boomerang, Maxima, and DASI analyses we assume uniform priors as in [1], with the parameters confined in the range $B \in (0, 3)$, $\Delta \in (0, \pi)$, $n \in (0.6, 1.4)$, $\omega_b \in (0.0025, 0.08)$, $\omega_c \in (0.05, 0.4)$, and $\Omega_\Lambda \in (0, 0.9)$. As extra priors, the value of h is confined in the range $(0.45, 0.9)$ and the universe age is limited to >10 Gyr as in [1]. A grid of $\sim 10\,000$ multipole CMB spectra is used as a database over which we interpolate to produce the likelihood function.

Figure 1 shows one of the best cases in our database, corresponding to $(B, \Delta, n, \omega_b, \omega_c, \Omega_\Lambda) = (0.63, \pi/4, 0.9, 0.0225, 0.1, 0.7)$. The adiabatic ($[C_l]_{\rm ad}$), entropy ($B^2[C_l]_{\rm iso}$), and correlated ($2B\cos\Delta[C_l]_{\rm corr}$) components are shown. The primary effect of adding a positively correlated component is to *reduce* the height of the low-l plateau relative to the acoustic peaks [15]. This is in contrast to the uncorrelated case, where the addition of entropy perturbations *increases* the plateau height relative to the peaks. Isocurvature perturbations have only a significant effect on intermediate angular scales for strongly blue-tilted spectra. They have a minimal effect on the peak structure for $n < 1$. Thus we find a near degeneracy between B and n when $\Delta = 0$: the effect of adding maximally correlated isocurvature perturbations mimics an increase in the primordial slope. This makes clear the importance of varying n when studying correlated isocurvature perturbations: a lower n allows a larger B to be consistent with the CMB data.

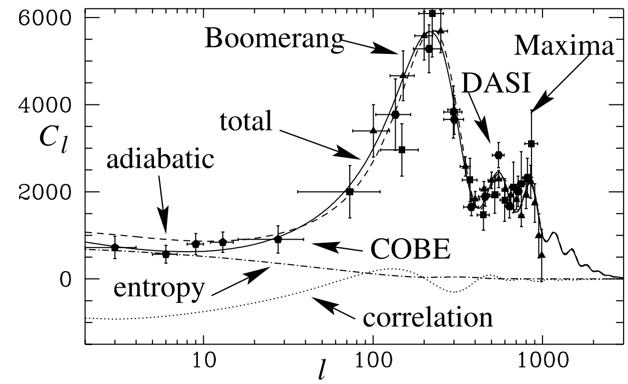

FIG. 1. Best-fit spectrum (solid line) and the component spectra, shown with the data with one sigma error bars, using maximum-likelihood normalization and unadjusted relative calibration.

This degeneracy is not perfect due to the different effect of n and B on the detailed shape of the angular power spectrum. In Fig. 2 we plot the likelihood for B and $\cos\Delta$, having marginalized over the other parameters. The plot shows that the marginalized likelihood peak occurs for $B = 0.4$, $\cos\Delta = 0.7$, although the pure adiabatic case $B = 0$ is well within one sigma. It is remarkable that, when a nonzero correlation is allowed, quite large values of B become acceptable, up to $B = 1.5$ (to 95% C.L.) when $\cos\Delta \approx 0.8$. Anticorrelation, on the other hand, reduces the range of B. We also show the likelihood contours possible in a future Planck-like experiment with zero calibration uncertainty and accuracy limited only by cosmic variance for $l < 1000$. This shows that future CMB data alone could detect a finite isocurvature contribution around the current peak of likelihood.

We found that the contour lines of the cosmological parameters ω_b and ω_c are almost parallel to B for $B < 1$. This means that the isocurvature perturbations do not

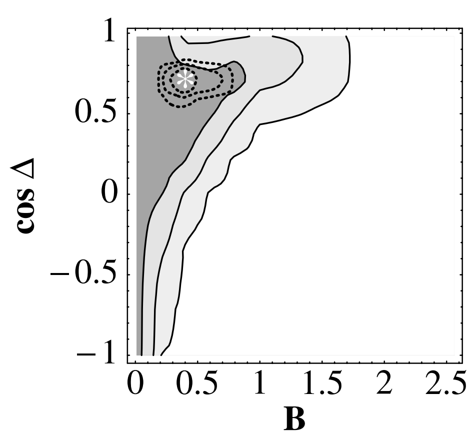

FIG. 2. Contour plot of the two-dimensional likelihood for B and $\cos\Delta$. The solid contours enclose 39%, 86%, and 99% of the likelihood and the star marks the peak. Dotted contours are for a future Planck-like experiment.

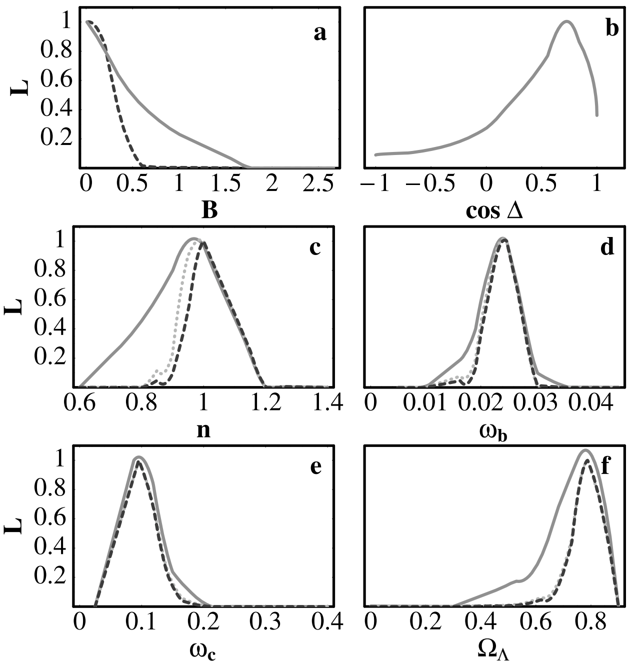

FIG. 3. One-dimensional likelihood functions in arbitrary units. The (light) dotted lines for the purely adiabatic models ($B = 0$); dashed lines for uncorrelated fluctuations ($\cos\Delta = 0$); solid lines for correlated fluctuations. See text for further explanation.

alter significantly the best estimates for these cosmological parameters. On the other hand, increasing B enlarges the region of confidence for Ω_Λ and for n toward smaller values.

Figure 3 summarizes our results: we plot the one-dimensional likelihood functions obtained by marginalizing all the remaining parameters. Figure 3a shows that the contribution of isocurvature perturbations can be as large as the adiabatic perturbations, or even larger: we find that $B < 1.3$ to 95% C.L. In contrast, uncorrelated isocurvature perturbations cannot exceed $B < 0.5$ to the same C.L. The likelihood functions for n and Ω_Λ extend toward smaller values, as anticipated, while the CDM and the baryon density estimates remain quite unaffected. The average values are $n = 0.94 \pm 0.1$, $\omega_b = 0.023 \pm 0.004$, $\omega_c = 0.1 \pm 0.03$, and $\Omega_\Lambda = 0.72 \pm 0.11$.

By contrast, Enqvist *et al.* [14] found that a large uncorrelated isocurvature contribution is only consistent with blue-tilted slopes. The reason for this difference is that correlations can cause the acoustic peak height to increase relative to the Sachs-Wolfe plateau (see Fig. 1), unlike the case of independent perturbations where the relative height always decreases. Trotta *et al.* [16] found that the CMB data were not consistent with a significant CDM isocurvature contribution because they restricted the primordial slope, n, to be unity.

As can be seen from Fig. 3 our estimates of ω_b and ω_c are virtually unaffected by the addition of correlated CDM isocurvature perturbations. Thus, in our model, the nature of the isocurvature component can be investigated almost independently of the composition of the matter component.

The main conclusion of this Letter is that the current CMB data are consistent with a large correlated CDM isocurvature perturbation contribution when the spectral slopes are allowed a tilt to the red ($n < 1$). The higher precision of future satellite data has the potential to detect the isocurvature contribution, if any, thereby showing that inflation was not a single-field process.

The authors are grateful to David Langlois, Roy Maartens, and Carlo Ungarelli for useful discussions. C. G. was supported by the ORS, and D. W. by the Royal Society.

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