Left-Handed Materials Do Not Make a Perfect Lens

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By means of an analysis on evanescent waves in left-handed materials (LHM), we show that within a slab of such a medium, sandwiched between two positive refraction media, there is amplification of evanescent waves in ideal lossless, dispersiveless media; however, contrary to previous claims, this is limited to a finite width of the slab so that it prevents their restoration and perfect focusing. We illustrate this by considering their coupling to propagating waves through a tunnel barrier containing a slab of LHM. Further, we show that the effect of absorption, necessarily present in such materials, may drastically change any evanescent amplifying wave into a decaying one.

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The resolution of optical signal detection is limited to half the wavelength λ of the radiation in use. This is well known to be due to the loss of evanescent components of the wave field emanating from the object, as it propagates up to the detection plane [1,2,3]. It has recently been proposed [4], however, that a slab of left-handed material (LHM), (i.e., of a medium with negative permittivity, permeability, and refractive index, initially discussed by Veselago [5]) could restore such evanescent wave-field components, being thus termed a *perfect lens*. The nature of evanescent waves is subtle, nevertheless, and it is easy to reach inconsistencies and divergencies that contradict the basic mathematical properties of scattered wave fields, specifically, radiation condition and square integrability, which represent their behavior as spherical waves at infinity and their finite energy, respectively. Such inconsistencies, present in Ref. [4], have further lead to comments [6] on [4], which we have also found to be either incomplete or incorrect.

We present in this Letter a detailed analysis of evanescent waves in an ideal lossless and dispersionless LHM with constitutive parameters whose values at a certain frequency are opposite those of vacuum. Such parameters were those for which the perfect lens was proposed. This shows errors in the analysis of [4] as regards a LHM slab. Also, we illustrate how absorption present in dispersive left-handed metamaterials may change these modes from amplifying into decaying.

The electric vector $\mathcal{E}$ of a wave field emanating from an object, propagating in a source-free or homogeneous half space, is well known to be represented by its angular spectrum $\mathbf{A}(k_x, k_y)$ of plane wave components of wave vector $\mathbf{k} = (k_x, k_y, k_z)$, $k_x^2 + k_y^2 + k_z^2 = (2\pi/\lambda)^2$, [2,3,7]

$$\mathcal{E}(\mathbf{r}) = \int_{-\infty}^{\infty} \mathbf{A}(k_x, k_y) \exp(i\mathbf{k} \cdot \mathbf{r}) \, dk_x \, dk_y \,. \qquad (1)$$

This expression contains both propagating and evanescent plane wave components. In what follows we shall focus our attention into each of those evanescent waves $\mathbf{E}(k_x, k_y, \mathbf{r}) = \mathbf{A}(k_x, k_y) \exp(i\mathbf{k} \cdot \mathbf{r})$. (From now on, unless explicitly stated, the k_x, k_y dependence of $\mathbf{E}$ will not be included in the notation). Let $\mathbf{E}^{(i)}$ be the electric field of an evanescent component of an S-polarized electromagnetic wave in the half space $z < 0$ occupied by vacuum, incident on the plane $z = 0$ that limits a LHM, filling the region $z > 0$, of dielectric permittivity $-\epsilon$, magnetic permeability $-\mu$, and refractive index $-n$ ($\epsilon > 0$, $\mu > 0$, and $n > 0$). A time harmonic dependence $\exp(-i\omega t)$ will be assumed throughout. We then choose the geometry such that $\mathbf{E}^{(i)}(z \leq 0) = (A^{(i)}, 0, 0) \exp(i\mathbf{k}^i \cdot \mathbf{r})$.

With the wave vector $\mathbf{k}^i = (0, k_y^i, k_z^i)$, $k_z^i = \pm iK_i$, $K_i = \sqrt{k_y^{i2} - k_0^2}$, $k_0 = \omega/c = 2\pi/\lambda$. The sign of the square root is discussed next.

The corresponding reflected and transmitted waves $\mathbf{E}^{(r)}$ and $\mathbf{E}^{(t)}$, respectively, are $\mathbf{E}^{(r)}(z \leq 0) = (A^{(r)}, 0, 0) \exp(i\mathbf{k}^r \cdot \mathbf{r})$, $\mathbf{E}^{(t)}(z \geq 0) = (A^{(t)}, 0, 0) \exp(i\mathbf{k}^t \cdot \mathbf{r})$.

With wave vectors: $\mathbf{k}^r = (0, k_y^i, k_z^r)$, $\mathbf{k}^t = (0, -nk_y^t, -nk_z^t)$; and $k_z^r = \pm iK_i$, $K_i = \sqrt{k_y^{i2} - k_0^2}$, and $k_z^t = \pm iK_t$, $K_t = \sqrt{k_y^{t2} - k_0^2}$. Of course, $A^{(i)}$, $A^{(r)}$, and $A^{(t)}$ are functions of k_y. Matching conditions at $z = 0$: $[E_x^{(i)} + E_x^{(r)}]_{z=0} = [E_x^{(t)}]_{z=0}$ and $\partial_x[E_x^{(i)} + E_x^{(r)}]_{z=0} = (-1/\mu)\partial_x[E_x^{(t)}]_{z=0}$ impose that $k_y^i = -nk_y^t$, which characterizes negative refraction. As regards the signs of k_z^i, k_z^r, and k_z^t, three kinds of modes may then be considered in this system:

(i) $\mathbf{E}^{(i)}$ and $\mathbf{E}^{(t)}$ are both evanescent components, decaying as $z \to -\infty$ and as $z \to \infty$, respectively. When $\mathbf{E}^{(r)}$ is evanescent decaying as $z \to -\infty$, then one must choose: $k_z^i = -iK_i$, $k_z^r = -iK_i$, and $k_z^t = -iK_t$. This ensures that $\mathbf{E}^{(i)}$, $\mathbf{E}^{(r)}$, and $\mathbf{E}^{(t)}$ are square integrable in $\mathbf{k}$ space, and so are the corresponding fields $\mathcal{E}$ in $\mathbf{r}$ space. $A^{(r)} = rA^{(i)}$ and $A^{(t)} = tA^{(i)}$, r and t being the reflection and transmission coefficients. As considered in Ref. [4], let us address the case in which at a certain frequency ω the LHM has $\epsilon = \mu = n = 1$, then $K_t = K_i$. Matching conditions at $z = 0$ lead to the relationship between r and t: $1 + r = t$. Thus, further constraints are required to determine them. An important situation arises in the choice of $\mathbf{E}^{(r)}$ evanescent growing as $z \to -\infty$, then, when $\epsilon = \mu = n = 1$, one has that $r = 0$ and $t = 1$, i.e., no reflected wave, thus there being one evanescent wave at each side of the interface: $\mathbf{E}^{(i)}(z \leq 0) = (A^{(i)}, 0, 0)\exp(ik_y^i y + K_i z)$, decaying as $z \to -\infty$ in $z < 0$, and $\mathbf{E}^{(t)}(z \geq 0) = (A^{(i)}, 0, 0)\exp(ik_y^i y - K_i z)$, decaying as $z \to \infty$ in $z > 0$.

(ii) $\mathbf{E}^{(i)}$ is evanescent growing as $z \to -\infty$, $\mathbf{E}^{(t)}$ is evanescent decaying as $z \to \infty$, and $\mathbf{E}^{(r)}$ is evanescent decaying as $z \to -\infty$. This is the situation initially considered in Ref. [4]. Then, one must choose $k_z^i = iK_i$, $k_z^r = -iK_i$, and $k_z^t = -iK_t$. It should be noticed, however, that at fixed k_y, $A^{(i)}$ cannot be a constant in this case, as assumed in Ref. [4], since then $\mathbf{E}^{(i)}$ would be unbounded in its definition domain: $z < 0$, as it increases without limit as either $z \to -\infty$, or at any given $z < 0$, as $k_y^i \to \infty$. Therefore, this choice for k_z^i cannot correspond to any physically realizable field [3,7], which should be square integrable (see also [3,7–9]). Proper normalization of $\mathbf{E}^{(i)}$ now imposes to take the incident amplitude: $A^{(i)}\exp(-K_i z_0)$, and restrict the definition domain of $\mathbf{E}^{(i)}$ to the strip: $-z_0 \leq z \leq 0$. This makes sense physically, as then $\mathbf{E}^{(i)}$ represents an evanescent wave component, created by some means (e.g., scattering or total internal reflection) at $z = -z_0$. The case in which at a certain frequency ω the LHM has: $\epsilon = \mu = n = 1$, however, now leads to divergent reflection and transmission coefficients, which excludes the possibility of transmission of an evanescent wave into such a semi-infinite LHM.

(iii) $\mathbf{E}^{(i)}$, is evanescent growing as $z \to -\infty$, $\mathbf{E}^{(r)}$ is evanescent decaying as $z \to -\infty$, $\mathbf{E}^{(t)}$ is evanescent growing as $z \to \infty$. Now, like in case (ii), proper normalization of $\mathbf{E}^{(i)}$ imposes that it be restricted to the strip $-z_0 \leq z \leq 0$, and has amplitude: $A^{(i)}\exp(-K_i z_0)$. Then the matching conditions at $z = 0$ yield $r = \frac{-\mu K_i + K_t}{-\mu K_i - K_t}\exp(-K_i z_0)$, $t = \frac{-2\mu K_i}{-\mu K_i - K_t}\exp(-K_i z_0)$.

If at a certain frequency ω: $\epsilon = \mu = n = 1$, then $K_t = K_i$ and the coefficients become $r = 0$ and $t = \exp(-K_i z_0)$. Therefore, the waves transmitted into the LHM are

$$\mathbf{E}^{(t)}(k_y, y, z \geq 0) = (A^{(i)}(k_y^i), 0, 0) \times \exp[ik_y^i y + K_i(z - z_0)]. \quad (2)$$

Equation (2) shows that the evanescent wave components transmitted into the LHM are now amplifying as z increases, but since all objects are limited in space, the angular spectrum $A^{(i)}$ decreases as $k_y^i \to \pm\infty$ as an inverse power of k_y^i (obviously this is also true for the k_x dependence of $A^{(i)}$ if any other polarization were chosen) [3]; also as energy should be conserved, $\mathcal{E}^{(t)}(x, y, z \geq 0)$ and $\mathbf{E}^{(t)}(k_y^i, y, z \geq 0)$ must be square integrable in y and k_y^i, respectively [3,8]. This and the radiation condition for $\mathcal{E}$ as $z \to \infty$ imposes that $z \leq z_0$. Namely, the waves given by (2) exist only within a strip $0 \leq z < d$, where $d \leq z_0$, in the LHM half space. Notice that this constraint for z was not realized in the amplifying solution obtained inside the LHM in Ref. [4]. Therefore, such a mode cannot exist in the LHM if $d > z_0$, and in that case the evanescent component transmitted into $z > 0$ must be zero, which implies that $t = 0$ and $r = -\exp(-iK_i z_0)$. The physical interpretation of all this is that the formation of a surface state requires an infinite transient time and hence involves infinite energy density when $z > z_0$.

We shall next show that, since $r = 0$ when $d \leq z_0$, the transmitted mode (2) of a semi-infinite LHM, is also the solution inside a LHM slab of width $d \leq z_0$ that at a certain frequency ω has $\epsilon = \mu = n = 1$. Let this layer be embedded in vacuum, limited by the planes $z = 0$ and $z = d$, and let an evanescent wave decaying as $z \to 0$ in $z < 0$ be incident on the interface at $z = 0$. One then has in each of the three regions: $\mathbf{E}(-z_0 \leq z \leq 0) = (A^{(i)}, 0, 0)\exp[ik_y^i y - K_i(z + z_0)] + (rA^{(i)}, 0, 0) \times \exp[ik_y^i y + K_i z]$, $\mathbf{E}(0 \leq z \leq d) = (A, 0, 0)\exp[ik_y^i y - K_i z] + (B, 0, 0)\exp[ik_y^i y + K_i z]$, $\mathbf{E}(z \geq d) = (tA^{(i)}, 0, 0)\exp[ik_y^i y - K_i z]$.

Where r and t are the reflection and transmission coefficients at $z = 0$ and $z = d$, respectively, and A and B depend on k_y. The incident evanescent wave in $z < 0$ has been normalized again to $\exp[-K_i z_0]$ and, as before, it is restricted to the strip $-z_0 \leq z \leq 0$ in order to ensure that it be square integrable. The matching conditions at $z = 0$ and $z = d$ give $r = A = 0$, $B = A^{(i)}\exp[-K_i z_0]$, and $t = A^{(i)}\exp[K_i(2d - z_0)]$. Hence, the resulting waves in each of the three regions are the following: $\mathbf{E}(-z_0 \leq z \leq 0) = (A^{(i)}, 0, 0)\exp[ik_y^i y - K_i(z + z_0)]$, $\mathbf{E}(0 \leq z \leq d) = (A^{(i)}, 0, 0)\exp[ik_y^i y + K_i(z - z_0)]$, and $\mathbf{E}(z \geq d) = (A^{(i)}, 0, 0)\exp[ik_y^i y - K_i(z + z_0 - 2d)]$. This, as mentioned, shows that the wave $\mathbf{E}(0 \leq z \leq d)$ inside the LHM is again as in Eq. (2). Its proper normalization imposes once again that $d \leq z_0$, otherwise this wave function will be zero. Hence there is no transmitted evanescent component inside the LHM slab when $d > z_0$. Notice that the current density that characterizes the energy transport along OY for each evanescent component inside the slab diverges when $z > z_0$ as: $J_y = Ck_y^i \exp[2K_i(z - z_0)]$, C being a constant.

As in case (iii) before, a width $d = 2z_0$ would imply a wave function $\mathbf{E}(k_y^i, 0 \leq z \leq d)$ inside the slab containing a factor $\exp[K_i(z - z_0)]$ that prevents it from being square integrable in k_y^i at values of z: $z > z_0$. Hence, the exit of the slab $z = d$ is at best (i.e., when $d = z_0$) equivalent to the plane $z = -z_0$ at which the evanescent wave was created. It should also be remarked that, contrary to the results of [4], *there are no multiple reflections within the LHM slab,* simply because as shown, the reflection coefficients r are zero.

As an illustration, we use the following example: consider an object at $z = -z_0$, consisting of two spikes separated a distance $2y_0$ from each other, from which the wave field, given by Eq. (1), propagates towards increasing z. Its angular spectrum is therefore: $2\cos(y_0 k_y)$, notice that those two peaks may have a finite width rather than being two δ functions, then modeling them by two rectangular functions of width $b < y_0$, the former angular spectrum would be multiplied by $2\sin(bk_y)/k_y$. At any point $0 \leq z \leq d$, each x-plane wave component of $\mathcal{E}$ is $E_x(k_y, 0 \leq z \leq d) = 2\cos(y_0 k_y)\exp[ik_z(z - z_0)]\exp(ik_y y)$. Therefore, inside the slab:

$$\mathcal{E}_x(y, 0 \leq z \leq d) = 2\int_{-\infty}^{\infty}\cos(y_0 k_y)\exp[ik_z(z - z_0)] \times \exp(ik_y y)\, dk_y\,.$$

In the evanescent region, $\exp[ik_z(z - z_0)] = \exp[\sqrt{k_y^2 - (2\pi/\lambda)^2}(z - z_0)]$. Thus the above integral diverges when $z > z_0$, and hence it cannot represent any physically realizable wave field. Notice that this also happens if the two peaks have a finite width, since then the corresponding angular spectrum $2\cos(y_0 k_y)2\sin(bk_y)/k_y$ decreases as k_y^{-1} when $k_y \to \pm\infty$ which is slower than the increase of the exponential $\exp[ik_z(z - z_0)]$. The same happens for any other object, which should be limited in space. (In particular, if the object were, e.g., a dipole source, the angular spectrum would be proportional to k_z^{-1}, which again decreases as k_y^{-1} when $k_y \to \pm\infty$). Thus, if for instance, $d = 2z_0$ as proposed in [4], since as shown before, the evanescent modes do not exist within the slab, the image of the two spikes at $z = z_0 + d$ will be $\mathcal{E}_x(y, z_0 + d) = \sin[(2\pi/\lambda)(y - y_0)]/(y - y_0) + \sin[(2\pi/\lambda)(y + y_0)]/(y + y_0)$, namely, two peaks of width λ separated by a distance $2y_0$, which is the same image as with an ideal conventional lens (infinite aperture and no aberrations).

We now illustrate all the above with the effect of a LHM slab on evanescent waves in a tunneling barrier between two dielectric semi-infinite media. This situation is relevant since ultimately, in the detection process, evanescent waves have to couple into propagating waves at a certain interface. The system to study is then composed of five regions: the space $0 < z \leq a$ and $a + d < z \leq 2a + d$ are air gaps, and $a < z \leq a + d$ is occupied by the LHM that at a certain frequency ω has: $\epsilon = \mu = n = 1$. The dielectric semi-infinite regions are $z < 0$ and $z > 2a + d$, respectively. Let us consider one of the plane propagating components of a wave field $\mathcal{E}$ that is incident on $z = 0$ from the first dielectric medium of refractive index $n_d > 0$ in $z < 0$. Therefore, in this medium, the wave function is $\mathbf{E}(z \leq 0) = (A^{(i)}, 0, 0)\exp(i\mathbf{k}^i \cdot \mathbf{r}) + (rA^{(i)}, 0, 0)\exp(i\mathbf{k}^r \cdot \mathbf{r})$, where $\mathbf{k}^i = (0, k_y^i, k_z^i)$, $k_z^i = \sqrt{k_0^2 n_d^2 - k_y^{i2}}$.

And the transmitted wave into the second dielectric medium is $\mathbf{E}(z \geq 2a + d) = (tA^{(i)}, 0, 0)\exp(i\mathbf{k}^i \cdot \mathbf{r})$.

Matching conditions at the interfaces, denoting $K = \sqrt{k_y^{i2} - k_0^2}$, lead to the reflection and transmission coefficients r and t:

$$t = \frac{4iKk_z^i \exp[-ik_z^i(2a + d)]\exp[K(d - 2a)]}{(K + ik_z^i)^2\exp[2K(d - 2a)] - (K - ik_z^i)^2}\,, \quad (3)$$

$$r = \frac{(k_z^{i2} + K^2)(1 - \exp[2K(d - 2a)])}{(K + ik_z^i)^2\exp[2K(d - 2a)] - (K - ik_z^i)^2}\,. \quad (4)$$

Of course, when $d = 0$, Eqs. (3) and (4) become those of t and r for the usual tunnel effect in an air gap of width $2a$. Also, notice that when $d = 2a$ then $t = 1$ and $r = 0$. At first sight, this seems to support the results of Ref. [4]. However, when the width d of the LHM slab is larger than that of the air gap a, although the current density of energy transport along OZ remains conserved, the current density parallel to the interfaces, i.e., along OY, becomes unbounded, as before, as d increases, or at fixed d, as K increases. This is seen at once from the wave function inside the LHM slab, which reads

$$\mathbf{E}(a \leq z \leq a + d) = [A^{(i)}\exp(ik_y^i y), 0, 0]\frac{2ik_z^i}{(K + ik_z^i)^2\exp[2K(d - 2a)] - (K - ik_z^i)^2} \times \{(K - ik_z^i)\exp[K(z - 2a)] + (K + ik_z^i)\exp[-K(z + 2a - 2d)]\}\,. \quad (5)$$

Once again, the wave function given by Eq. (5) is not square integrable in k_y when $d > a$. In fact, when $d = 2a$ the energy density inside the LHM slab diverges as $\cosh[2K(z - 2a)]$ for $z > 2a$ when either z or K increase. We see, therefore, that these divergencies prevent the LHM slab from restoring the evanescent waves. Then, since d must be $d \leq a$, according to Eq. (3), $t \neq 1$. Hence, the coupling of the evanescent waves with the propagating waves at $z > d + 2a$, which is necessarily involved in the detection process, unavoidably leads to an amplitude of the transmitted propagating wave different from that of the corresponding component of the incident field. This conveys an unavoidable aberration in the resultant angular spectrum of the wave field transmitted into $z \geq 2a + d$ and detected.

Similar results are obtained for p polarization, by exchanging μ by ϵ in the matching conditions.

So far, ideal lossless media have been addressed. However, the positivity of the electromagnetic energy imposes that the LHM presents frequency dispersion [5]. This, in turn, conveys that actual LH metamaterials be absorbing [10,11]. We shall next see how the presence of absorption, even if small, drastically changes the nature of the waves in the LHM, as it produces an evanescent wave field, decaying as z increases, instead of an amplifying wave. We once again consider a slab of LHM, surrounded by in vacuum, limited by the planes $z = 0$ and $z = d$. We address an evanescent component of a wave-field incident on the slab from $z < 0$. This evanescent wave decays as z increases. Let the refractive index of the LHM at some frequency ω be $n = -1 + in_2$, with $0 < n_2 \ll 1$. Where we have made use of: $n = -\sqrt{(-\epsilon_r + i\epsilon_i)(-\mu)}$ $(\epsilon = \epsilon_r + i\epsilon_i)$. When $\mu = 1$, and $\epsilon_r = 1$, one has: $n = -\sqrt{1 - i\epsilon_i} \simeq -(1 - i\epsilon_i/2)$, so that $n_2 = \epsilon_i/2$. Then, in the LHM: $K = \sqrt{k_y^2 - k_0^2}$ becomes $K(1 - in_2)$.

Normalizing the incident evanescent wave to $\exp(-K_i z_0)$, as before, one can make use of Eqs. (3)–(5) by writing $k_z^i = iK_i$ (incident evanescent wave on the slab), substituting K by $K_i(1 - in_2)$, and making $a = 0$ (no air gaps). Then, providing that $n_2 \exp(K_i d) \gg 2$, the second term in the denominator of Eqs. (3)–(5) can be neglected. Thus, the fields read $\mathbf{E}(z_0 \leq z \leq 0) = (A^{(i)} \exp(-K_i z_0), 0, 0)[\exp(ik_y^i y - K_i z) - \frac{n_2 + 2i}{n_2} \times \exp(ik_y^i y - K_i z)]$,

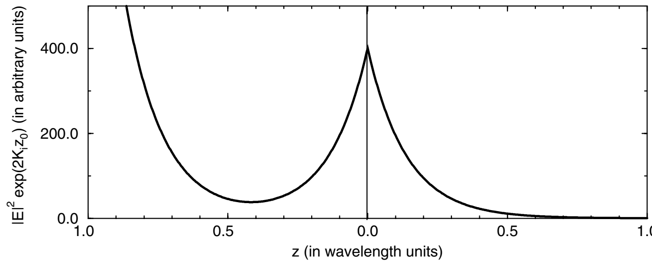

FIG. 1. Distribution $|\mathbf{E}(z)|^2 \exp(2K_i z_0)$, of an evanescent wave intensity, both in vacuum ($z < 0$), and inside a left-handed material ($z > 0$) that presents absorption.

$$\mathbf{E}(0 \leq z \leq d) = (A^{(i)} \exp(-K_i z_0), 0, 0) \left\{ \frac{2}{n_2^2} (2 - in_2) \exp[ik_y^i y + K_i(z - 2d) + iK_i n_2(2d - z)] - in_2 \exp(ik_y^i y - K_i z + iK_i n_2 z) \right\}, \tag{6}$$

$\mathbf{E}(z \geq d) = (A^{(i)} \exp(-K_i z_0), 0, 0) \frac{4(1 - in_2)}{n_2^2} \exp(ik_y^i y - K_i z) \exp(iK_i n_2 d)$. Notice the remarkable fact that the existence of some absorption, which should be accounted for when $n_2 \exp(K_i d) \gg 2$, gives rise in the LHM slab to a decaying evanescent wave, stemming from the second term of Eq. (6). This is in striking contrast with the amplifying wave transmitted into the LHM in the absence of such absorption. Also, there is now a wave reflected at $z = d$, represented by the first term of Eq. (6). In fact, when $K_i d \gg 1$, the second term of Eq. (6) dominates near $z = 0$, whereas the first term contributes near $z = d$.

For example, for $n_2 = 0.1$, $K_i d = 10$, $d = 2.8\lambda$, the intensity at the exit of the slab is $|\mathbf{E}(z = d)|^2 = 32 \times 10^{-5} \exp(-2K_i z_0)$, whereas if $n_2 = 0 |\mathbf{E}(z = d)|^2 = e^{20} \exp(-2K_i z_0) \simeq 5 \times 10^8 \exp(-2K_i z_0)$. On the other hand, the intensity at the entrance of the slab is $|\mathbf{E}(z = 0)|^2 = |1 - (n_2 + 2i)/n_2|^2 = 4 \times 10^2 \exp(-2K_i z_0)$. This quantity is remarkably larger than the value $\exp(-2K_i z_0)$ of the incident field intensity at $z = 0$ (which also coincides with the total field at $z = 0$) when $n_2 = 0$. This increase is due to the contribution $(n_2 + 2i)/n_2$ of the reflected wave at $z = 0$. Figure 1 shows $|\mathbf{E}(z)|^2 \exp(2K_i z_0)$, both in vacuum ($z < 0$) and inside the LHM, ($z > 0$). This plot, that has been limited to the interval $-1 < z/\lambda < 1$, illustrates the above remarks.

In conclusion, although evanescent waves in an ideal lossless dispersiveless LHM slab become amplifying, the width of this slab is limited, so that their restoration is physically meaningless as it involves infinite energy. It is neither true, as stated in [4], that multiple reflections within the LHM slab are the cause of the amplifying wave, simply because such multiple reflections do not exist since the reflection coefficients at the slab interfaces are zero for the constitutive parameters considered. In addition, absorption, which linked to dispersion, is always present in real LHM, transforms any amplified wave into a decaying one inside this medium.

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[1] M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, United Kingdom, 1999).
[2] J. W. Goodman, *Introduction to Fourier Optics* (McGraw Hill, New York, 1968).
[3] R. P. Boas, *Entire Functions* (Academic Press, New York, 1954); M. Nieto-Vesperinas, *Scattering and Diffraction in Physical Optics* (J. Wiley, New York, 1991); E. Wolf and M. Nieto-Vesperinas, J. Opt. Soc. Am. A **2**, 886 (1985).
[4] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[5] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[6] G. V. t Hooft, Phys. Rev. Lett. **87**, 249701-1 (2001); J. M. Williams, *ibid.* **87**, 249703-1 (2001).
[7] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
[8] G. C. Sherman and H. J. Bremermann, J. Opt. Soc. Am. **59**, 146 (1969).
[9] C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (J. Wiley, New York, 1977).
[10] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1963).
[11] N. Garcia and M. Nieto-Vesperinas, "Is There an Experimental Verification of a Negative Index of Refraction Yet?," Opt. Lett. (to be published). Here it is shown that absorption precludes observation of negative refraction in so far proposed LMHs.