Field-Tuned Superconductor-Insulator Transition with and without Current Bias

E. Bielejec and Wenhao Wu
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 14 August 2001; published 6 May 2002)

The magnetic-field-tuned superconductor-insulator transition has been studied in ultrathin beryllium films quench condensed near 20 K. In the zero-current limit, a finite-size scaling analysis yields the scaling exponent product $\nu z = 1.35 \pm 0.10$ and a critical sheet resistance, R_c, of about $1.2R_Q$, with $R_Q = h/4e^2$. However, in the presence of dc bias currents that are smaller than the zero-field critical currents, νz becomes 0.75 ± 0.10. This new set of exponents suggests that the field-tuned transitions with and without a dc bias current belong to different universality classes.

DOI: 10.1103/PhysRevLett.88.206802 PACS numbers: 73.50.–h, 74.25.Dw, 74.40.+k, 74.76.–w

A quantum system can undergo a continuous phase transition at $T = 0$ by varying a parameter in its Hamiltonian rather than the temperature [1]. At finite temperatures, a quantum phase transition exhibits scaling behavior with the temperature and the appropriate tuning parameter. The superconductor-insulator (SI) transition in thin films [2] and arrays of Josephson junctions [3] are examples of quantum phase transitions. The SI transition in thin films occurs when film thickness [4] or applied magnetic field [5–7] is varied through a critical value. There are two types of models for the SI transition in thin films: ones that rely on fluctuations in the phase [8–11] and others that rely on fluctuations in the amplitude [12] of the order parameter. For models based on phase fluctuations, Cooper pairs are assumed to be present on both sides of the transition. In the field-tuned transition, the interplay between disorder and vortex-vortex interactions plays an essential role. At $T = 0$, vortices are pinned at low fields, but for fields above a critical value vortices Bose condense into a vortex superfluid as the vortex density is increased. There is a duality between the field-tuned transition and the thickness-tuned transition, for which adding thickness results in a Cooper-pair condensate. In the vicinity of the quantum critical point, the resistance of a film is predicted to obey the scaling law [9,10]:

$$R(\delta, T) \propto R_c f(\delta t), \tag{1}$$

where $t(T) = T^{-1/\nu z}$, and δ is the deviation of a tuning parameter from its critical value. For field-tuned transitions, $\delta = |B - B_c|$ with B and B_c being the applied magnetic field and the critical field, respectively. The critical resistance R_c is predicted to have a universal value of $R_Q = h/4e^2 \approx 6.5\ \mathrm{k}\Omega/\square$. Scaling arguments set a lower bound on the correlation length exponent $\nu \geq 1$ [1] and give the value of the dynamical critical exponent $z = 1$ [9,10]. Experimentally, results consistent with these predictions have been found in scaling analyses of the field-tuned transitions in InO_x [5] and MoGe [6]. However, the predicted universal critical resistance is not observed in these experiments.

In this Letter, we report a scaling analysis of the field-tuned SI transition in ultrathin Be films quench condensed near 20 K. We have found that, for data measured in the zero-current limit, the scaling exponent product $\nu z = 1.35 \pm 0.10$. This agrees with results from relatively thicker InO_x [5] and MoGe [6] films, but disagrees with $\nu z = 0.7$ found in ultrathin Bi/Ge films [7,13]. We have also observed that R_c is significantly lower than R_Q in relatively thin and marginally superconducting Be films. However, R_c increases with increasing film thickness and reaches a nearly thickness-independent value of $1.2R_Q$ in robust superconducting Be films. Such behavior is very different from that observed in the MoGe [6] and Bi/Ge [7,13] systems, for which R_c decreases with increasing film thickness. We have also carried out for the first time studies of the field-tuned transition in the presence of a dc bias current. The applied dc bias current should exert a force of $\mathbf{j} \times \mathbf{B}$ on the vortices in the direction perpendicular to both the applied magnetic field $\mathbf{B}$ and the current density $\mathbf{j}$. We have observed that the νz becomes 0.75 ± 0.10 in the presence of dc bias currents, suggesting that the field-tuned transitions with and without dc bias current belong to different universality classes.

In Fig. 1, we plot the temperature dependence of the sheet resistance measured in the zero-current limit for one of our Be films at various magnetic fields applied

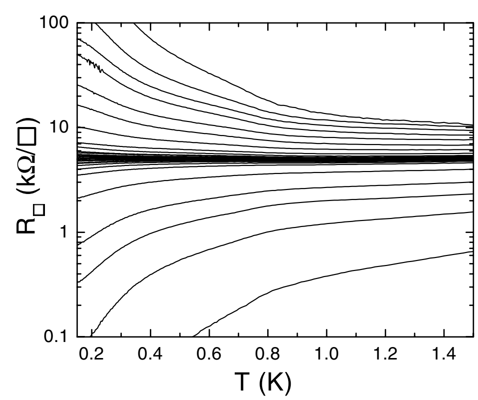

FIG. 1. Sheet resistance vs temperature measured on one marginally superconducting film at various magnetic field values. For curves from bottom to top, the field increased from 0.05 to 2.25 T. The critical field was 0.66 T. In zero field, $T_c \sim 2$ K.

 0031-9007/02/88(20)/206802(4)$20.00

perpendicular to the plane of the film. These Be films were quench condensed onto bare glass substrates which were held near 20 K during evaporations, under UHV conditions inside a dilution refrigerator. This *in situ* progressive evaporation setup allowed for systematic studies of the SI transition as film thickness was varied. We deposited each set of films in fine steps. We carefully monitored $R_\square$ during each evaporation step until a desirable value of $R_\square$ was reached. The films were very close to 10 Å in thickness, however, our quartz thickness monitor was not sensitive enough to pick up the small thickness increments after each step. Film resistance was measured in a standard four-terminal geometry using a PAR-124A lock-in amplifier operating at 27 Hz. The ac probe current was fixed at 1 nA. At the finite measuring temperatures in the vicinity of the field-tuned transition, the *I-V* characteristics in this low-current regime were linear. More details regarding these Be films have been published elsewhere [14]. Quench-condensed Be films are chosen for this study because such films are nearly amorphous [14]. These Be films undergo a transition from insulating to superconducting [14] when the normal state sheet resistance, R_N, is reduced below $\sim 10\ \mathrm{k}\Omega/\square$ with increasing film thickness. We have now studied the field-tuned SI transition in several films of R_N between 5.6 and 12 $\mathrm{k}\Omega/\square$. The superconducting transition temperature, T_c, of these films varied between 0.5 to 4 K in zero field.

The film in Fig. 1, with $R_N = 10.7\ \mathrm{k}\Omega/\square$ at 15 K, is considered marginally superconducting, as we will discuss later. With increasing field, corresponding to curves from the bottom to the top in Fig. 1, this film was driven from superconducting to insulating, with a rather flat $R_\square$ vs T curve at a critical field of $B_c = 0.66$ T. The main part of Fig. 2(a) shows $R_\square$ vs B at various temperatures for the same film as shown in Fig. 1. In the vicinity of a quantum critical point, the resistance of a two-dimensional system is predicted to obey the scaling law in Eq. (1). Determining the critical exponents involves plotting the $R_\square$ vs B data at various temperatures according to the scaling law. The good crossing point, over one decade in temperature, in the $R_\square$ vs B plot in the main part of Fig. 2(a) identifies $B_c = 0.66$ T and $R_c = 4.4\ \mathrm{k}\Omega/\square$ for this film. We have used two methods to determine the scaling exponent νz. First, we can find νz by evaluating the derivative of the $R_\square$ vs B curve at the critical field. In this case, we have the following scaling equation:

$$\frac{\partial R}{\partial B}\Big|_{B_c} \propto R_c T^{-1/\nu z} f'(0). \tag{2}$$

A plot of $\frac{\partial R}{\partial B}|_{B_c}$ vs T^{-1} on a log-log scale, shown in the inset to Fig. 2(a), yields a straight line with a slope equal to $1/\nu z$, from which we determine $\nu z = 1.36 \pm 0.10$. Alternatively, we can plot R/R_c vs $|B - B_c|t$ and treat $t(T)$ as an unknown variable. The values of $t(T)$ at various temperatures are determined by obtaining the best collapse of the data. Following this procedure presented in recent papers [7,13] by Markovic *et al.*, we obtain the temperature dependence of $t(T)$ which we plot on a log-log scale in the inset to Fig. 3(a). The collapse of the data is shown in the main part of Fig. 3(a) in a R/R_c vs $|B - B_c|t$ plot. Figure 3(a) shows good collapse of the data over 3 orders of magnitude in $|B - B_c|t$ and 2 orders of magnitude in R/R_c. The straight line in the inset to Fig. 3(a) shows a power-law fit, as expected by the scaling function, Eq. (1). The slope of the line in the inset gives $-1/\nu z$, from which we find $\nu z = 1.34 \pm 0.10$. The agreement between the above two methods shows the consistency of the scaling analysis. The scaling exponents obtained in our Be films in the zero-bias limit appear to be in good agreement with the predictions of theories based on the "dirty boson" model [9,10], and with renormalization group theories [15,16] and Monte Carlo [11] simulations.

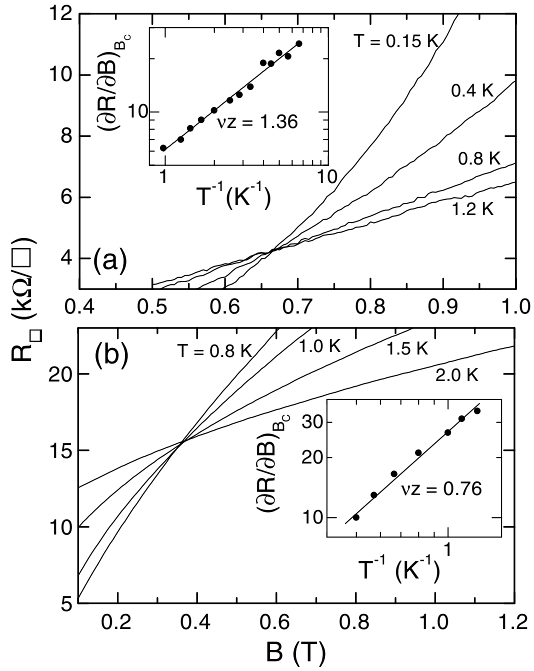

FIG. 2. Main figures show sheet resistance vs field at a number of temperatures as labeled in the figures, with $I_{\mathrm{bias}} = 0$ in (a) and $I_{\mathrm{bias}} = 2.5\ \mu$A in (b). The crossing points identify the critical fields and critical resistances. Insets show the power-law relations between $\frac{\partial R}{\partial B}|_{B_c}$ and T^{-1} for the corresponding I_{bias} values, which determine the values of νz.

Another important prediction of the bosonic model is the universal critical resistance $R_c = R_Q = h/4e^2 \approx 6.5\mathrm{k}\Omega/\square$ at the quantum critical point. This remains a controversial issue since only in the Bi/Ge system [7,13] has a critical sheet resistance close to the predicted value of R_Q been observed. In fact, careful investigations in the Bi/Ge system have revealed that R_c is $1.1R_Q$ to $1.2R_Q$ and decreases in a narrow range as R_N is reduced with increasing film thickness. It was suggested [7] that the slightly larger R_c than R_Q was consistent with calculations in two dimensions for a bosonic model including Coulomb interactions [17], which predicted $R_c \sim 1.4R_Q$, as well as Monte Carlo simulations in the (2+1)-dimensional XY model without disorder [18], which found $R_c =$

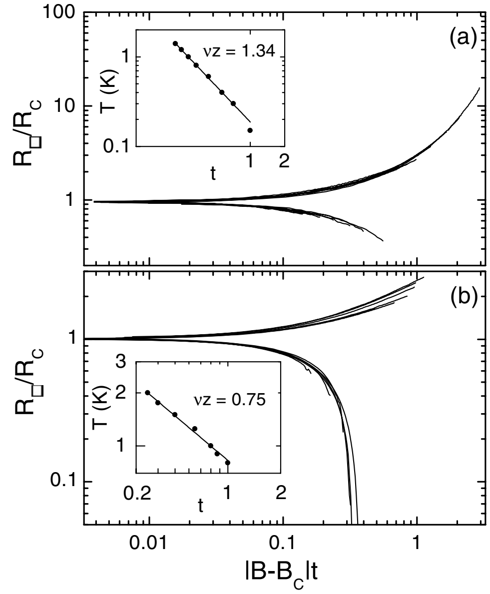

FIG. 3. Main figures show the scaling plots of R/R_c vs $|B - B_c|t$, with $I_{\text{bias}} = 0$ in (a) and $I_{\text{bias}} = 2.5$ μA in (b). Insets show the power-law relations between the parameter t and the temperature, which determine the values of νz.

$1.2R_Q$. The small variation of R_c with R_N could be explained [13] as a finite-temperature effect, since strictly speaking the critical resistance is predicted to be universal only at $T = 0$. In other systems, such as MoGe [6] films, R_c varies in a much wider range among films of varying R_N. In order to explain the low R_c of their MoGe films, Yazdani and Kapitulnik proposed [6] a two-channel model in which the conductance due to normal electrons adds to the conductance due to Cooper pairs. It has been noted [13] that the resulting film resistance can be either larger or smaller than R_Q since the magnetoresistance contribution [19] from the unpaired electrons can be either positive, if the spin-orbit coupling is strong, or negative, if the spin-orbit coupling is very weak.

Our Be films show a zero-field SI transition as R_N is reduced below $\sim$10 k$\Omega/\square$ with increasing film thickness [14]. Films of R_N near 11 k$\Omega/\square$ are considered marginally superconducting since they have near-zero critical field values. With increasing film thickness, the critical field, B_c, increases with decreasing R_N, as shown in Fig. 4(a). In Fig. 4(b), we plot how R_c in the field-tuned transition varies with R_N. For marginally superconducting films of R_N between 9 to 12 k$\Omega/\square$, R_c was significantly smaller than R_Q. Nevertheless, we see clearly that, as R_N is reduced by increasing film thickness, R_c increases. For thicker films having robust critical field values, R_c appears to saturate to a value of 7.1 $\sim$ 8.0 k$\Omega/\square$, which is about $1.2R_Q$. This value is very close to that observed in Bi/Ge films [13]. However, we note that the dependence of R_c on R_N observed in our films is the opposite of what was observed in MoGe [6] and Bi/Ge [7,13] films, for which R_c decreased with decreasing R_N. In addition, our results disagree with the suggestion [13] that the magnetoresistance of the unpaired electrons caused the discrepancy between R_c and R_Q. Since Be has the weakest spin-orbit coupling among metals, the magnetoresistance should be negative, leading to $R_c < R_Q$ following the two-channel model. We note that in Figs. 1, 2(a), and 3(a) we have shown data measured on a marginally superconducting film of $R_N = 10.7$ k$\Omega/\square$ at 15 K. Although the critical resistance of this film, $R_c \sim 4.4$ k$\Omega/\square$, is significantly smaller than R_Q, we have found that, for all the marginally as well as robust superconducting films of R_N ranging from 5.6 to 12 k$\Omega/\square$, the critical exponents are the same with $\nu z = 1.35 \pm 0.10$ in the zero-current limit.

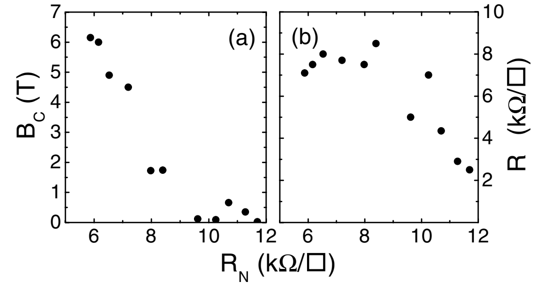

FIG. 4. Figures show (a) the critical field B_c, and (b) the critical resistance R_c, as functions of the normal-state sheet resistance R_N, measured at 15 K.

Below, we describe results of the field-tuned SI transition in the presence of a dc bias current, I_{bias}, which was varied between 125 nA and 2.5 μA and kept below the zero-field critical current ($\sim$15 μA). For each fixed I_{bias}, we used a magnetic field to tune a film from superconducting to insulating. We believe that joule heating was insignificant based on the following arguments. First, we have performed extensive *I-V* measurements over the entire temperature and magnetic field range of our experiments, with the time span for the *I-V* sweeps ranging from 10 min to 1 h. The *I-V* curves were completely reproducible without any observable hysteresis and independent of the sweep rate. Second, we can estimate the temperature increase, ΔT, on the Be films due to joule heating. The thin Be films were deposited on glass microslide substrates of thickness 0.23 mm, which were attached to a copper sample holder by a very thin layer of grease. The glass substrate was the dominant source of heat resistance, with a thermal conductivity of $\sim$ 0.0003 W/Km at 100 mK [20]. For a typical film square of size 3×3 mm^2 and resistance 10 kΩ, joule heating for $I_{\text{bias}} = 125$ nA is about 0.15 nW, resulting in $\Delta T \sim 0.012$ mK at 100 mK. For $I_{\text{bias}} = 2.5$ μA, $\Delta T \sim 5.0$ mK at 100 mK. We note that the heating power at 2.5 μA was 400 times larger than the heating power at 125 nA. If joule heating were significant, the data obtained with $I_{\text{bias}} = 2.5$ μA should show a flattening of the data in the low temperature region when compared to the data obtained with $I_{\text{bias}} = 125$ nA. The fact that the scaling results at 125 nA and 2.5 μA agree well suggests that heating was insignificant. This also argues against the existence of significant electron heating decoupled from the lattice.

We plot in Figs. 2(b) and 3(b), for $I_{\mathrm{bias}} = 2.5\ \mu$A, the results of a scaling analysis based on the two previously described methods. In Table I, we list the parameters from a scaling analysis of the field-tuned transition at various I_{bias}. Results from the data collapse method are presented for I_{bias} values of 250 nA and 2.5 μA, for which the amount of data taken was adequate for such an analysis. It appears that $\nu z \sim 0.75 \pm 0.10$, showing no systemic change with I_{bias} in the range we have studied. Nevertheless, it is significantly smaller than the νz found in the zero-current limit. We can only speculate that the bias current could lead to a symmetry breaking, resulting in different critical behavior at the transition. We note that experiments need to be carried out in other systems in order to find out whether this result is universal. In addition, experiments at low I_{bias} values should be carried out to determine whether νz changes abruptly or gradually as I_{bias} is increased from zero. Such experiments can probe the threshold I_{bias} for the change in the scaling exponents, and have the potential of revealing whether the new scaling exponents are produced by certain nonlinear effects in the vortices under a bias current. We also need to consider the possibility of conduction being dominated by narrow superconducting filaments, leading to a change in the dimensionality of the system. While it is difficult to determine to what extent film inhomogeneity affects our experiments, we would like to comment on the critical currents measured in our films. For the film shown in Figs. 2(b) and 3(b), the critical current in zero field was about 1.5×10^{-5} A. This film was about 10 Å in thickness and 3 mm in width. Thus, the critical current density would be about $5 \times 10^{6}\ \mathrm{A/m^2}$ if the current were uniformly distributed in the bulk of the film. On the other hand, if we assume that the current runs through a few filaments of 100 Å in total width, the critical current density would be $1.5 \times 10^{12}\ \mathrm{A/m^2}$. The critical currents of amorphous Be films have been measured by other groups, for example by Okamoto *et al.* [21]. Their Be films had a resistivity of $3.6 \times 10^{-6}\ \Omega$m. This value gives rise to $R_\square \sim 3.6\ \mathrm{k}\Omega/\square$ for a film of the same thickness as ours (≈ 10 Å), meaning that their films were roughly 3 times less resistive than ours. They found that their films had critical current density of about $1 \times 10^{8}\ \mathrm{A/m^2}$. Compared to their critical current density, the assumption in our films of conduction dominated by a few narrow filaments leads to an unreasonably large critical current density. In addition, we can compare our marginally superconducting films with robust superconducting films which are unlikely to be dominated by narrow filaments. As we have described, these two types of films showed an identical scaling exponent product νz in the limit of $I_{\mathrm{bias}} = 0$. It is possible that film inhomogeneity has lead to a depressed R_c in marginally superconducting films, but has not changed the effective dimensionality of the system. Therefore, the change in the scaling exponents with a dc bias is unlikely to be due to a dimensional crossover as a result of filamentary superconductivity.

TABLE I. Bias currents and derived parameters.

I_{bias} (nA)	B_c (T)	R_c (k$\Omega/\square$)	R_N (k$\Omega/\square$)	νz[a]	νz[b]
125	0.33	15.5	9.36	0.81	···
250	0.35	16.1	9.36	0.76	0.73
1000	0.36	15.5	9.36	0.73	···
2500	0.38	15.0	9.36	0.77	0.75

[a] νz obtained by the $\frac{\partial R}{\partial B}|_{B_c}$ method.
[b] νz obtained by the data collapse method.

In conclusion, we have studied the field-tuned SI transition in quench-condensed Be films with and without a dc bias current. Our scaling analysis has shown that the application of a bias current leads to a new set of scaling exponents, suggesting that the field-tuned transitions with and without dc bias belong to different universality classes.

We gratefully acknowledge S. Teitel and Y. Shapir for numerous useful discussions.

[1] S. L. Sondhi *et al.,* Rev. Mod. Phys. **69**, 315 (1997).
[2] A. M. Goldman and N. Markovic, Phys. Today, **51**, No. 11, 39 (1998).
[3] R. Fazio and H. S. J. van der Zant, Phys. Rep. **355**, 235 (2001).
[4] D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. **62**, 2180 (1989).
[5] A. F. Hebard and M. A. Paalanen, Phys. Rev. Lett. **65**, 927 (1990); M. A. Paalanen, A. F. Hebard, and R. R. Ruel, *ibid.* **69**, 1604 (1992)..
[6] Ali Yazdani and Aharon Kapitulnik, Phys. Rev. Lett. **74**, 3037 (1995); D. Ephron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, *ibid.* **76**, 1529 (1996).
[7] N. Markovic, C. Christiansen, and A. M. Goldman, Phys. Rev. Lett. **81**, 5217 (1998).
[8] M. P. A. Fisher *et al.,* Phys. Rev. B **40**, 546 (1989).
[9] M. P. A. Fisher, Phys. Rev. Lett. **65**, 923 (1990).
[10] M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Phys. Rev. Lett. **64**, 587 (1990).
[11] M.-C. Cha *et al.,* Phys. Rev. B **44**, 6883 (1991).
[12] J. M. Valles, Jr., R. C. Dynes and J. P. Garno, Phys. Rev. Lett. **69**, 3567 (1992); S-Y. Hsu, J. A. Chervenak, and J. M. Valles, Jr., *ibid.* **75**, 132 (1995)..
[13] N. Markovic *et al.,* Phys. Rev. B **60**, 4320 (1999).
[14] E. Bielejec and J. Ruan, and Wenhao Wu, Phys. Rev. B **63**, 100 502 (2001); E. Bielejec and J. Ruan, and Wenhao Wu, Phys. Rev. Lett. **87**, 036801 (2001)..
[15] L. Zhang and M. Ma, Phys. Rev. B **45**, 4855 (1992).
[16] K. G. Singh and D. S. Rokhsar, Phys. Rev. B **46**, 3002 (1992).
[17] Igor F. Herbut, Phys. Rev. Lett. **81**, 3916 (1998).
[18] M.-C. Cha and S. M. Girvin, Phys. Rev. B **49**, 9794 (1994).
[19] G. Bergmann, Phys. Rep. **107**, 1 (1984).
[20] G. K. White, *Experimental Techniques in Low Temperature Physics* (Clarendon Press, Oxford, 1979), 3rd ed., p. 320.
[21] M. Okamoto, K. Takei, and S. Kubo, J. Appl. Phys. **62**, 212 (1987).