

Naturalness Bounds on Dipole Moments from New Physics

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Assuming naturalness that the quantum corrections to the mass should not exceed the order of the observed mass, we derive and apply model-independent bounds on the anomalous magnetic moments and electric dipole moments of leptons and quarks due to new physics.

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In spite of its splendor of the phenomenological successes, the standard model of the elementary particles still leaves unanswered many fundamental questions, such as the origin of the quark-lepton generations, the curious pattern of their mass spectrum, and the unnatural fine tuning in the Higgs mass renormalization [1]. People expect that some new physics at some nearby high energy scale, such as compositeness [2], broken supersymmetry [3], extra dimensions [4], or brane worlds [5], would open ways to resolve these problems. At its early stage, signatures of the new physics might reveal themselves through effective nonrenormalizable interactions such as anomalous magnetic moments [6–10] and electric dipole moments [11]. The quantum corrections to the masses, due to these effects, diverge badly with an effective momentum cutoff at the new-physics scale. On the other hand, masses of the quarks, leptons, gauge bosons, and Higgs scalar are observed to be small or very small in comparison with the expected new-physics scale. It is unnatural that the large quantum corrections accidentally cancel its large bare mass to give the small or very small observed masses, unless it is protected by some dynamical mechanism which does not work at the tree level. This last exception is very unlikely. Thus, we can assume that the quantum contribution δm_{new} from the new physics should not exceed the order of the observed mass m_{obs} :

$$|\delta m_{\text{new}}| \leq O(m_{\text{obs}}) \quad (\text{naturalness bound}). \quad (1)$$

The δm_{new} on the left-hand side of (1) is written in terms of the new-physics parameters (the effective coupling constants, the cutoff scales, the heavy state masses, etc.) and other known quantities, and, consequently, it imposes a bound on the new-physics parameters. In fact, a relation of the type (1) for Higgs scalar mass is used to advocate the necessity of some new physics [1]. An argument with (1) for excited states in the composite model was given in Ref. [8] some time ago. Marciano has been using similar arguments for a long time [12]. In Ref. [13], two of the present authors (K.A. and K.K.) considered a model where the naturalness bound with (1) is saturated solely by the effects of anomalous magnetic moment from some new physics. In this paper, we apply the naturalness bound (1) to effective magnetic and electric dipole moments of fermions, which are expected in many of the

new-physics candidates [6–11], and we derive many useful phenomenological bounds.

We suppose that the new physics induces the anomalous magnetic moment μ and/or electric dipole moments d of quark or lepton ψ at low energies in comparison with new-physics scale Λ . The latter violates CP invariance. The effective Lagrangian for the interaction is given by

$$\mathcal{L} = -\frac{1}{2}\mu\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi - \frac{1}{2}id\bar{\psi}\sigma_{\mu\nu}\gamma_5F^{\mu\nu}\psi, \quad (2)$$

where $F^{\mu\nu}$ is the field strength of photon A^μ . Though (2) is a low-energy approximation for the real physics, we need to take into account its quantum effects up to its characteristic scale. If (2) were a fundamental interaction, the diagram in Fig. 1 would give rise to a quadratically divergent contribution to the fermion mass, which severely violates renormalizability. Now we argue that the internal line momenta of the diagram are, in many cases, effectively cut off at the characteristic scale of the new physics. For example, in the composite models, or the brane world models, the interaction for the momenta much higher than the inverse size of the composite particle extension or the brane world width can no longer be expressed in the form (2). Even though the effects of the high momenta should be taken into account by some other way, it is not through (2). Thus, we cut off the momenta as far as (2) is concerned. In the supersymmetric models, no real quadratic divergence in the diagram in Fig. 1 exists because they are canceled by those from the diagrams with the super partner internal lines. The symmetry, however, is broken, and the contributions of the order of the breaking scale do not cancel, while those much higher than the scale cancel. Thus, the momenta are cut off at the breaking scale.

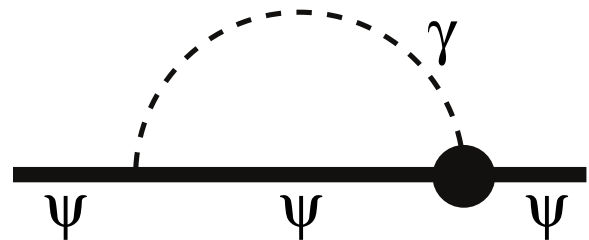


FIG. 1. Self-energy diagram for charged fermion with an anomalous vertex part, which is indicated by the blob.

As an approximation to this existing mechanism of the momentum cutoff, we insert the cutoff function,

$$(1 - \Lambda^2/q^2)^{-2}, \quad (3)$$

at the photon propagator, where Λ is the new-physics scale, and q is the photon momentum. The approximation with (3) is sufficient for our purpose, because we are concerned with the order-of-magnitude relation (1). If one wants, the cutoff (3) can be done in a gauge covariant way, by introducing the covariant derivative regularization to the photon kinetic terms. Then, it is straightforward to see that, due to the quantum effects via Fig. 1, the fermion mass term acquires the correction $\bar{\psi}(\delta m + i\delta m_5 \gamma_5)\psi$ with

$$\delta m = -3eQ\mu\Lambda^2/8\pi^2, \quad \delta m_5 = -3eQd\Lambda^2/8\pi^2, \quad (4)$$

where e is the electromagnetic coupling constant, Q is the electric charge of the fermion ψ , and we have neglected small contributions compared with Λ^2 . The bare Lagrangian, in general, should include a $\bar{\psi}\gamma_5\psi$ term:

$$L = \bar{\psi}(m_0 + im_5\gamma_5)\psi, \quad (5)$$

where m_0 and m_5 are the bare mass parameters. The physical mass m is defined as the coefficient of the $\bar{\psi}\psi$ term in the effective Lagrangian in the chirally transformed frame where the $\bar{\psi}\gamma_5\psi$ term vanishes. Then we have

$$m = \sqrt{(m_0 + \delta m + \dots)^2 + (m_5 + \delta m_5 + \dots)^2}. \quad (6)$$

where “...” indicates the other quantum corrections. The naturalness implies $\delta m, \delta m_5 < O(m)$, so that

$$3e|Q\mu|\Lambda^2/8\pi^2 < O(m), \quad 3e|Qd|\Lambda^2/8\pi^2 < O(m). \quad (7)$$

The relations in (7) have three interesting ways of phenomenological applications.

(i) We know from many existing experiments that the new physics scale is, roughly, at least greater than $\Lambda_{\min} = 1$ TeV [14]. Then we have the model-independent upper bounds,

$$|\mu|, |d| < O(8\pi^2 m/3e|Q|\Lambda_{\min}^2). \quad (8)$$

If we have an experimental value greater than the bound (8), we would be faced with a serious fine-tuning problem. They often render the most stringent phenomenological bounds for μ and d .

(ii) If we know the experimental upper bound $|\mu|_{\max}$ or $|d|_{\max}$ for $|\mu|$ or $|d|$, we have

$$|\kappa|\Lambda < O(\sqrt{8\pi^2 m|\kappa|_{\max}/3e|Q|}) \quad (\kappa = \mu \text{ or } d). \quad (9)$$

The quantity $|\kappa|\Lambda$ serves as the dimensionless coupling constant in perturbation expansion with the interaction Lagrangian (2), and its smallness is desired.

(iii) If we have real evidence that the dipole moment μ or d deviates from the standard-model predictions, and know the experimental lower bound $|\mu|_{\min}$ or $|d|_{\min}$ for $|\mu|$ or $|d|$, then the naturalness sets the model-independent *upper bound* for the responsible new-physics scale: Λ :

$$\Lambda < O(\sqrt{8\pi^2 m/3e|Q||\kappa|_{\min}}) \quad (\kappa = \mu \text{ or } d). \quad (10)$$

Now we apply the bounds (8)–(10) to the individual cases of the leptons and quarks. We indicate the quantities for each fermion by subscripts such as μ_e, d_μ , etc. Following conventions in the literature, we use $\delta a = \mu/(eQ/2m)$ instead of μ itself for the anomalous magnetic moment of charged leptons.

Muon.—The bound (8) with $\Lambda_{\min} = 1$ TeV implies

$$|\delta a_\mu| < O(6 \times 10^{-6}), \quad |d_\mu| < O(6 \times 10^{-19} e \text{ cm}), \quad (11)$$

where the former is much less stringent than the experimental deviation [MUON ($g - 2$) Collaboration [15]] from the standard-model expectation [16]:

$$\delta a_\mu = (25 \pm 16) \times 10^{-10}, \quad (12)$$

and the latter is comparable with the experimental bound [17],

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19} e \text{ cm}. \quad (13)$$

Then we use the bound (9) with the 95% C.L. (confidence level) upper bounds from (12) and (13) to get

$$|\mu_\mu|\Lambda < O(0.0003), \quad |d_\mu|\Lambda < O(0.012), \quad (14)$$

which justifies the perturbation expansions in μ_μ and d_μ .

In the near future, the experimental group will report the result of the more precise analysis which is now underway [15]. If an evidence would be found, for example, at the order of the central value of (12), the bound (10) would imply

$$\Lambda < O(50 \text{ TeV}), \quad (15)$$

on the scale Λ of the new physics responsible for the anomalous magnetic moment.

Electron.—The bound (8) with $\Lambda_{\min} = 1$ TeV implies

$$|\delta a_e| < O(1.5 \times 10^{-10}), \quad |d_e| < O(3 \times 10^{-21} e \text{ cm}), \quad (16)$$

where the former is less stringent than the experimental-theoretical result [18],

$$\delta a_e = (-1.2 \pm 2.8) \times 10^{-11}, \quad (17)$$

and the latter is much less stringent than the experimental bound [19],

$$d_e = (1.8 \pm 1.6) \times 10^{-27} e \text{ cm}. \quad (18)$$

Using the bound (9) and the 95% C.L. upper bounds from (17) and (18), we get

$$|\mu_e|\Lambda < O(0.00003), \quad |d_e|\Lambda < O(6 \times 10^{-8}), \quad (19)$$

which justifies the perturbation expansions in μ_e and d_e .

Tau-lepton.—The bound (8) with $\Lambda_{\min} = 1$ TeV implies

$$|\delta a_\tau| < O(0.002), \quad |d_\tau| < O(1.0 \times 10^{-17} e \text{ cm}), \quad (20)$$

which are more stringent than the experimental results (95% C.L.) [20],

$$-0.052 < \delta a_\tau < 0.058, \quad (21)$$

$$-3.1 \times 10^{-16} e \text{ cm} < d_\tau < 3.1 \times 10^{-16} e \text{ cm}. \quad (22)$$

From (9) with the 95% C.L. upper bounds from (21) and (22), we get

$$|\mu_\tau| \Lambda < O(0.9), \quad |d_\tau| \Lambda < O(0.9). \quad (23)$$

For the tau-lepton, the experimental bounds for weak dipole moments are also available. It is straightforward to extend our method to the electroweak theory. We have only to replace Q in the results by $(\pm 1/4 - Q \sin^2 \theta) / \cos \theta \sin \theta$ for the Z boson, and by $1/2\sqrt{2} \sin \theta$ for the W boson, where θ is the Weinberg angle.

Neutrinos.—Because $Q = 0$ for neutrinos, the diagram in Fig. 1 is absent, and we do not have the relations (7)–(10). Instead we should evaluate the two-loop diagrams in Fig. 2. This may require not only complex calculations, but also careful considerations about renormalization of the severely divergent nonrenormalizable diagrams. We will perform the investigation in another place. For the present purpose of the order-of-magnitude relations, it is sufficient to combine the typical one-loop calculations to guess the result,

$$\begin{aligned} \delta m &= -3eg^2 c \mu \Lambda^2 / 64\pi^4, \\ \delta m_5 &= -3eg^2 c d \Lambda^2 / 64\pi^4, \end{aligned} \quad (24)$$

where c is a numerical constant of $O(1)$, and g is the gauge coupling constant of $SU(2)_L$. Then, we obtain the

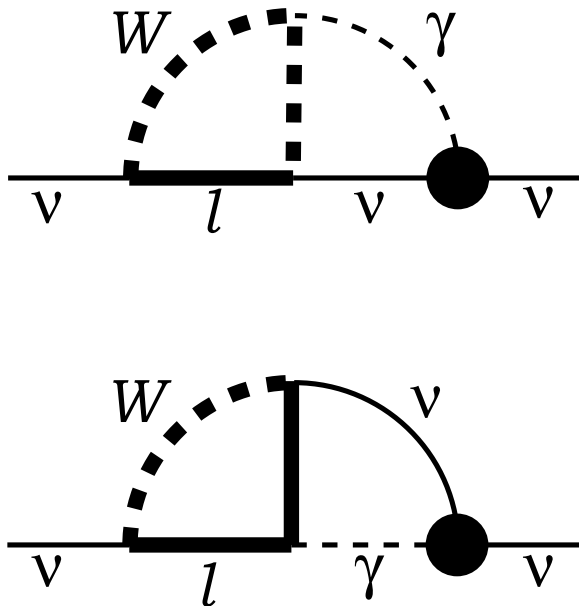


FIG. 2. Self energy diagrams for neutrino with an anomalous vertex part, which is indicated by the blob.

naturalness bound,

$$3eg^2 |c\kappa| \Lambda^2 / 64\pi^4 < O(m) \quad (\kappa = \mu \text{ or } d). \quad (25)$$

Here we again have three interesting phenomenological applications corresponding to (8)–(10).

(i) With the nearest new-physics scale, $\Lambda_{\min} = 1$ TeV,

$$|\mu|, |d| < O(64\pi^4 m / 3eg^2 |c| \Lambda_{\min}^2). \quad (26)$$

(ii) If we know the experimental upper bound $|\mu|_{\max}$ or $|d|_{\max}$ for $|\mu|$ or $|d|$, we have

$$|\kappa| \Lambda < O(\sqrt{64\pi^4 m |\kappa|_{\max} / 3eg^2 |c|}) \quad (\kappa = \mu \text{ or } d). \quad (27)$$

(iii) If we know the experimental lower bound $|\mu|_{\min}$ or $|d|_{\min}$ for $|\mu|$ or $|d|$, we have

$$\Lambda < O(\sqrt{64\pi^4 m / 3eg^2 |c| |\kappa|_{\min}}) \quad (\kappa = \mu \text{ or } d). \quad (28)$$

We use (26) with $\Lambda_{\min} = 1$ TeV and the experimental upper bounds [14],

$$\begin{aligned} m_{\nu_1} &< 3.0 \text{ eV}, & m_{\nu_2} &< 0.19 \text{ MeV}, \\ m_{\nu_3} &< 18.2 \text{ MeV}, \end{aligned} \quad (29)$$

where ν_1 , ν_2 , and ν_3 are the mass eigenstates of ν_e , ν_μ , and ν_τ . Then we get the naturalness bounds,

$$\begin{aligned} |\mu_{\nu_1}| &< O(1.7 \times 10^{-13} \mu_B), & |d_{\nu_1}| &< O(3 \times 10^{-24} e \text{ cm}), \\ |\mu_{\nu_2}| &< O(1.1 \times 10^{-8} \mu_B), & |d_{\nu_2}| &< O(2 \times 10^{-19} e \text{ cm}), \\ |\mu_{\nu_3}| &< O(1.1 \times 10^{-6} \mu_B), & |d_{\nu_3}| &< O(2 \times 10^{-17} e \text{ cm}), \end{aligned} \quad (30)$$

which are compared with the experimental or the phenomenological bounds [21–24],

$$\begin{aligned} |\mu_{\nu_1}| &< 1.5 \times 10^{-10} \mu_B, & |\mu_{\nu_2}| &< 7.4 \times 10^{-10} \mu_B, \\ |\mu_{\nu_3}| &< 18.2 \times 10^{-7} \mu_B, & |d_{\nu_3}| &< 5.2 \times 10^{-17} e \text{ cm}, \end{aligned} \quad (31)$$

where the first three are at 90% C.L., and the last, at 95% C.L. If we use the naturalness bound (27) and the experimental bounds (31), we get

$$\begin{aligned} |\mu_{\nu_1}| \Lambda &< O(1.5 \times 10^{-6}), & |\mu_{\nu_2}| \Lambda &< O(0.0008), \\ |\mu_{\nu_3}| \Lambda &< O(0.4), & |d_{\nu_3}| \Lambda &< O(0.5). \end{aligned} \quad (32)$$

The experimental results on the solar [25] and atmospheric [26] neutrinos suggest that the differences of m_{ν_1} , m_{ν_2} , and m_{ν_3} are much less than the order of eV [27], which implies that

$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3} < 3.0 \text{ eV}. \quad (33)$$

If we use (33) instead of (29), we have

$$\begin{aligned} |\mu_{\nu_1}|, |\mu_{\nu_2}|, |\mu_{\nu_3}| &< O(1.7 \times 10^{-13} \mu_B), \\ |d_{\nu_1}|, |d_{\nu_2}|, |d_{\nu_3}| &< O(3 \times 10^{-24} e \text{ cm}), \end{aligned} \quad (34)$$

instead of (30).

Quarks.—Though the magnetic and electric dipole moments of quarks are not directly measurable, they could

affect hadron phenomenology, for example, through scaling violation in deep inelastic lepton-hadron scattering or the electric dipole moments of nucleons. For quarks, we can again use the bounds (8)–(10), because they are electrically charged. The bound (8) with $\Lambda_{\min} = 1$ TeV and the phenomenological values of masses [14], $m_u = (1-5)$ MeV, $m_d = (3-9)$ MeV, $m_s = (75-170)$ MeV, $m_c = (1.15-1.35)$ GeV, $m_b = (4.0-4.4)$ GeV, and $m_t = (174.3 \pm 5.1)$ GeV, lead to

$$\begin{aligned} |\mu_u| &< O(4 \times 10^{-6} \mu_N), & |d_u| &< O(4 \times 10^{-20} e \text{ cm}), \\ |\mu_d| &< O(1.5 \times 10^{-5} \mu_N), & |d_d| &< O(1.5 \times 10^{-19} e \text{ cm}), \\ |\mu_s| &< O(0.0003 \mu_N), & |d_s| &< O(3 \times 10^{-18} e \text{ cm}), \\ |\mu_c| &< O(0.0011 \mu_N), & |d_c| &< O(1.1 \times 10^{-17} e \text{ cm}), \\ |\mu_b| &< O(0.007 \mu_N), & |d_b| &< O(7 \times 10^{-17} e \text{ cm}), \\ |\mu_t| &< O(0.14 \mu_N), & |d_t| &< O(1.5 \times 10^{-15} e \text{ cm}), \end{aligned} \quad (35)$$

where $\mu_N = e/2m_p$ with the proton mass m_p is the nuclear magneton.

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- [1] L. Susskind, Phys. Rev. D **20**, 2619 (1979); G. 't Hooft, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft *et al.*, NATO Advanced Study Institutes, Ser. B, Vol. 59 (Plenum Press, New York, 1980), p. 135; L. Maiani, in Proceedings of the Gif-sur-Yvette Summer School, Paris, 1980, p. 3; N. Sakai, Z. Phys. C **11**, 153 (1981); E. Witten, Nucl. Phys. **B188**, 513 (1981); M. Veltman, Acta Phys. Pol. B **12**, 437 (1981); R. K. Kaul, Phys. Lett. **109B**, 19 (1982); Pramana **19**, 183 (1982); R. K. Kaul and P. Majumdar, Nucl. Phys. **B199**, 36 (1982).
- [2] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); K. Akama and H. Terazawa, INS-Report 257, 1976; H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D **15**, 480 (1977).
- [3] Yu. A. Gol'fand and E. P. Likhtman, JETP Lett. **13**, 323 (1971); D. V. Volkov and V. P. Akulov, Phys. Lett. **46B**, 109 (1973); J. Wess and B. Zumino, Phys. Lett. **49B**, 52 (1974).
- [4] Th. Kaluza, Sitzungsberichte der K. Preussischen Akademie der Wissenschaften zu Berlin, 1921, p. 966; O. Klein, Z. Phys. **37**, 895 (1926).
- [5] M. D. Maia, Braz. J. Phys. **8**, 429 (1978); J. Math. Phys. **25**, 2090 (1984); K. Akama, Lect. Notes Phys. **176**, 267 (1983); V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983); M. Visser, Phys. Lett. **159B**, 22 (1985); M. Pavšič, Classical Quantum Gravity **2**, 869 (1985); G. W. Gibbons and D. L. Wiltshire, Nucl. Phys. **B287**, 717 (1987).
- [6] K. Akama, Prog. Theor. Phys. **51**, 1879 (1974).
- [7] K. Akama, Prog. Theor. Phys. **64**, 1494 (1980); G. L. Shaw, D. Silverman, and R. Slansky, Phys. Lett. **94B**, 57 (1980); S. J. Brodsky and S. D. Drell, Phys. Rev. D **22**, 2236 (1980); H. Terazawa, M. Yasuè, K. Akama, and M. Hayashi, Phys. Lett. **112B**, 387 (1982).
- [8] K. Akama, I. Ito, H. Komatsu, and M. Yasuè, Prog. Theor. Phys. **73**, 167 (1985).
- [9] S. Ferrara and E. Remiddi, Phys. Lett. **53B**, 347 (1974); P. Fayet, in *Unification of the Fundamental Particle Interactions*, edited by S. Ferrara, J. Ellis, and P. Van Nieuwenhuizen (Plenum, New York, 1980).
- [10] P. Nath and M. Yamaguchi, Phys. Rev. D **60**, 116006 (1999); R. Casadio, A. Gruppuso, and G. Venturi, Phys. Lett. B **495**, 378 (2000); M. L. Graesser, Phys. Rev. D **61**, 074019 (2000).
- [11] J. Ellis, S. Ferrara, and D. Nanopoulos, Phys. Lett. **114B**, 231 (1982); W. Buchmuller and D. Wyler, Phys. Lett. **121B**, 321 (1983); F. del Aguila, M. Gavela, J. Grifols, and A. Mendez, Phys. Lett. **126B**, 71 (1983); J. Frere and M. Gavela, Phys. Lett. **132B**, 107 (1983); J. Polchinski and M. Wise, Phys. Lett. **125B**, 393 (1983).
- [12] W. J. Marciano, in *Radiative Corrections: Status and Outlook*, edited by B. F. L. Ward (World Scientific, Singapore, 1995), pp. 403–414; W. Marciano, in *Particle Theory and Phenomenology*, edited by K. Lassila *et al.* (World Scientific, Singapore, 1996), p. 22.
- [13] K. Akama and K. Katsura, hep-ph/9807534.
- [14] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
- [15] Muon ($g - 2$) Collaboration, H. N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).
- [16] A. Czarnecki and W. J. Marciano, Nucl. Phys. (Proc. Suppl.) **76**, 245 (1999); M. Knecht and A. Nyffeler, hep-ph/0111058; M. Knecht, A. Nyffeler, M. Perrottet, and E. de Rafael, hep-ph/0111059; M. Hayakawa and T. Kinoshita, hep-ph/0112102; J. Bijnens, E. Pallante, and J. Prades, hep-ph/0112255.
- [17] J. Bailey *et al.*, Nucl. Phys. **B150**, 1 (1979).
- [18] R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. **59**, 26 (1987); H. Dehmelt, Phys. Scr. **T22**, 102 (1987); T. Kinoshita, Phys. Rev. Lett. **75**, 4728 (1995).
- [19] E. D. Commins, S. B. Ross, D. DeMille, and B. C. Regan, Phys. Rev. A **50**, 2960 (1994).
- [20] L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **434**, 169 (1998).
- [21] J. F. Beacom and P. Vogel, Phys. Rev. Lett. **83**, 5222 (1999).
- [22] D. A. Krakauer *et al.*, Phys. Lett. B **252**, 177 (1990).
- [23] A. M. Cooper-Sarkar, S. Sarkar, J. Guy, W. Venus, P. O. Hulth, and K. Hultqvist, Phys. Lett. B **280**, 153 (1992).
- [24] R. Escribano and E. Masso, Phys. Lett. B **395**, 369 (1997).
- [25] R. Davis, Jr., D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968); J. N. Bahcall, N. A. Bahcall, and G. Shaviv, Phys. Rev. Lett. **20**, 1209 (1968).
- [26] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); Phys. Lett. B **436**, 33 (1998).
- [27] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, J. High Energy Phys. **105**, 15 (2001).