

Breakdown of Energy Equipartition in a 2D Binary Vibrated Granular Gas

Klebert Feitosa* and Narayanan Menon†

Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003-3720

(Received 20 November 2001; published 24 April 2002)

We report experiments on the equipartition of kinetic energy in a mixture of pairs of different types of grains vibrated in two dimensions. In general, the two types of grains do not attain the same granular temperature, $T_g = 1/2m\langle v^2 \rangle$. However, the temperature ratio is constant in the bulk of the system and independent of the vibration velocity. The ratio depends strongly on the ratio of mass densities of the grains, but not on their inelasticity. Also, the temperature ratio is insensitive to compositional variables such as the number fraction of each component and the total number density. We conclude that a single granular temperature, as traditionally defined, does not characterize a multicomponent mixture.

DOI: 10.1103/PhysRevLett.88.198301

PACS numbers: 83.80.Fg, 05.20.Jj, 81.05.Rm, 83.10.Pp

When two different gases are put in thermal contact with each other they eventually reach thermal equilibrium, with equipartition of kinetic energy between the gas molecules and a single temperature for the system as mandated by the zeroth law of thermodynamics. In this Letter, we present an experiment designed to test the validity of energy equipartition in a mixture of two types of macroscopic grains driven into a nonequilibrium steady state by external vibration. Kinetic theories for granular systems have long used the idea of a granular temperature defined as the average kinetic energy of the particles, $T_g = 1/2m\langle v^2 \rangle$; the value of T_g in a given system is determined by a balance between the power input by external driving sources and the energy lost in interparticle collisions. The results of such calculations of transport in single-component systems have been validated in both simulations [1,2] and experiments [3]. It is natural to ask whether the variable T_g obeys a zeroth law and, in particular, whether a mixture of grains is characterized by a single, shared, granular temperature.

The earliest theoretical developments of kinetic theories for binary mixtures of spheres [4,5] or of disks [6] start with a clearly identified assumption that a single temperature variable, T_g , characterizes the entire mixture. Most subsequent work preserves this assumption. An important exception is the work of Garzo and Dufty [7] who study theoretically the “cooling” of a binary granular mixture from an initial distribution of particle velocities. They conclude that the cooling rate of each species in the mixture is the same, although their temperatures are different throughout the cooling process. Monte Carlo simulations of the Boltzmann equation agree with this calculation [8]. Experiments and simulations on binary mixtures of granular materials have primarily focused on the segregation and mixing of different types of grains [9] and, for the most part, have not considered the issue of equipartition. However, an experiment on a vibrated monolayer of grains by Losert *et al.* [10] found that grains of two different sizes and masses do not attain the same value of T_g . In this Letter, we show that such differences persist in the bulk of a 2D system and that, in general, the two components of a

mixture arrive at a ratio of granular temperatures, $\gamma \neq 1$. We explore the dependence of the temperature ratio, γ , on particle properties as well as on compositional parameters of the mixture. Our data argue for a redefinition of the granular temperature in order to accommodate an extension to a multicomponent system.

We make binary mixtures of spherical glass balls with aluminum, steel, or brass balls. The two relevant material properties being varied are mass and inelasticity (see Table I), and the two compositional parameters being varied are the number fraction of each component, x , and the number density, specified in terms of the average occupied area fraction, ρ_{avg} , of the particles in the cell. To determine the importance of particle material properties we hold the average number density and number fraction fixed and compare mixtures of steel (heavy-elastic) and glass (light-elastic), brass (heavy-inelastic) and glass (light-elastic), and aluminum (light-inelastic) and glass (light-elastic) in terms of the T_g of each component and the temperature ratio γ . Next, to determine whether failure of the equipartition is a bulk effect, we hold the number fraction fixed for a steel-glass mixture and measure the temperature ratio as the number density of balls is varied. Finally, to test the dependence of the temperature ratio on the relative fraction concentration, we hold the number density fixed for a

TABLE I. Some material properties of the spheres used in the experiment.

Particle	Mass [mg]	Effective inelasticity ^a	Mass ratio w/glass
Glass	5.24	0.17	—
Aluminum	5.80	0.31	0.92
Steel	15.80	0.21	0.33
Brass	18.00	0.39	0.28

^aThe effective inelasticity quoted is the fraction of energy lost in a collision in a single component system, averaged over all collisions. Since the relative velocity of the spheres, the impact parameter, and rotational motion all affect the inelasticity of a given collision, the number quoted above is the mean of a wide distribution.

brass-glass pair, and measure the temperature ratio of the components as the number fraction of brass is varied.

The balls (diameter $d = 1.600 \pm 0.002$ mm) are confined to a vertical, rectangular, Delrin cage ($32d$ high \times $48d$ wide \times $1.1d$ thick) sandwiched between two parallel plates of antistatic coated Plexiglas. An electromechanical shaker (LDS 500L) vibrates the cage vertically at a frequency of 60 Hz and amplitudes up to $2.4d$, producing maximum accelerations, Γ , and velocities, v_0 , of 56 g and 1.45 m/s, respectively. The motions of the balls are recorded with a monochrome high-speed camera (Kodak Motioncorder) at a rate of 2000 frames/s. Each ball is located with a precision of $0.03d$ ($\approx 40 \mu\text{m}$). Glass balls are unambiguously distinguished from their metallic partners by using a combination of reflected and transmitted light, and by thresholding appropriately. Most of the results we discuss here are taken in a rectangular ($10d \times 20d$) window centered in the geometrical center of the cell.

The data presented in this Letter are all taken at relatively large values of excitation, corresponding to $\Gamma > 32$ g and $v_0 > 0.86$ m/s. Beyond this scale of excitation, the vertical density profile does not vary much and tends to an asymptotic profile that is symmetric about the center of the cell, as shown in Fig. 1. The density profile is identical for two different species when their masses are matched, even if they have different inelasticity, as in the case of the aluminum-glass mixture shown in Fig. 1(a). In contrast, the profiles of the other two mixtures (steel-glass and brass-glass) in Fig. 1(b) show a slightly higher concentration of heavy particles in the center of the cell and a more uniform distribution of light particles throughout the cell. However, in all three mixtures, while there are differences in the overall density distribution, the particles in the mixture are locally well mixed, with no obvious tendency for balls of

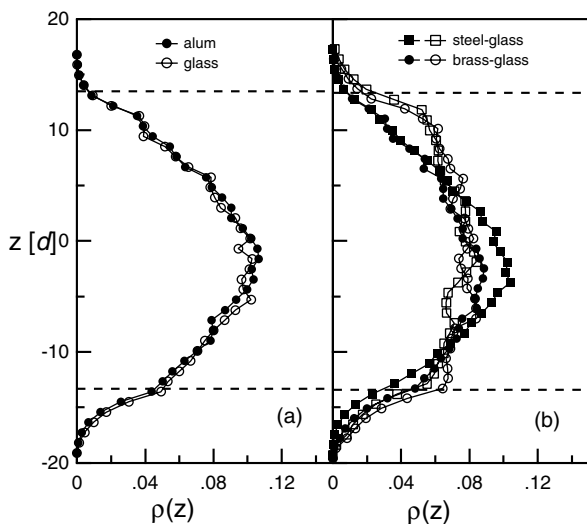


FIG. 1. (a) Density profile, ρ , of aluminum-glass ($\rho_{\text{avg}} = 0.096$) versus height, z (in d). (b) Density profile, ρ , of steel-glass ($\rho_{\text{avg}} = 0.096$), and brass-glass ($\rho_{\text{avg}} = 0.049$, scaled) versus height, z . The dashed lines represent the limits of the vibrating walls. $\Gamma = 56$ g and $x = 1/2$ for all three pairs.

a particular type to cluster [11] when vigorously excited as in this case. Finally, we have not observed any gradients of velocity or density in the horizontal direction.

The principal observation of our experiment is that the two components of a binary mixture do not always equilibrate to the same granular temperature. Figure 2(a) shows T_g of three pairs of materials as a function of the squared vibration velocity of the cell, v_0^2 , with the number density being held fixed at $\rho_{\text{avg}} = 0.096$ and the number fraction of glass balls fixed at $x = 1/2$. The granular temperature T_g is proportional to the squared vibration velocity v_0^2 with the proportionality constant being a function of number density ρ_{avg} and inelasticity of the balls (for instance, the temperature of glass is lower in the more inelastic glass-brass mixture than in the glass-steel mixture). As shown in Fig. 2(b), T_g equilibrates for the aluminum-glass mixture, but does not equilibrate for the steel-glass and brass-glass mixtures. In terms of particle properties, equipartition is achieved when the mass ratio is close to one ($m_{\text{glass}}/m_{\text{alum}} = 0.92$), and fails when the masses are mismatched ($m_{\text{glass}}/m_{\text{steel}} = 0.29$, $m_{\text{glass}}/m_{\text{brass}} = 0.33$). Even when the temperatures of the two components do not equilibrate, the temperature ratios do not vary as a function of v_0^2 within the error bars, as shown in Fig. 2(b). Furthermore, the temperature ratios are very close to one another ($\gamma_{g-s} = T_{\text{glass}}/T_{\text{steel}} = 0.66 \pm 0.06$ and $\gamma_{g-b} = T_{\text{glass}}/T_{\text{brass}} = 0.69 \pm 0.09$), even though the inelasticity of the metal partner in each case is quite different (Table I). We conclude that the temperature ratio differs from one when the mass ratio departs significantly from one, but

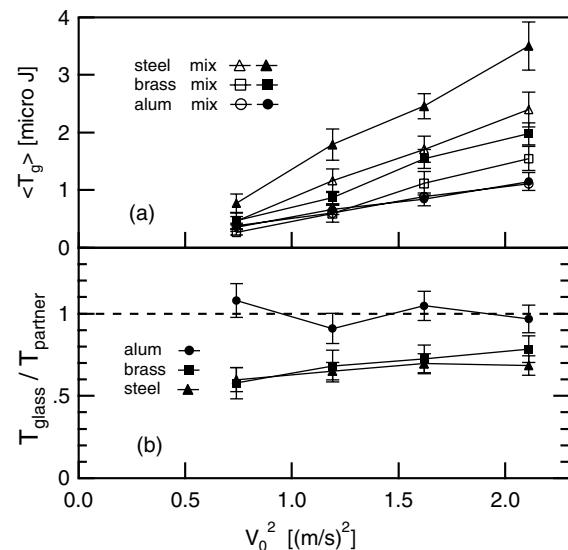


FIG. 2. (a) Average granular temperature $\langle T_g \rangle$ versus squared vibration velocity v_0^2 for aluminum-glass, brass-glass, and steel-glass mixtures. The open symbols and filled symbols represent T_g for glass and the metal partner, respectively. $\rho_{\text{avg}} = 0.096$ and $x = 1/2$ for all mixtures. (b) Ratio between glass temperature and partner temperature, $\gamma = T_{\text{glass}}/T_{\text{partner}}$, versus the squared vibration velocity, v_0^2 .

does not depend sensitively on the relative value of the inelasticity of the two components. Moreover, the temperature ratio is not equal to the ratio between the temperatures measured in the pure systems.

The failure of equipartition is not merely a boundary effect, nor is the value of γ that we report in Fig. 2(b) an inhomogeneous average over different regions of the cell. This is shown in Fig. 3, where the temperature profiles of steel and glass are plotted as a function of the height of the cell, $z[d]$, for $v_0 = 1.45$ m/s. In a collision with a moving wall which is effectively infinitely massive, both species pick up the same velocity, which implies that the heavier species (steel) picks up greater kinetic energy. The temperature ratio γ is thus different from one at the walls; this is analogous to the previous observations of Losert *et al.* [10]. Remarkably, however, the temperature ratio quickly reaches a constant value ($\gamma = 0.69$) in the interior of the cell, even as the T_g of each component continues to decrease toward the middle of the cell as a result of losses from inelastic collisions. Therefore the effect of inelastic collisions is to drive the value of γ in the interior of the cell toward a value different from one, which, as we mentioned above, depends on the mass ratio of components, but is insensitive to their inelasticity, and to the value of v_0 .

We pursue further the possibility that the value of γ obtained is due to boundary effects by increasing the number density, ρ_{avg} , of a steel-glass mixture holding the number fraction fixed at $x = 1/2$. While this is not exactly equivalent to increasing the system size, it does tune the ratio of the system size to the mean-free path, so that we vary the number of collisions experienced by balls in transferring energy from the boundary to the interior. The temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$ is plotted against the squared vibration velocity, v_0^2 , in Fig. 4, for mixtures whose number density, ρ_{avg} , varies by a factor of 4. Once again, the ratio of the two temperatures is essentially independent of v_0^2 . Importantly, the temperature ratio maintains an average value of $\gamma = 0.60 \pm 0.06$, with no obvious systematic trend as ρ_{avg} changes. This result further confirms that the failure of equipartition is a bulk effect, and also suggests

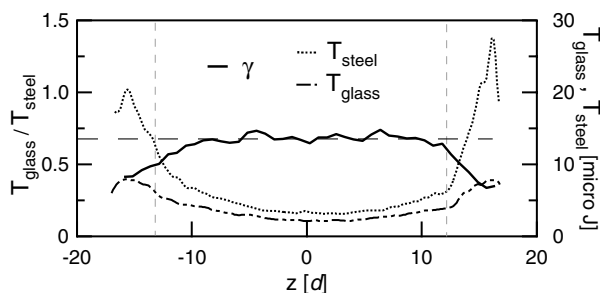


FIG. 3. Vertical temperature profiles for a steel-glass mixture. The dotted and dot-dashed lines represent T_{steel} and T_{glass} and the solid line represents the ratio between the glass and steel temperatures. The dashed vertical lines represent the limits of the vibrating walls. For this system $\rho_{\text{avg}} = 0.096$, $x = 1/2$, and $\Gamma = 56$ g.

that the value of γ is insensitive to the number density of the mixture.

Finally, we study the effect on equipartition of varying number fractions in a mixture at fixed density. Figure 5 shows the temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{brass}}$, of a brass-glass mixture as a function of v_0^2 , for three different number fractions of brass, x : $1/2$, $1/4$, and $1/8$. The temperature ratios for the different relative concentrations are all the same within experimental errors. This demonstrates that the components do not reach thermal equilibrium, even though the relative number of collisions with glass balls increases for a given brass ball, as the number fraction of brass is decreased. It has been predicted [12] that equipartition does not generally occur even in the tracer limit of a single foreign particle in a sea of other particles. This is consistent with our observation that the temperature ratio γ does not depend on number fraction.

As mentioned earlier, Garzo and Dufty [7] study a freely cooling binary system, and find a temperature ratio dependence on material parameters as well as on compositional parameters. Since there are no comparable predictions for a driven system, we have compared our data with these predictions and find quite good agreement with our results as a function of material parameters. However, they predict a strong dependence on the number fraction of the components, where none is observed. The comparison between theory and experiment encourages further theoretical development in driven systems.

In conclusion, we have observed that energy equipartition does not generally hold for a binary vibrated granular system. However, the ratio between the steady state values of the granular temperatures of the two components does robustly satisfy some general trends. The temperature ratio is constant in the interior of a system, and is insensitive to vibration velocity and inelasticity, while depending strongly on mass density. Furthermore, the ratio is insensitive to compositional parameters such as the number fraction of each component and average number density. This opens the possibility that, just as in the work of

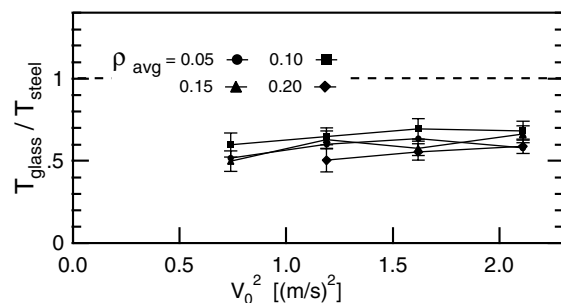


FIG. 4. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{steel}}$, in a steel-glass mixture plotted against squared vibration velocity, v_0^2 . Different markers represent different number densities of the mixture. The number fraction is fixed at $x = 1/2$. The horizontal dashed line represents equipartition ($\gamma = 1$).

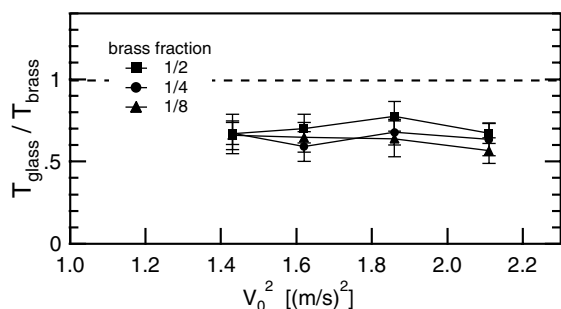


FIG. 5. Temperature ratio, $\gamma = T_{\text{glass}}/T_{\text{brass}}$, in a brass-glass mixture versus the squared vibration velocity of the cell, v_0^2 . Different markers represent different number fractions of brass for the same total number of particles ($\rho_{\text{avg}} = 0.049$). The horizontal dashed line represents equipartition ($\gamma = 1$).

Garzo and Dufty [7] for a cooling system, there is a well-defined and calculable ratio of temperatures achieved in a steady state system. Thus a modified definition of granular temperature, with a prefactor depending chiefly on material properties, may adequately describe a multicomponent system. Numerical simulations could play a useful role in that material properties may be continuously tuned, as opposed to experiments in which only a limited pool of materials is readily available.

We gratefully acknowledge support from NSF DMR CAREER 987433, and ACS PRF G33730. We also thank J.L. Machta for useful conversations and F. Rouyer for contributions to this work.

Note added.—After submission of this Letter, we became aware of the closely related work of Wildman and

Parker [13] that also studies the granular temperature in a binary vibrated system.

*Electronic address: kfeitosa@physics.umass.edu

†Electronic address: menon@physics.umass.edu

- [1] K. Helal, T. Biben, and J. P. Hansen, *Physica (Amsterdam)* **240A**, 361 (1997).
- [2] C. Bizon, M. D. Shattuck, J. B. Swift, and H. L. Swinney, *Phys. Rev. E* **60**, 4340 (1999).
- [3] F. Rouyer and N. Menon (to be published); E. Rericha, C. Bizon, M. D. Shattuck, and H. Swinney, cond-mat/0104474; X. Yang *et al.*, cond-mat/0108256.
- [4] M. Farrell, C. K. K. Lun, and S. B. Savage, *Acta Mech.* **63**, 45 (1986).
- [5] J. Jenkins and F. Mancini, *Phys. Fluids A* **1**, 2050 (1989).
- [6] J. Jenkins and F. Mancini, *J. Appl. Mech.* **54**, 27 (1987).
- [7] V. Garzo and J. Dufty, *Phys. Rev. E* **60**, 5706 (1999).
- [8] J. M. Montanero and V. Garzo, *Granular Matter* (to be published).
- [9] J. M. Ottino and D. V. Khakhar, *Annu. Rev. Fluid Mech.* **32**, 55 (2000).
- [10] W. Losert *et al.*, *Chaos* **9**, 682 (1999).
- [11] By contrast, at low accelerations, there is a clear tendency toward complete segregation, as in the simulation of S. McNamara and S. Luding [*IUTAM Symposium on Segregation in Granular Flows*, edited by A. D. Rosato and D. L. Blackmore (Kluwer, Dordrecht, 2000), p. 305]; S. Luding *et al.*, *ibid.*, p. 297.
- [12] A. Santos and J. W. Dufty, *Phys. Rev. Lett.* **86**, 4823 (2001); P. A. Martin and J. Piasecki, *Europhys. Lett.* **46**, 613 (1999).
- [13] R. D. Wildman and D. J. Parker, *Phys. Rev. Lett.* **88**, 064301 (2002).