# Size Scaling of Turbulent Transport in Magnetically Confined Plasmas

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Transport scaling with respect to device size in magnetically confined plasmas is critically examined for electrostatic ion-temperature-gradient turbulence using global gyrokinetic particle simulations. It is found, by varying device size normalized by ion gyroradius while keeping other dimensionless plasma parameters fixed, that fluctuation scale length is microscopic in the presence of zonal flows. The local transport coefficient exhibits a gradual transition from a Bohm-like scaling for device sizes corresponding to present-day experiments to a gyro-Bohm scaling for future larger devices.

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Transport levels in magnetically confined plasmas are generally observed to be well above those associated with collisional relaxation processes. This anomalous transport is believed to arise from microscopic turbulence driven by pressure gradients. The balance between turbulent transport and heating power determines the performance of magnetic fusion plasmas. Therefore, an accurate prediction of the expected transport level is critical for the design of fusion reactors. At present, the reactor design studies [1] rely on extrapolations of turbulent transport properties from present-day tokamak experiments to larger devices. These estimates are based in large part on some forms of empirical scaling, particularly device size scaling, for the global energy confinement time. These empirical scaling estimates are not always compatible with theoretical constraints from transformation invariants of fundamental plasma equations [2]. In this work, transport scaling with respect to device size is critically examined using first-principles gyrokinetic particle simulations for electrostatic toroidal ion temperature gradient (ITG) turbulence [3], which is a leading candidate to account for anomalous ion thermal transport in the tokamak core region. These large scale nonlinear simulations have recently been enabled by advances in efficient algorithms and by effective utilization of tera-scale massively parallel computers.

In the absence of a fundamental, first-principles turbulence theory, heuristic, mixing length rules are often utilized to estimate size scaling of turbulent transport [3]. This approach invokes a random walk type of picture for diffusive processes using the scale length of turbulent eddies as the step size and the linear growth time of the instability as the step time. It predicts that if the eddy size increases with device size, the transport scaling is Bohm-like, i.e., local ion heat diffusivity is proportional to $\chi_B = cT/eB$. Here $c$, $T$, $e$, $B$ are, respectively, speed of light, electron temperature, electric charge of electrons, and magnetic field amplitude. On the other hand, if the eddy size is microscopic (on the order of the ion gyroradius), the transport scaling is gyro-Bohm, i.e., local ion heat diffusivity is proportional to $\chi_{\mathrm{GB}} = \rho^* \chi_B$. Here, $\rho^* = \rho_i/a$ is the ion gyroradius $\rho_i$ normalized by the tokamak minor radius $a$. If transport is not diffusive (e.g., large transport events dominate the contribution to energy fluxes) the scaling can also be Bohm-like. Most theories [3] and local (or flux-tube) direct simulations [4] of ITG turbulence predict a gyro-Bohm scaling for ion transport since they assume fluctuations on a microscopic scale length and ignore pressure gradient profile variations. The gyro-Bohm scaling is often the implied scaling in reactor designs [1], and is clearly beneficial for larger devices since it predicts that transport coefficient decreases when the device size increases. However, trends from experimental observations have been more complicated. Transport scalings in low confinement regimes ($L$ mode) have always been observed to be Bohm or worse than Bohm in major tokamaks [5,6]. In particular, dimensionless scaling studies on the DIII-D tokamak found that ion transport and energy confinement time exhibit Bohm-like behavior, while fluctuation characteristics suggest a gyro-Bohm scaling [7] for transport. In the high confinement regime ($H$ mode), transport scalings have been reported to be either Bohm [8] or gyro-Bohm in limited operational parameter space [6]. The uncertainty here may, in part, reflect the difficulty in varying $\rho^*$ while keeping all other dimensionless parameters fixed (e.g., Mach number of toroidal rotation in $H$ mode).

An effective tool for scaling studies is full torus gyrokinetic particle simulations [9]. In these large scale calculations, kinetic effects and global profile variations are treated rigorously, and $\rho^*$ can be varied over a wide range while all other dimensionless parameters are fixed. In previous global gyrokinetic simulations of electrostatic ITG turbulence, Bohm-like transport scaling was observed due to radially elongated eddies associated with the global structure of linear toroidal eigenmodes [10]. However, those scaling studies did not properly deal with turbulence-driven zonal flows. Our more realistic simulations in which zonal flows are self-consistently included found that the global mode structure is destroyed by the random shearing action of the zonal flows. This results predominantly in the reduction of the radial correlation length and subsequently the turbulence level [11]. This finding that the shearing of zonal flows is the dominant saturation mechanism represents a new nonlinear paradigm that is fundamentally different from that of the Hasegawa-Mima system [12], which has been popular because of its simplicity as a nonlinear paradigm for understanding drift wave turbulence. This motivated us to carefully study $\rho^*$ scaling with self-generated zonal flows using large-scale simulations with device-size scans.

In this Letter, we report simulation results which show that the fluctuation scale length is microscopic and independent of device size, that test particle transport is diffusive, and that local transport coefficient exhibits a gradual transition from a Bohm-like scaling for device sizes corresponding to present-day tokamak experiments to a gyro-Bohm scaling for future larger devices. The device size where this transition occurs is much larger than that expected from linear theory based on pressure gradient profile variations. These findings show that extrapolations based on empirical scalings or mixing length rules can be unreliable and that full device nonlinear simulations can play a key role in complementing and then eventually replacing extrapolation methods by directly addressing parameter regimes inaccessible through conventional analytic or experimental approaches.

This study employed a well bench-marked, massively parallel full torus gyrokinetic toroidal code (GTC) [11] and used representative parameters of DIII-D tokamak $H$-mode core plasmas which have a peak ion temperature gradient at $r = 0.5a$ with the following local parameters: $R_0/L_T = 6.9$, $R_0/L_n = 2.2$, $q = 1.4$, $\hat{s} \equiv (r/q)(dq/dr) = 0.78$, $T_e/T_i = 1$, $\epsilon \equiv r/R_0 = 0.18$. Here $R_0$ is the major radius, $r$ is the minor radius, $L_T$ and $L_n$ are the temperature and density gradient scale lengths, respectively, $T_i$ and $T_e$ are the ion and electron temperature, and $q$ is the safety factor. These parameters [13] give rise to a strong ITG instability with a linear threshold of $(R_0/L_T)^{\rm crit} = 4.0$. These global simulations used fixed boundary conditions with electrostatic potential $\delta\phi = 0$ enforced at $r < 0.1a$ and $r > 0.9a$. The size of the tokamak is varied up to $a = 1000\rho_i$ with $\rho_i$ measured at $r = 0.5a$ and other key dimensionless parameters fixed. The simplified physics model includes the following: a parabolic $q$ profile, a pressure gradient profile of $\exp\{-[(r - 0.5a)/0.3a]^6\}$, a circular cross section, no impurities, and electrostatic fluctuations with an adiabatic electron response. Externally driven plasma flows and collisions [14] are not treated in these simulations. In the full torus nonlinear simulation of $a = 1000\rho_i$, we calculated 7000 orbital time steps of $1 \times 10^9$ particles (guiding centers), and interactions of these particles with self-consistent electrostatic potential represented on $125 \times 10^6$ spatial grid points to address realistic reactor-grade plasma parameters covering disparate spatial and temporal scales. These large scale simulations became feasible only recently with the implementation of an efficient global field-aligned mesh using magnetic coordinates, which reduces computational requirements by 2 orders of magnitude, and with the access to a multiteraflop massively parallel computer, the fastest civilian computer in the world at the time of this study [15].

Each of these simulations starts with very small random fluctuations which grow exponentially due to the ITG instability. Zonal flows are then generated through modulational instability [16,17] and saturate the toroidal ITG eigenmodes through random shearing [18]. Finally, the nonlinear coupling of ITG-zonal flows leads to a fully developed turbulence with a steady state transport level that is insensitive to initial conditions. The fluctuations in the steady state are nearly isotropic in radial and poloidal directions. In contrast, when zonal flows are suppressed in the simulation, the radial spectra is narrower (dominated by low $k_r$ components) for larger device sizes [19], and the radially extended eddies may induce large scale transport events which give rise to Bohm-like transport [10].

First, we quantify the fluctuation scale length. Radial correlation functions for the fieldline-averaged fluctuation quantities (density perturbations, etc.) are calculated using $r = 0.5a$ as a reference position, and averaged in toroidal direction because of axisymmetry and over a few eddy turnover times assuming statistically steady state. The correlation functions for density perturbations (or electrostatic potential excluding zonal flow component) are found to be self-similar for different tokamak size (Fig. 1), and suggest a turbulent eddy size of $\sim 7\rho_i$ which is independent of device size. The correlation functions for temperature perturbations are very similar to that of the density perturbations, and the correlation functions for heat flux show a correlation length about half of those for density and temperature perturbations. All these correlation functions decay exponentially and no significant tails at large radial separations exist. We conclude that fluctuation scale length is microscopic, i.e., on the order of ion gyroradius and independent of device size.

Next we examine whether transport is diffusive using the probability distribution function for the radial diffusion of test particles (passive particles that do not affect the turbulence). After nonlinear saturation, $6 \times 10^6$ test particles are initiated around $r = 0.5a$ with a uniform poloidal distribution. The probability distribution function of radial displacement after a few eddy turnover times

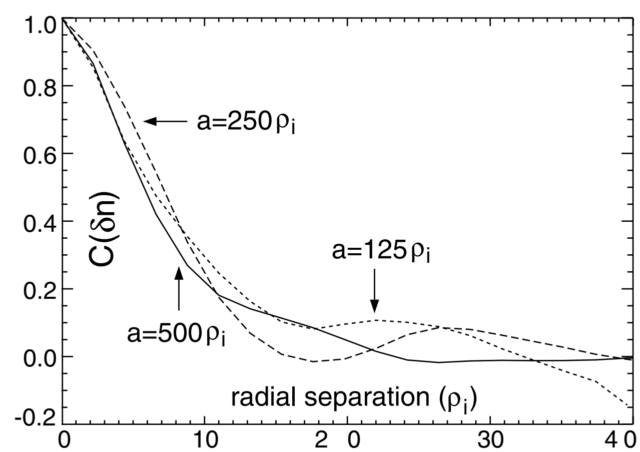


FIG. 1. Radial correlation functions for density perturbations.

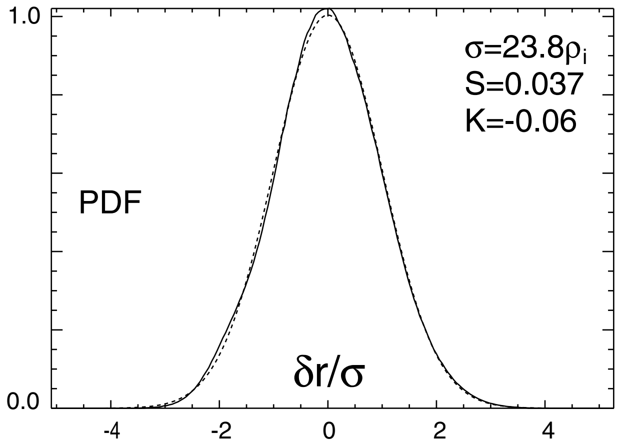


FIG. 2. Probability density function of test particle radial displacement (solid line) is very close to a Gaussian (dotted line). $\sigma$, $S$, $K$ are, respectively, the standard deviation, skewness, and kurtosis of the measured data.

is found to be very close to a Gaussian (Fig. 2). Further examination of the deviation from the Gaussian reveals no singular structure in either pitch angle or energy space. This indicates that there is no sharp resonance in the wave-particle interactions. Since the radial motion of test particles is diffusive rather than ballistic, the wave does not trap or convect the particles, but only scatters the particle orbits. We can calculate ion heat conductivity based on the random walk model of test particle heat flux, $Q$, due to the energy-dependent diffusivity $D = \sigma^2/2\tau$, $Q = -\int \frac{1}{2}v^2 D\partial f/\partial r d^3v$, where $\sigma$ is the standard deviation for radial displacement at time $\tau$ after the initiation of test particles. We also measure the self-consistent heat flux, $Q = \int \frac{1}{2}v^2 \delta v_{E\times B}\delta f d^3v$, where $v$ is particle velocity, $\delta f$ is the perturbed distribution function, and $\delta v_{E\times B}$ is the radial component of gyrophase-averaged $\mathbf{E} \times \mathbf{B}$ drift. We found that the test particle heat flux is very close to the self-consistent heat flux. This suggests that wave transport, where the wave extracts energy from ions in the hot region and deposits it back to ions in the cold region, does not play a significant role. Furthermore, the probability distribution functions for the electrostatic potential, temperature fluctuations, and heat fluxes all decay exponentially with no significant tails at large amplitudes. This is observed for large devices where the transport scaling is gyro-Bohm, and where there are a large number of data samplings for adequate statistics. We conclude that the heat flux is carried by the radial diffusion of particles, and that large transport events, where heat pulses propagate ballistically, are apparently absent over this simulation time.

Now that the fluctuation scale length is found to be microscopic and test particle transport be diffusive, we might expect the transport scaling is gyro-Bohm. Surprisingly, local ion heat conductivity (Fig. 3) measured at $r = 0.5a$ in this scan exhibits Bohm-like scaling for plasmas corresponding to present-day tokamak experiments ($a < 400\rho_i$) even though turbulence eddy size is independent of device size. This result is consistent with recent dimensionless scaling studies on the DIII-D tokamak which found that ion transport and energy confinement time exhibit Bohm-like behavior while fluctuation characteristics suggest a gyro-Bohm scaling [7]. As we increase the device size further (up to $a = 1000\rho_i$), there is a gradual transition from Bohm-like scaling to gyro-Bohm scaling. Interestingly, recent transport studies of the JET tokamak [8] and a scan of power thresholds for the formation of internal transport barriers [20] show a similar trend. These findings show that extrapolations from present-day experiments to larger devices based on empirical scalings or mixing length rules can be unreliable.

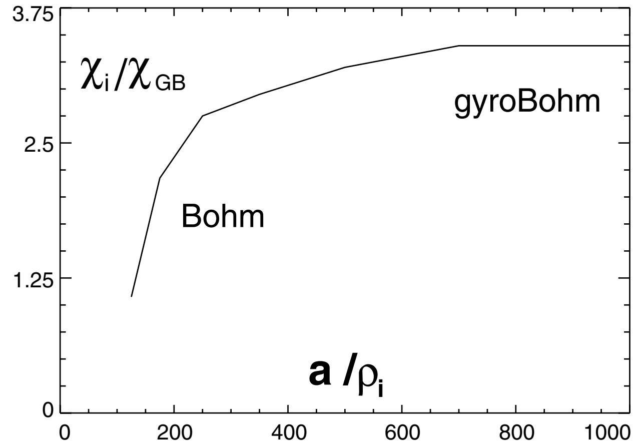


FIG. 3. Ion heat conductivity vs tokamak minor radius.

Possible mechanisms for the transition from Bohm scaling to gyro-Bohm scaling need to be identified. The device size where the transition occurs in the present studies is much larger than that expected from the linear ITG theory of pressure gradient profile variations. It is well known that strong profile variations in a small device can reduce the linear growth rate of ITG mode [3]. However, the results shown on Fig. 4 indicate that this effect is important only for $a < 100\rho_i$. The linear growth rates are found to become independent of device size when $a > 100\rho_i$ for both the most unstable linear mode ($k_\theta\rho_i \sim 0.45$) and the

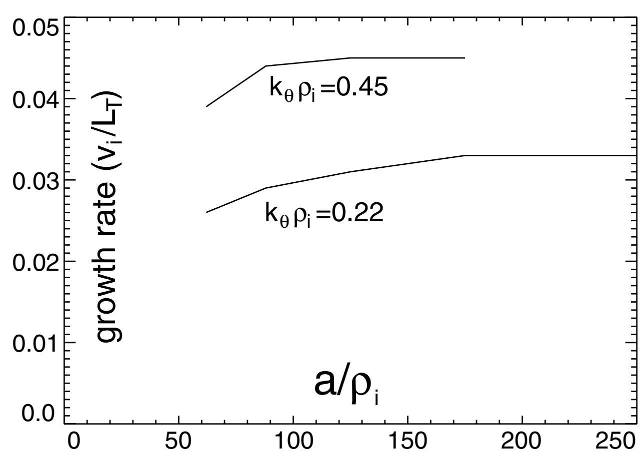


FIG. 4. Linear growth rate vs device size.

dominant nonlinear mode ($k_\theta \rho_i \sim 0.22$). Therefore, the transition from Bohm to gyro-Bohm should be governed by nonlinear processes. Note that the gyro-Bohm growth rate and transport level are higher than those of Ref. [13] because of finite aspect ratio effects.

Two-dimensional fluid simulations of toroidal ITG modes have found that Bohm-like transport can be driven when the diamagnetic flow shear is a significant fraction of the linear growth rate near the ITG threshold [21], and that the scaling is always gyro-Bohm far away from marginality. However, zonal flows are not properly treated in that study. In the present simulations with zonal flows included, we have scanned the pressure gradient down toward the linear threshold, and found that turbulence is completely suppressed by zonal flows [13] before the shear of diamagnetic rotation becomes a significant fraction of the linear growth rate. In the simulation results shown in Fig. 3, the plasma is far away from linear marginality. In fact, the linear growth rates $\gamma$ are comparable to mode real frequencies $\omega_r$ ($\gamma \sim \omega_r/2$ for $k_\theta \rho_i = 0.45$, and $\gamma \sim \omega_r/3$ for $k_\theta \rho_i = 0.22$). Profile relaxation has been observed in full torus simulations [22], which can drive the system toward marginality. To prevent this unrealistic relaxation, we use an effective collision operator for energy diffusion to model a heat bath: $\delta f_c = -f_0[(v/v_{\mathrm{T_i}})^2 - 3/2]\delta T/T_i$, where $\delta T$ is ion temperature perturbation averaged on flux surface and over minor radius range of a few eddy sizes. The effective collision time of this operator is on the order of ion energy confinement time, which is much longer than the turbulence decorrelation time. Ion temperatures are restored to their initial value using this heat source/sink. Thus pressure profiles are kept fixed throughout the simulations. Therefore, the Bohm-like scaling for small device size produced in our simulations is not due to marginality or profile relaxation.

It is found that the fluctuation amplitude (excluding zonal flows) scales as $\delta v_{E\times B} \propto v_{\mathrm{dia}}/\sqrt{\rho^*}$ in the Bohm regime for small devices, and $\delta v_{\mathbf{E}\times\mathbf{B}} \propto v_{\mathrm{dia}}$ in the gyro-Bohm regime for larger devices, where $v_{\mathrm{dia}} = v_i \rho_i / R_0$. This $\delta v_{\mathbf{E}\times\mathbf{B}}$ scaling, together with the fact that test particle transport is diffusive, indicates that the effect of sharp profile variations of the pressure gradient in a relatively small size plasma reduces the fluctuation amplitude through nonlinear processes and leads to Bohm-like transport. A plausible mechanism for this effect is the radial penetration of fluctuations from the unstable region to the linearly stable region [21,22]. Indeed, it is observed that in the nonlinearly saturated phase, fluctuations spread radially toward each boundary (edge and axis) from the unstable region (with an extent of $\sim a/2$) for a distance on the order of $25\rho_i$, independent of the device size. If we assume that total fluctuation energy content is not affected by this radial expansion, then the fluctuation intensity scales as $(\delta\phi)^2 \simeq \delta\phi_{\mathrm{GB}}^2/(1 + 50\rho_*)^2$, where $\delta\phi_{\mathrm{GB}} \simeq \rho^* T_e/e$ is the gyro-Bohm scaling for $\rho^* \to 0$. Since $\chi_i \propto |\delta\phi|^2$ has previously been observed [14], the heat conductivity should then scale as $\chi_i \simeq \chi_{\mathrm{GB}}/(1 + 50\rho_*)^2$. Interestingly, this heuristic scaling formula fits well the simulation results presented in Fig. 3.

In future studies, effects of kinetic electrons [23,24] and collisions [14] will be included in the global simulations. Externally driven plasma flows, which have been found experimentally to be a key factor in determining transport scaling in $H$ mode [6] will also be investigated.

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