Probing the High Density Behavior of the Nuclear Symmetry Energy with High Energy Heavy-Ion Collisions

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(Received 14 December 2001; published 23 April 2002)

High energy heavy-ion collisions are proposed as a novel means to constrain stringently the high density (HD) behavior of nuclear symmetry energy. Within an isospin-dependent hadronic transport model, it is shown for the first time that the isospin asymmetry of the HD nuclear matter formed in high energy heavy-ion collisions is uniquely determined by the HD behavior of the nuclear symmetry energy. Experimental signatures in two sensitive probes, i.e., π^- to π^+ ratio and neutron-proton differential collective flow, are also investigated.

The high density (HD) behavior of the nuclear symmetry energy $E_{\text{sym}}(\rho)$ is very important for understanding many interesting astrophysical phenomena, but it is also subject to the largest uncertainty among all properties of dense nuclear matter [1]. To study the HD behavior of $E_{\text{sym}}(\rho)$ has been a longstanding goal of extensive research with various microscopic and/or phenomenological models over the last few decades, e.g., [2]; for a recent review, see, e.g., [3]. The model predictions are extremely diverse and often contradictory. The results can be roughly classified into two groups, i.e., a group where the $E_{\text{sym}}(\rho)$ rises and one in which it falls with the increasing density. For instance, within the Hartree-Fock approach using about 25 Skyrme and Gony effective interactions that have been widely used successfully in studying saturation properties of symmetric nuclear matter and nuclear structures near the β stability valley, the calculated symmetry energies were found to fall approximately equally into the two groups [4,5]. The density dependence of nuclear symmetry energy, especially at high densities, has many profound consequences for various studies in astrophysics [6,7]. In particular, an increasing $E_{\text{sym}}(\rho)$ leads to a relatively more proton-rich neutron star, whereas a decreasing one would make the neutron star a pure neutron matter at high densities. Consequently, the chemical composition and cooling mechanisms of protoneutron stars [8,9], critical densities for Kaon condensations in dense stellar matter [10,11], mass-radius correlations [12,13], as well as the possibility of a mixed quark-hadron phase [14] in neutron stars will all be rather different. The fundamental cause of the extremely uncertain HD behavior of $E_{\text{sym}}(\rho)$ is the complete lack of terrestrial laboratory data to constrain directly the model predictions. In this Letter, high energy heavy-ion collisions are proposed as a novel means to constrain stringently the HD behavior of $E_{\text{sym}}(\rho)$. The upper bound of HD symmetry energy effects on high energy heavy-ion collisions is explored. It is shown for the first time that the neutron/proton ratio of HD nuclear matter formed in high energy heavy-ion collisions is uniquely

determined by the HD behavior of $E_{\text{sym}}(\rho)$. Experimental signatures of the HD behavior of $E_{\text{sym}}(\rho)$ in two sensitive observables, i.e., the π^-/π^+ ratio and the neutron-proton differential collective flow, are also investigated.

We use here the parabolic approximation of the equation of state (EOS) for isospin asymmetric nuclear matter [3]

$$e(\rho, \delta) = e(\rho, 0) + E_{\text{sym}}(\rho)\delta^2, \qquad (1)$$

where $e(\rho, 0)$ is the energy per nucleon in symmetric nuclear matter, and $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry in terms of the neutron (ρ_n) and proton (ρ_p) density, respectively. For the isoscalar part we use the simplest, momentum-independent parametrization

$$e(\rho, 0) = \frac{a}{2}u + \frac{b}{1+\sigma}u^\sigma + \frac{3}{5}e_F^0 u^{2/3}, \qquad (2)$$

where $u \equiv \rho/\rho_0$ is the reduced density and $e_F^0 = 36$ MeV is the Fermi energy. The parameters $a = -358.1$ MeV, $b = 304.8$ MeV, and $\sigma = 7/6$ are determined by saturation properties and a compressibility $K_\infty = 201$ MeV of isospin symmetric nuclear matter. Our conclusions in this work are independent of the particular form of $e(\rho, 0)$. The symmetry energy is [1,3]

$$E_{\text{sym}}(\rho) \equiv e(\rho, 1) - e(\rho, 0) = \frac{5}{9}E_{\text{kin}}(\rho, 0) + V_2(\rho),$$
$$(3)$$

where $E_{\text{kin}}(\rho, 0)$ is the kinetic energy per nucleon in symmetric nuclear matter and $V_2(\rho)$ is the deviation of the interaction energy of pure neutron matter from that of symmetric nuclear matter. The $E_{\text{sym}}(\rho)$ becomes negative if the condition $V_2(\rho) \leq -\frac{5}{9}E_{\text{kin}}(\rho, 0)$ is reached at high densities. Thus a pure neutron matter could become most stable, leading to the isospin separation instability in HD neutron-rich matter. Consequently, pure neutron domains or neutron bubbles surrounding isolated protons can be formed in neutron stars [1]. To represent the two groups of model predictions, we use for the $E_{\text{sym}}(\rho)$

$$E_{sym}^a(\rho) \equiv E_{sym}(\rho_0)u \text{ and}$$

$$E_{sym}^b(\rho) \equiv E_{sym}(\rho_0)u \frac{u_c - u}{u_c - 1}, \quad (4)$$

where $E_{sym}(\rho_0) = 30$ MeV is the symmetry energy at normal nuclear matter density, and $u_c = \rho_c/\rho_0$ is the reduced critical density at which the $E_{sym}^b(\rho)$ crosses zero and becomes negative at higher densities. The predicted value of u_c ranges from about 2.7 (Hartree-Fock with the Skyrme interaction Sp [5]) to 9 (variational many-body approach with the UV14 + UVII interaction [2]). These two forms of the $E_{sym}(\rho)$ (inset in the lower window) together with the corresponding EOS for isospin asymmetric nuclear matter are shown in Fig. 1. With the linearly increasing $E_{sym}^a(\rho)$, the EOS (upper window) becomes stiffer with the increasing δ. The isospin symmetric nuclear matter remains to be the ground state at all densities. This is in stark contrast to the situation using the $E_{sym}^b(\rho)$. The EOS obtained with the $E_{sym}^b(\rho)$ and $u_c = 3$ (lower window), is softened instead of being stiffened by the increasing isospin asymmetry δ at densities higher than $3\rho_0$. At these high densities, the pure neutron matter and the isospin

symmetric nuclear matter thus becomes the most stable and unstable form of HD nuclear matter, respectively.

High energy heavy-ion collisions provide the only terrestrial situation where the HD neutron-rich matter can be formed. Moreover, fast radioactive heavy-ion beams to be available at the planned rare isotope accelerator (RIA) in the United States can make the HD nuclear matter even more neutron rich. The HD behavior of $E_{sym}(\rho)$ affects properties of the HD nuclear matter formed in high energy heavy-ion collisions. Moreover, it leads to interesting precursory phenomena in the experimental observables of high energy heavy-ion collisions. We investigate these effects and phenomena within an isospin-dependent hadronic transport model [15,16]. Evolutions of the phase-space distribution functions of nucleons, Delta resonances, and pions with their explicit isospin degrees of freedom are solved numerically by using the test-particle approach [17,18]. Both the isoscalar and isovector mean-field potentials are derived consistently from the nuclear EOS given above. Isospin-dependent total and differential cross sections among all particles are taken either from the elementary particle scattering data or obtained by using the detailed balance. Explicitly isospin-dependent Pauli blockings for fermions are also employed. For a review of the model, we refer the reader to Ref. [19].

Shown in Fig. 2 is the correlation between the isospin asymmetry δ and the baryon density ρ/ρ_0 at the instant of 20 fm/c in the reaction of ^{132}Sn + ^{124}Sn at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm. This very reaction will be available at the RIA facility. The
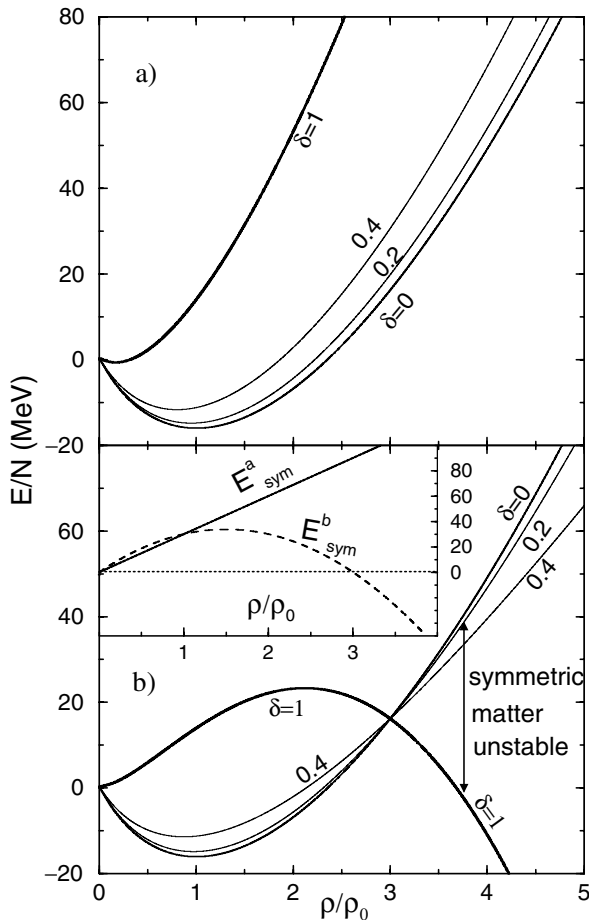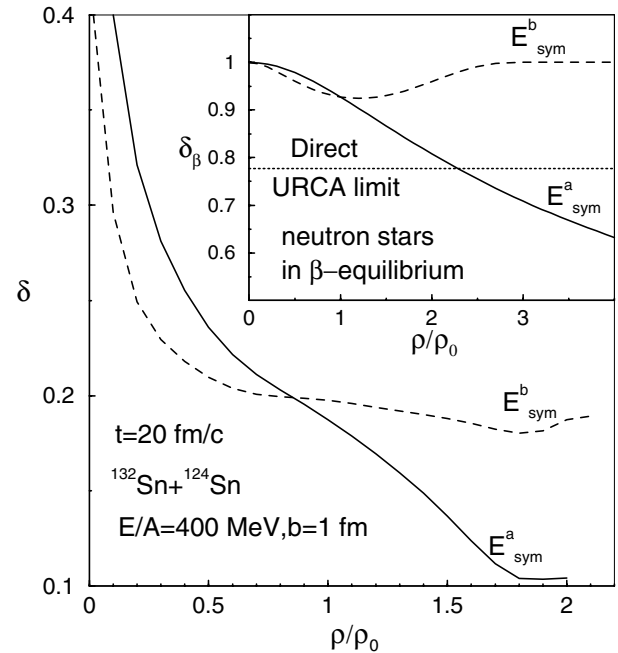

FIG. 1. The upper (lower) window is the equation of state of isospin-asymmetric nuclear matter corresponding to the nuclear symmetry energy parametrization E_{sym}^a (E_{sym}^b) shown in the inset of the lower window.

FIG. 2. The isospin asymmetry-density correlation at $t = 20$ fm/c over the whole space in the ^{132}Sn + ^{124}Sn reaction with the nuclear symmetry energy E_{sym}^a and E_{sym}^b, respectively. The corresponding correlation in neutron stars is shown in the inset.
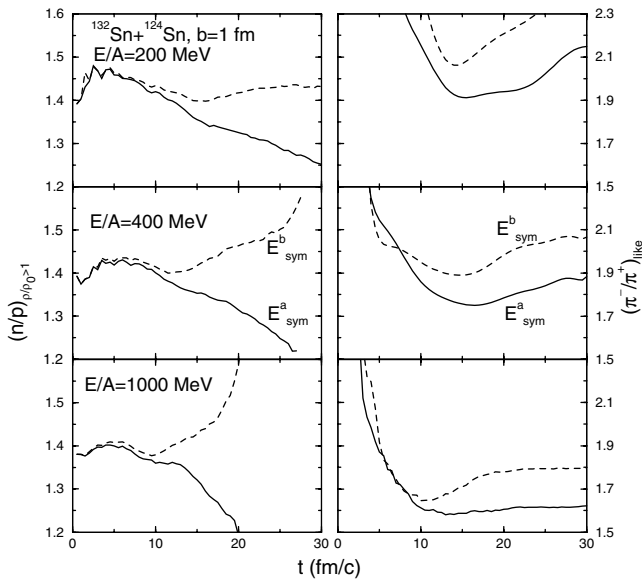
FIG. 3.   Left panels: the neutron/proton ratio of nuclear matter with density higher than normal nuclear matter density as a function of time with the nuclear symmetry energy $E_{\rm sym}^a$ and $E_{\rm sym}^b$, respectively. Right panels: $(\pi^-/\pi^+)_{\rm like}$ ratios in the same reactions.

overall rise of δ at very low densities is due to the neutron skins of the colliding nuclei. One also notices that the $E_{\rm sym}^b(\rho)$ leads to a slightly higher compression due to its relatively softening effect on the nuclear EOS. Dramatic effects due to the different symmetry energies are clearly revealed especially at high densities. For a comparison, the corresponding correlation in neutron stars at β equilibrium is shown in the inset. As a reference, an estimate of the direct URCA limit of δ_β (below which the fast cooling process can happen in protoneutron stars and thus results in also higher neutrino flux) is indicated with the dotted line [8]. To a good approximation perfectly suitable for this study, a neutron star can be considered as consisting of neutrons, protons, and electrons. At β equilibrium the proton fraction $x_\beta \equiv \rho_p/(\rho_n + \rho_p)$ in neutron stars is then determined by [8]

$$\hbar c(3\pi^2 \rho x_\beta)^{1/3} = 4E_{\rm sym}(\rho)(1 - 2x_\beta). \qquad (5)$$

The equilibrium isospin asymmetry $\delta_\beta = 1 - 2x_\beta$ is therefore entirely determined by the $E_{\rm sym}(\rho)$. With the $E_{\rm sym}^b(\rho)$, the δ_β is 1 for $\rho/\rho_0 \geq 3$, indicating that the neutron star has become a pure neutron matter at these high densities. On the contrary, with the $E_{\rm sym}^a(\rho)$, the neutron star becomes so proton rich that the direct URCA process can happen at densities higher than about $2.3\rho_0$. An astonishing similarity is seen in the resultant $\delta - \rho$ correlations for the neutron star and the heavy-ion collision. In both cases, the symmetry energy $E_{\rm sym}^b(\rho)$ makes the HD nuclear matter more neutron rich than the $E_{\rm sym}^a(\rho)$ and the effect grows with the increasing density. Of course, this is no surprise since the same nuclear EOS is at work in both cases. As shown in Fig. 1, in stark contrast to the situation with the $E_{\rm sym}^a(\rho)$,
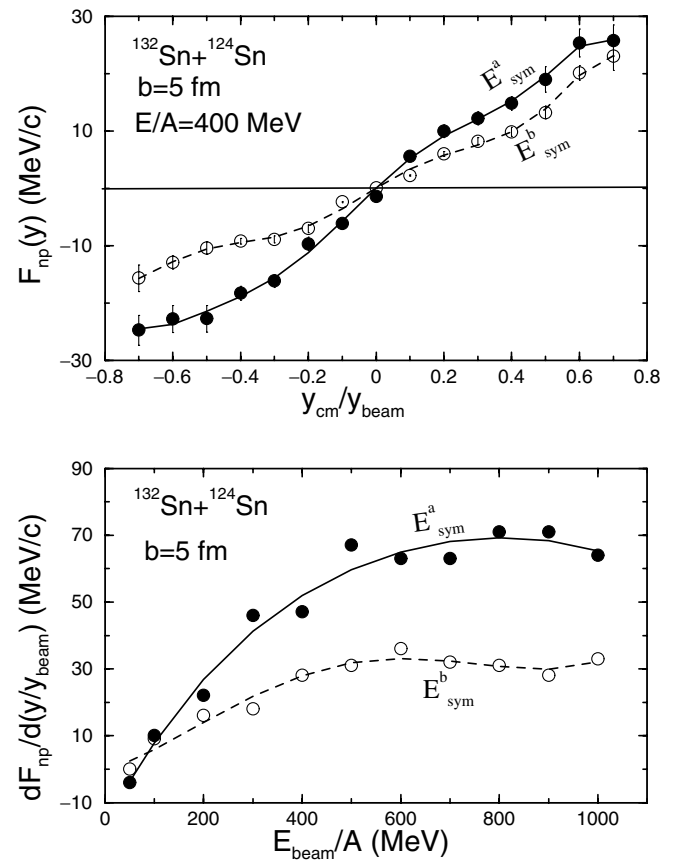

FIG. 4.   Upper window: the neutron-proton differential collective flow in the ^{132}Sn + ^{124}Sn reaction with the nuclear symmetry energy $E_{\rm sym}^a$ and $E_{\rm sym}^b$, respectively.   Lower window: excitation function of the slope parameter of the differential flow for the ^{132}Sn + ^{124}Sn reaction. The solid (with the $E_{\rm sym}^a$) and dashed (with the $E_{\rm sym}^b$) lines are drawn to guide the eye.

the decreasing $E_{\rm sym}^b(\rho)$ above $1.5\rho_0$ makes it energetically more favorable to have the denser region more neutron rich.

   To further investigate the HD behavior of $E_{\rm sym}(\rho)$, the average n/p ratio of the HD region with $\rho/\rho_0 \geq 1$ is examined as a function of time and beam energy in the left panels of Fig. 3. The effect on $(n/p)_{\rho/\rho_0 \geq 1}$ due to the different $E_{\rm sym}(\rho)$ is seen to grow with both the reaction time and the beam energy. This is because of the higher densities reached with the more energetic beams and after longer times of compression.

   How to probe experimentally the HD behavior of $E_{\rm sym}(\rho)$ in high energy heavy-ion collisions? Among the observables we have explored, the π^-/π^+ ratio and the neutron-proton differential collective flow are found to be most promising. The former measures sensitively the neutron to proton ratio n/p of the HD nuclear matter and thus indirectly the HD behavior of $E_{\rm sym}(\rho)$. At beam energies below about 2 GeV/nucleon, pions are mostly produced through the decay of $\Delta(1232)$ resonances. The primordial π^-/π^+ ratio is approximately quadratic in n/p according to the branching ratios of single pion production via Δ resonances in nucleon-nucleon collisions

$$\pi^-/\pi^+ = \frac{5n^2 + np}{5p^2 + np} \approx (n/p)^2. \qquad (6)$$

Pion reabsorptions and rescatterings are expected to complicate the above relationship. Nevertheless, very high sensitivity to the n/p ratio is retained in the final π^-/π^+ ratio as indicated in the experimental data of high energy heavy-ion collisions [20]. Shown in the right panels of Fig. 3 are the $(\pi^-/\pi^+)_{\text{like}}$ ratio

$$(\pi^-/\pi^+)_{\text{like}} \equiv \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+} \qquad (7)$$

as a function of time. This ratio naturally becomes the final π^-/π^+ ratio when the reaction time t is much longer than the lifetime of the delta resonance τ_Δ. The $(\pi^-/\pi^+)_{\text{like}}$ ratio is rather high in the early stage of the reaction because of the large numbers of neutron-neutron scatterings near the surfaces of the colliding nuclei. By comparing the two sides in Fig. 3, it is seen that a variation of about 30% in the $(n/p)_{\rho/\rho_0 \geq 1}$ ratio due to the different $E_{\text{sym}}(\rho)$ results in about a 15% change in the final π^-/π^+ ratio. The later thus has an appreciable response factor of about 0.5 to the variation of HD n/p ratio, and it is approximately beam energy independent.

The neutron-proton differential collective flow is measured by [21]

$$F_{np}(y) \equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_{x_i} \tau_i, \qquad (8)$$

where $N(y)$ is the total number of free nucleons at the rapidity y, p_{x_i} is the transverse momentum of particle i in the reaction plane, and τ_i is $+1$ and -1 for neutrons and protons, respectively. The $F_{np}(y)$ combines constructively the in-plane transverse momenta generated by the isovector potentials while reducing significantly influences of the isoscalar potentials of both neutrons and protons. Thus, it can reveal more directly the HD behavior of $E_{\text{sym}}(\rho)$ in high energy heavy-ion collisions. A typical result for the ^{132}Sn + ^{124}Sn reaction is shown in the upper window of Fig. 4. A clear signature of the HD behavior of $E_{\text{sym}}(\rho)$ appears at both forward and backward rapidities. To characterize the effect, the slope $dF_{np}/d(y_{cm}/y_{\text{beam}})$ at midrapidity is shown as a function of beam energy in the lower window. A striking difference of about a factor of 2 exists in the reactions at $E_{\text{beam}} \geq 200$ MeV/nucleon. This large effect can be very easily observed by using available detectors at several heavy-ion facilities in the world [22,23]. Compared with the π^-/π^+ ratio, the neutron-proton differential collective flow is more directly affected by and is thus also a more sensitive probe of the HD behavior of $E_{\text{sym}}(\rho)$.

In conclusion, the HD behavior of $E_{\text{sym}}(\rho)$ has been puzzling physicists for decades. In this work, high energy heavy-ion collisions are proposed as a novel means to solve this longstanding problem. The upper bound of HD symmetry energy effects on high energy heavy-ion collisions is explored. For the first time, it is shown that the isospin asymmetry of HD nuclear matter formed in high

energy heavy-ion collisions is uniquely determined by the HD behavior of $E_{\text{sym}}(\rho)$. Both the π^-/π^+ ratio and the neutron-proton differential collective flow can serve as sensitive probes. Measurements of these observables will provide the first terrestrial data to constrain stringently the HD behavior of nuclear symmetry energy.

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