# Model and Observations of Schottky-Noise Suppression in a Cold Heavy-Ion Beam

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Some years ago it was found at GSI in Darmstadt that the momentum spread of electron-cooled beams of highly charged ions dropped abruptly to very low values when the particle number decreased to 10 000 or less. This has been interpreted as an ordering of the ions, such that they line up after one another in the ring. We report observations of similar transitions at CRYRING, including an accompanying drop in Schottky-noise power. We also introduce a model of the ordered beam from which the Schottky-noise power can be calculated numerically. The good agreement between the model calculation and the experimental data is seen as evidence for a spatial ordering of the ions.

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Coulomb crystals, ordered systems of charged particles confined by electromagnetic fields, were already observed in electromagnetic traps more than 40 years ago using dust particles [1] and later also with atomic ions [2]. For a long time, there has been speculation and discussions about the possibility to achieve such ordering in beams in accelerators or storage rings. Some 20 years ago, observations of ordering were reported from the NAP-M storage ring in Novosibirsk [3], although there have been discussions about the conclusions drawn from these observations. A phenomenon with a different signature was discovered more recently at the ESR ring at GSI, Darmstadt [4]. Here, the momentum spread of electron-cooled beams of highly charged ions was seen to drop abruptly when the particle number decreased to a value between 1000 and 10 000 as the beam decayed through collisions with residual-gas molecules. A reduction in Schottky-noise power immediately after the transition was also seen for particle numbers between 1000 and 10 000. The Schottky noise, also known as shot noise, is the statistical noise in the beam current. It is due to the discreteness of the beam or the fact that it is made up of individual particles, and this makes the instantaneous current fluctuate around its macroscopic average.

It has been assumed that the momentum spread drops because intrabeam scattering becomes strongly suppressed. This in turn is interpreted as the effect of beam ordering, in such a way that the ions become lined up after one another without being able to pass each other due to the strong repulsion between the highly charged ions and small energy spread due to the cooling [5]. Also in laser-cooled beams of singly charged ions effects that may be attributed to the suppression of intrabeam scattering have been observed [6,7].

We here describe a model of such an ordered beam, and we compare the model predictions with observations of an abruptly decreasing momentum spread and a simultaneous reduction of the Schottky-noise power made at CRYRING [8] using $^{129}Xe^{36+}$ ions at 7.4 MeV per nucleon. Although this ion is highly charged and not fully stripped, it has a sufficiently long lifetime in the ring in order to allow reasonable observation times. With no electron current in the cooler, the lifetime is approximately 190 s, determined by electron capture from collisions with rest-gas molecules and, to a smaller extent, electron loss. Turning on the electron cooler causes ions and electrons to recombine. This reduces the lifetime down to 29 s for the highest electron density used. The effect of a sudden reduction in momentum spread has also been seen at CRYRING with $Xe^{36+}$ ions at 3.1 MeV/u and with $Ni^{17+}$ and $Pb^{53-55+}$, but this report will concentrate on the results with 7.4-MeV/u $Xe^{36+}$ ions.

The ions, produced in an electron-beam ion source (EBIS) and preaccelerated in a radio-frequency quadrupole (RFQ), are injected into the synchrotron/storage ring and accelerated to the desired energy in about 1 s. The ions are then stored and continuously electron cooled for up to 25 min while the Schottky signal from the beam is recorded by a spectrum analyzer. Figure 1 shows an example of how the Schottky signal evolves as a function of time, and it is seen how the momentum spread suddenly

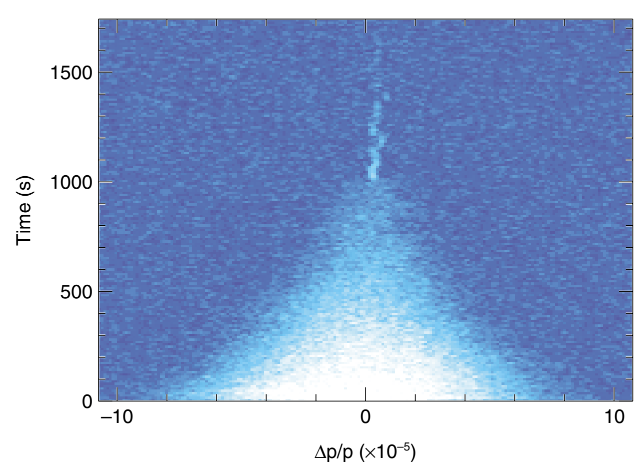


FIG. 1 (color). Schottky signal from a beam of $Xe^{36+}$ ions. At $t = 0$, about 30 000 particles are injected, and it is seen how the relative momentum spread $\Delta p/p$ suddenly shrinks approximately 1000 s after the injection.

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becomes very small after about 1000 s. Guided by earlier interpretations and the model presented in this Letter, we refer to the drop in momentum spread as beam ordering, although no direct experiment has been able to detect the positions of individual ions.

The signal was recorded at the 10th harmonic of the revolution frequency, about 7.27 MHz, and the relative momentum spread $\Delta p/p$ is obtained from the measured relative frequency spread through

$$\frac{\Delta p}{p} = \frac{1}{-\eta}\frac{\Delta f}{f}, \tag{1}$$

where $\eta$ is $-0.8$ for CRYRING.

The wiggles at the top of Fig 1 are caused by a small instability in one or several of the power supplies that determine the beam momentum or the length of the orbit.

Sequences such as the one in Fig. 1 were recorded for a number of different electron densities in the electron cooler, i.e., for different cooling rates. Figure 2 shows the particle number at the transition to the small momentum spread as a function of the electron density. This particle number is too small to measure directly, but is calculated from the injected current, the beam lifetime, and the time from injection to transition. At each injection, the beam current just after the acceleration, while the beam was still bunched, was determined from the intensity of a signal from an electrostatic pickup. This signal was calibrated to a dc current transformer in a separate measurement. The lifetime was determined from the count rate in a beam-profile monitor that counts ionized rest-gas molecules created by the ion beam. The uncertainty in the calibration of the absolute current of $\pm 10\%$ dominates the uncertainty in the absolute particle number. Relative particle numbers are more accurate since the pickup signal, the profile-monitor count rate, and the time to the transition could be measured with higher precision.

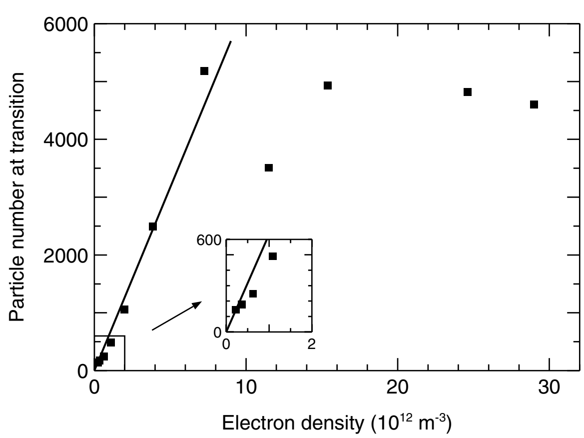


FIG. 2. The number of particles in the beam when the transition to the ordered state takes place as a function of the electron density in the cooler. The inset shows an enlargement of the lower left portion of the figure.

The lowest electron density where a transition to a small momentum spread was observed was $2.2 \times 10^{11}$ m$^{-3}$, corresponding to an electron current of 1.7 mA. Then there were only about 145 particles left in the ring, as seen from Fig. 2. This means that intrabeam scattering is effective even when the distance between the particles in the 52-m-circumference ring is as large as 35 cm. This distance can be compared with the diameter of the beam pipe, which is 10 cm.

As the electron density is increased, the particle number at the transition increases up to approximately 5000. Ordering is not seen to appear above that number even if the electron density is increased further. Although one could argue that the cooling conditions are less favorable at higher densities, the electron-beam quality and alignment can to some extent be checked by monitoring the ion-electron recombination rate through the beam lifetime. Except for the fact that the three highest points were taken with an electron-beam expansion [9] of a factor 25 instead of 100 as used otherwise, reducing the recombination rate to approximately one-half, the recombination rate was essentially proportional to the electron density. Nevertheless, more studies are needed before a definitive conclusion can be drawn about this apparent limit.

The relative momentum spread of the ordered beam, as obtained from the frequency spread in Schottky spectra such as those in Fig. 1, is found to be about $1 \times 10^{-6}$. This is only an upper limit of the true momentum spread, however, since ripple on the power supplies to the dipole magnets in the ring and to the cathode of the electron cooler (which defines the electron and thus the ion energy) can easily produce an apparent momentum spread of that size.

The power of the Schottky signal as a function of particle number is plotted in Fig. 3 for three different electron densities in the cooler. Normally, the Schottky power is proportional to the number of particles in the beam, and this proportionality is seen to hold to a very high accuracy in all cases, except for the ordered beam above some 1000 particles. Here, a suppression of the Schottky power by a factor of up to roughly 3 is seen. Looking carefully, it can also be seen that the absolute power, independently of particle number, is somewhat lower for higher electron densities (the vertical scale, although uncalibrated due to unknown gains, etc., in the Schottky amplifiers, is the same in the three plots). Above the transition, this effect is known to be caused by the appearance of density waves in the beam [10]. These give rise to Schottky spectra with a double-peak character which are routinely observed with cold beams, including ours at high electron densities. It looks as if this effect also exists for the ordered beam, although the existence of waves in the ordered beam is unproven.

The Schottky noise is detected by a capacitive pickup consisting of four 1.4-m-long plates surrounding the ion beam. The signal from this detector can be approximated by a delta pulse each time a particle passes through it. The total signal from $N$ particles can thus be written as [11]

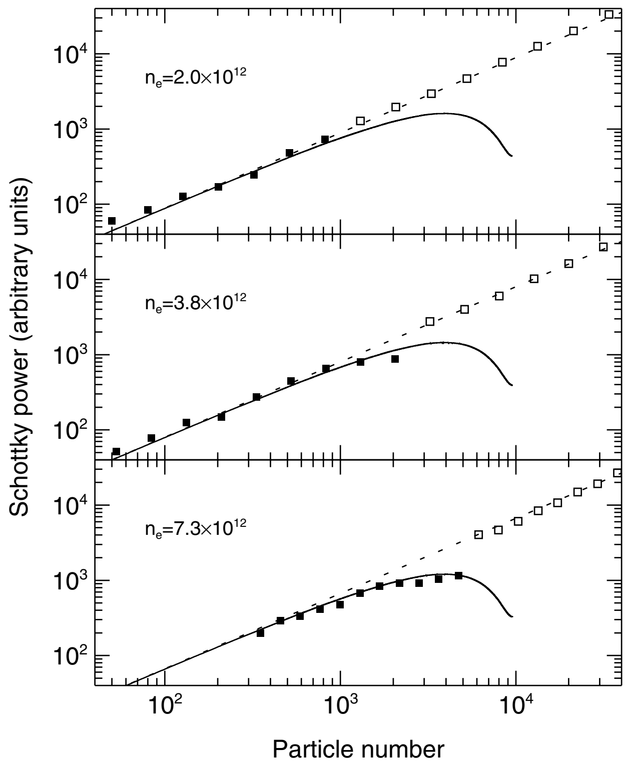


FIG. 3. Schottky power as a function of the particle number when the electron density in the cooler is, from above, $2.0 \times 10^{12}$, $3.8 \times 10^{12}$, and $7.3 \times 10^{12}$ m$^{-3}$. The open squares show the power before the transition and the filled squares represent the ordered beam. The curves are the results of the model calculations discussed in the text. Uncorrelated particles should have a Schottky power proportional to the particle number, as indicated by the dashed lines which have unit slope.

$$I(t) = Zq \sum_{a=1}^{N} \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{2\pi n + \theta_a}{\omega_a}\right), \qquad (2)$$

where $Zq$ is the charge of the ions, $\theta_a$ is the initial phase of ion $a$ in its motion around the ring, and $\omega_a$ is its revolution frequency. For uncorrelated particles, the spectral density of this signal is

$$S(\omega) = \frac{(Zq)^2}{2\pi} \sum_{a=1}^{N} \sum_{n=-\infty}^{\infty} \omega_a^2 \delta(\omega - n\omega_a) \qquad (3)$$

and is concentrated in bands around the revolution frequency and its harmonics. With $\omega_0$ being the nominal revolution frequency, the integrated noise power in each (nonoverlapping) band is then

$$P_n = \frac{(Zq)^2}{2\pi} \sum_{a=1}^{N} \omega_a^2 \approx \frac{N(Zq)^2\omega_0^2}{2\pi}, \qquad (4)$$

and it is thus proportional to the particle number, as mentioned above.

If there are correlations between the particles, however, the expressions (3) and (4) are no longer valid. In an attempt to describe the correlations present in the type of beam discussed here, and to find a modified expression for the integrated noise power, we introduce a phenomenological model of the ordered beam. The model is based on an algorithm for assigning positions of the beam particles around the ring, producing a "snapshot" of the particle distribution in the beam. We then let this static particle configuration "rotate" and calculate the noise power it gives rise to. Averaging over many snapshots, we obtain a noise power that can be compared with the experimental data to see whether the assumed particle configuration is consistent with that in the real beam.

Setting $\omega_a = \omega_0$ for all $a$ in expression (3) (which is what we mean by a static particle configuration), the correlations are represented by the values of $\theta_a$. It is then readily found that the integrated noise power becomes

$$P_n = \frac{(Zq)^2\omega_0^2}{2\pi}\left[\left(\sum_{a=1}^{N} \cos n\theta_a\right)^2 + \left(\sum_{a=1}^{N} \sin n\theta_a\right)^2\right]. \qquad (5)$$

The main feature of the model is the existence of a certain minimum distance between the particles. This distance is defined classically by setting kinetic and potential energies equal, giving

$$d_{\min} = \frac{(Zq)^2}{4\pi\epsilon_0 kT} \qquad (6)$$

if $kT$ is the ion temperature. Apart from the minimum distance, the particles have random positions along the ring circumference $C$.

The algorithm for choosing $\theta_a$ can thus be described by (i) choosing $\theta_1$ randomly between 0 and $2\pi$, (ii) choosing $\theta_2$ randomly between 0 and $2\pi$ also, but excluding the region $\theta_1 \pm 2\pi d_{\min}/C$, and (iii) repeating step (ii) the desired number of times, excluding the regions around all previously inserted particles. Numerically it is found that the maximum number of particles $N_{\max}$ that can be inserted into the ring in this way is approximately $0.747C/d_{\min}$ when $N$ is large, with a standard deviation of the order of $N^{1/2}$. With $N_{\max}$ particles in the ring all interparticle distances are between $d_{\min}$ and $2d_{\min}$. Note that the only free parameter in the model is the ion temperature $kT$ which determines both $d_{\min}$ and $N_{\max}$.

From direct measurements on the beam, only upper limits to the temperature of the ordered beam can be obtained. The momentum spread of $\leq 1 \times 10^{-6}$ gives 2 meV or less longitudinally. The transverse beam dimensions are below the resolution of our beam-profile monitors; a resolution of 0.5 mm FWHM gives 3 eV as an upper limit transversally. Since the ions are continuously electron cooled, however, we can assume that they are in temperature equilibrium with the cooler electrons in the absence of intrabeam scattering. The electron temperature was calculated from peak shapes in dielectronic-recombination spectra on many occasions, and typical values are 1–3 meV transversally and 0.05–0.15 meV longitudinally.

If we choose the temperature of the $Xe^{36+}$ ions equal to 0.5 meV, we get $d_{\min} = 4$ mm and an $N_{\max}$ of 9500 with the 52 m circumference of CRYRING. Using these

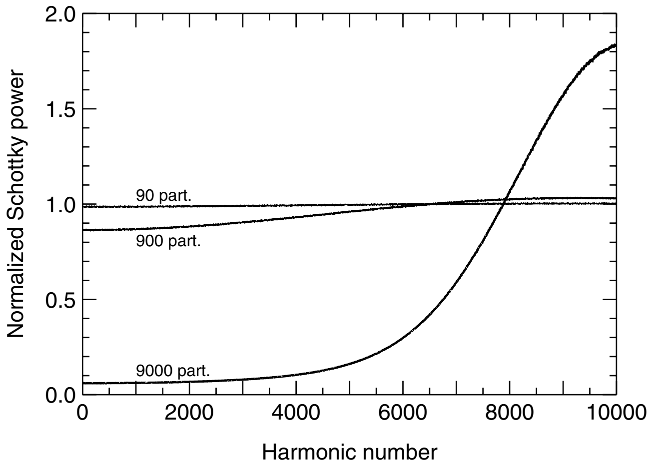


FIG. 4. Schottky power, calculated according to Eq. (5), as a function of harmonic number $n$ for 90, 900, and 9000 particles in the beam. The power is normalized such that it is equal to 1 for uncorrelated particles.

parameters, the Schottky power was calculated from expression (5) for particle numbers up to $N_{\max}$. The result is shown as the curves in Fig. 3. The power was calculated at the first harmonic of the revolution frequency and was averaged over 100 000 simulated particle distributions. The curves have been shifted to the position of the dashed line, allowing for the observed reduction in noise power due to density waves in the beam (although, again, the existence of such waves in the ordered beam is unproven). Choosing a different $kT$ essentially has the effect of sliding the curves along the dashed lines.

While the frequency spread of the Schottky signal gives information about the momentum spread of the beam, the Schottky power, according to our model, gives information about the *spatial* correlation between the ions. The good agreement between the model and the experimental data gives evidence for the existence of such a correlation or order, which becomes noticeable at particle numbers above 1000 or so.

One can also compare the predicted particle number $N_{\max} = 9500$, above which ordering should not appear, with the observed limit of 5000 particles. However, it must be understood that the model tries to describe the beam after the transition to the ordered state, but gives no information about how the actual transition occurs. It therefore does not predict at which particle number the transition really takes place, nor how this number depends on, e.g., the cooling rate, as depicted in Fig. 2. The discrepancy between the two numbers could thus be due to this lack of a description of the transition process. It could also be due to some deficiency of the model of the ordered beam, such as its static character, or of the experiment. This could be investigated further since it should be straightforward to improve the model by introducing some dynamics into it. Also, it would be interesting to extend calculations already performed on beam ordering at CRYRING [12] to the parameter range relevant for the experiment presented here.

A beam with long-range order should give rise to strong Schottky signals at harmonics equal to the particle number. Since our model only has short-range order (except when the particle number approaches the maximum value $N_{\max}$), such an effect should be absent. Figure 4 shows that this indeed is the case. Only at harmonics approaching the maximum particle number is there an enhancement of the Schottky signal, and then not to a single harmonic but to a broad range.

In conclusion, we have observed a reduction in both momentum spread and Schottky-noise power in beams of highly charged, heavy ions. These observations agree well with those made at the ESR storage ring. We have also presented a model of the particle distribution in an ordered beam and used it to calculate the Schottky power generated by such a beam. Comparing the calculations with the experimental results, we find evidence for the existence of spatial ordering in a high-energy ion beam.

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