Synchronization of Chaotic Semiconductor Laser Systems: A Vectorial Coupling-Dependent Scenario

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We demonstrate the influence of vectorial coupling on the synchronization behavior of complex systems. We study two semiconductor lasers subject to delayed optical feedback which are unidirectionally coherently coupled via their optical fields. Our experimental and numerical results demonstrate a characteristic synchronization scenario in dependence on the relative feedback phase leading cyclically from chaos synchronization to almost uncorrelated states, and back to chaos synchronization. Finally, we reveal the influence of the feedback phase on the dynamics of the solitary delay system.

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Synchronization phenomena of coupled nonlinear oscillators are encountered in physical, chemical, and biological systems, e.g., in mechanical oscillators, neural networks, in physiological interactions, and in laser systems [1]. Currently, the investigations of these phenomena focus on the understanding of fundamental scenarios related to the onset and loss of synchronization under variation of system parameters [2]. In this paper, we show that the nature of the coupling is important in this context by studying synchronization phenomena in optically coupled laser systems. A particularity of optical systems is the possibility of vectorial coherent coupling via amplitude and phase of the optical fields. We present experimental and numerical investigations on the influence of such coherent coupling on the dynamics and synchronization behavior of two unidirectionally coupled semiconductor lasers (SL) subject to delayed optical feedback, thus emitting chaotically; a system offering great advantages for our experimental studies, because of well-controllable parameters, and well-studied nonlinear behavior.

This system has been addressed by modeling [3,4], and also experimentally first successful chaos synchronization has been achieved [5], but detailed studies of the synchronization scenario are lacking. We demonstrate that a striking dynamical scenario mediating between chaos synchronization and weakly correlated states evolves under well-controlled variation of the optical feedback phase. For adjusted phase, we achieve excellent synchronization of the intensity dynamics in combination with coherence of the optical fields, despite the fast chaotic wavelength-fluctuations present in the dynamics of each subsystem. Variation of the phase leads to conspicuous changes in the intensity dynamics associated with drastically reduced correlation between the subsystems, until finally, the synchronized state is reached again for a phase shift of 2π. Our experiments substantiated by numerical simulations demonstrate that the synchronization scenario is closely linked to the vectorial nature of the coupling.

A scheme of our experimental setup is depicted in Fig. 1. Since well-matched parameters are essential for these synchronization experiments, we have selected two device-identical SLs (uncoated Hitachi HLP1400 Fabry-Perot SLs). Their optical spectra agree within 0.1 nm, slope efficiency within 3%, and threshold current within 7%. Each laser is pumped by a low-noise dc current source, and temperature stabilized to better than 0.01 K. We expose both lasers to delayed optical feedback from high reflecting mirrors with equal delays of $\tau_{\mathrm{delay},1,2} =$ 2.9 ns. Independently, we control the optical feedback phases $\Phi_{1,2}$ of the reflected light by changing the length of the cavities on subwavelength scale via piezo actuators (PZA). These two systems are coupled via the injection of a well-defined fraction of the optical field of laser 1 into the external cavity of laser 2. The combination of optical isolator (ISO), $\frac{\lambda}{2}$ plate, and polarizer (POL) guarantees a coherent coupling of the lasers via the dominant TE component of the optical fields. The coupling time τ_c is determined to 4.6 ns, though we note that τ_c is not of relevance in our experiment, since the coupling is unidirectional. We detect the intensity dynamics of both lasers simultaneously with 6 GHz photodetectors ($PD_{1,2}$) and analyze the signals using an oscilloscope of 4 GHz analog bandwidth, and an electrical spectrum analyzer (ESA). Hence our detection device resolves the intensity dynamics of both lasers simultaneously on the relevant sub-ns time scales. In addition, we monitor the optical spectra of both lasers with an optical spectrum analyzer (OSA) with 0.1 nm resolution, and detect the average output power ($PD_{1,2,3}$).

In our experiments both lasers are driven 1% above their solitary threshold current $I_{\mathrm{th}}^{\mathrm{sol}}$. We have minimized the

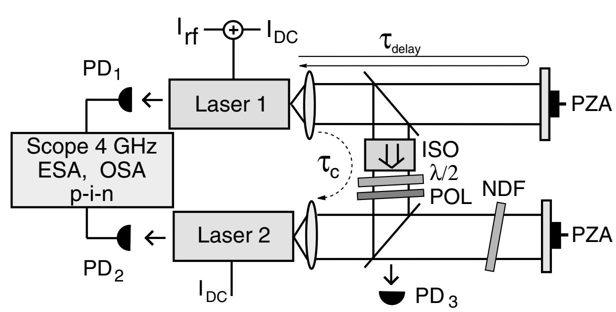

FIG. 1. Scheme of the experimental setup.

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detuning between the optical frequencies of the two lasers to less than 1 GHz by controlling the temperature of each laser appropriately. Furthermore, we have adjusted the amount of delayed optical feedback using a neutral density filter (NDF), such that the threshold reduction is 7% in laser 1 and 4% in laser 2. Accordingly, both lasers operate in the well-studied low frequency fluctuation (LFF) regime (see, e.g., [6] and references therein). In agreement with [5], we find that best synchronization results are achieved when the sum of the coupling intensity and the feedback intensity of laser 2 is larger than the feedback intensity of laser 1. For optimized conditions, we couple an amount equal to 40% of the feedback intensity laser 1 is subjected to into laser 2. The feedback of laser 2 is 70% compared to the feedback of laser 1 by the NDF placed in its cavity. We note, that under these conditions perfect synchronization cannot be achieved, due to the different injected intensities, nevertheless, almost identical emission dynamics is possible.

We find similar changes in the intensity dynamics of the coupled system, when changing the optical feedback phase either for laser 1 or for laser 2. Here we restrict ourselves to changing the feedback phase of laser 2, as the significant parameter is the relative optical feedback phase $\Phi_{\rm rel}$ between laser 1 and laser 2. We emphasize that changing the feedback phase in the solitary feedback system has no visible influence, neither on the intensity dynamics, nor on the rf and the optical spectra. Figure 2 depicts snapshots of the intensity time series of the coupled lasers for three different values of $\Phi_{\rm rel}$ under otherwise identical conditions. The phases are 0π (Fig. 2a), 0.7π (Fig. 2b), and 1.4π (Fig. 2c), respectively. The intensity time series of laser 1 are always represented by gray solid lines and that of laser 2 by black solid lines. Figure 2a depicts the time series for optimized synchronization between the subsystems. We assign the relative optical feedback phase being 0π to these conditions. We find maximum cross correlation between the two signals, if the time series of laser 2 is shifted forward in time by τ_c. Thus, as observed previously, the signal of laser 2 is lagging by the coupling time τ_c [5,7]. For ease of comparison, the lag of the time series of laser 2 has been compensated for in the figure. Note that both the intensity dropouts, as well as the fast intensity fluctuations are highly correlated. We obtain a cross correlation coefficient of approximately 0.90. Furthermore, we find excellent agreement of the rf spectra, and of the optical (multimode) spectra of lasers 1 and 2, thus confirming the synchronization.

Figure 3 depicts a 10 ns zoom of Fig. 2a, demonstrating the very high correlation of laser 1 and laser 2 dynamics, even on sub-ns time scale. Thus, Fig. 3 underlines the remarkable synchronization of the fast intensity fluctuations. We note that the phenomena reported in this paper are robust against reasonable variations of the coupling strength and injection current. We observe chaos synchronization even for considerably higher pump currents in the regime of the fully developed coherence collapse as well, but the detection with sufficient bandwidth is technically more demanding in this regime (see, e.g., [6]).

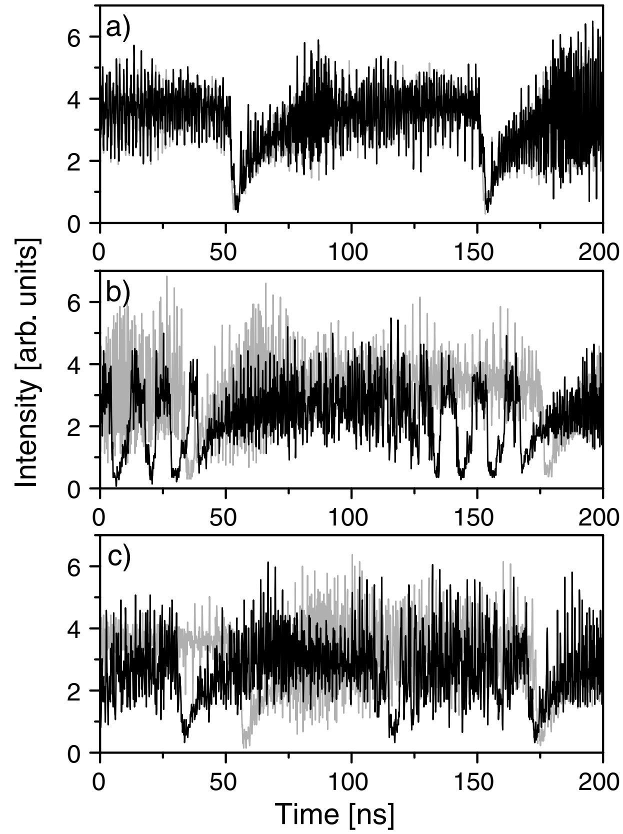

FIG. 2. Intensity time series of laser 1 (gray line) and laser 2 (black line) for different relative optical feedback phases: (a) $\Phi_{\rm rel} = 0\pi$ rad, (b) $\Phi_{\rm rel} = 0.7\pi$ rad, (c) $\Phi_{\rm rel} = 1.4\pi$ rad. The intensity time series of laser 2 are shifted by $\tau_c = 4.6$ ns forward in time. The conditions for laser 1 are kept constant.

We find that a gradual variation of $\Phi_{\rm rel}$ leads to intermittent loss of synchronization with still highly correlated dynamics of the two lasers for certain time intervals followed by sudden jumps to low correlated states. These jumps occur more frequently for increasing deviations of $\Phi_{\rm rel}$ from

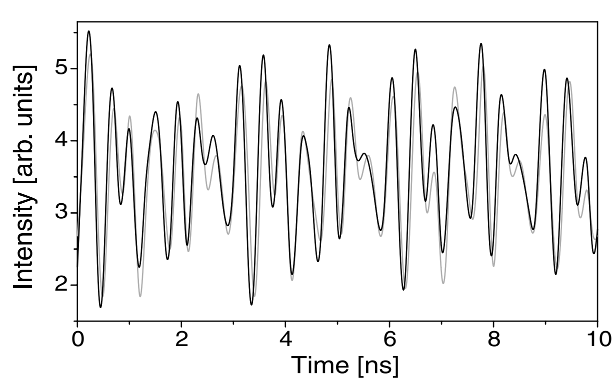

FIG. 3. 10 ns zoom of the intensity time series for adjusted relative optical feedback phase. The gray line represents laser 1, the black line laser 2.

$\Phi_{rel} = 0$ leading to less correlated states. For further increasing Φ_{rel} to 0.7π, we observe drastic changes in the intensity dynamics of laser 2, which are depicted in Fig. 2b. We find well pronounced oscillations consisting of several almost regular intensity dropouts occurring, when laser 1 reaches sufficient high intensities. These oscillations appear in the rf spectrum of laser 2 as a peak with its maximum at 88 MHz. For this interesting regime, we observe a drastic decrease of the correlation coefficient down to 0.2. The optical spectra of lasers 1 and 2 show substantial deviations in their relative intensities of the longitudinal modes. When increasing Φ_{rel} to 1.4π we find that the strong oscillations in the intensity dynamics of laser 2 vanish. Figure 2c depicts a corresponding time series for this regime. The correlation coefficient drops down to about 0.1. Lasers 1 and 2 seem to run independently. The optical spectra are qualitatively similar to the uncoupled case, but in the rf spectrum of laser 2 we observe additional peaks due to the injection of laser 1. Finally, we regain the synchronized state for increasing Φ_{rel} to 2π, depicted in Fig. 2a, while passing a small range of intermittent synchronization with longer and longer time intervals of highly correlated emission of lasers 1 and 2. For variation of the cavity length within a range of several wavelengths, Φ_{rel} is a cyclic parameter.

By monitoring the average intensities at the beam splitter of system 2, we can analyze interference effects among the coupling field and the field in the cavity of laser 2. In the case of synchronization we find a significantly reduced intensity at PD_3, giving evidence for constructive interference towards laser 2. Thus, the optical fields are coherent, despite the fast chaotic fluctuations in optical wavelength, which are typical for this LFF regime. In contrast, for less correlated states the optical fields add incoherently.

We have verified that our system exhibits chaos-pass filtering properties, contrasting genuine chaos synchronization to linear amplification [7]. In the case of synchronization, a small perturbation superimposed on the chaotic transmitter (laser 1) signal is filtered out by the receiver (laser 2) which selectively synchronizes to the transmitter chaos. In contrast, a linear amplifier would receive both the chaotic signal, and the perturbation in the same way. We have modulated the dc pump current of laser 1 with a sinusoidal signal of approximately 1% of I_{th}^{sol}, and monitored the effect on laser 2. For optimized synchronization, i.e., $\Phi_{rel} = 0$, we observe a suppression of the external perturbation of up to 20 dB. We find signal suppression for rf frequencies up to 2 GHz, noting that the maximum suppression ratios are achieved for frequencies of modulation near the cavity round trip resonances. These results prove chaos synchronization for our system. Furthermore, the high suppression ratios are attractive for utilizing them for receiver systems in high bit-rate chaos communication [3,7,8]. As expected, we did not observe signal suppression for Φ_{rel} deviating significantly from zero, as synchronization of the lasers breaks down.

We have performed numerical modeling for two identical, coupled lasers in order to obtain more detailed understanding of our experimental results, and to exclude relevant influence of the experimentally unavoidable slightly parameter mismatch between the lasers. We have extended the well known Lang-Kobayashi SL rate equations [9] for the slowly varying complex electrical field amplitudes $E_{1,2}$, and the carrier densities $n_{1,2}$ by an additional term $K_{1,2}(t)$ which is $K_1(t) = 0$ and $K_2(t) = \kappa E_1(t - \tau_c)e^{-i(\omega_0\tau_c)}$, accounting for the injection of the signal of laser 1 into laser 2. The equations read as follows:

$$\begin{aligned} \dot{E}_{1,2}(t) &= \frac{1}{2}(1 + i\alpha)\xi n_{1,2}(t)E_{1,2}(t) \\ &\quad + \gamma_{1,2}E_{1,2}(t - \tau)e^{-i(\omega_0\tau + \Phi_{1,2})} + K_{1,2}(t), \\ \dot{n}_{1,2}(t) &= (p - 1)\frac{I_{th}}{e} - \frac{n_{1,2}(t)}{T_1} \\ &\quad - [\Gamma_0 + \xi n_{1,2}(t)]|E_{1,2}(t)|^2. \end{aligned} \tag{1}$$

We assume that both lasers emit at the same optical frequency $\omega_0 = \omega_{1,2}$, and are pumped at $I_{1,2} = p \times I_{sol,th,1,2}$, where, $p = 1.05$ denotes the pumping parameter. The parameters used in the numerical simulations correspond to the experimental conditions, with the linewidth enhancement factor being $\alpha = 4$, the differential gain $\xi = 5 \times 10^{-7}\ ns^{-1}$, the carrier lifetime $T_1 = 1.0$ ns, and the photon decay rate $\Gamma_0 = 5.5 \times 10^2\ ns^{-1}$. The coupling parameters are the coupling rate $\kappa = 16\ ns^{-1}$, the delay times $\tau_{1,2} = \tau = 2.9$ ns, and the feedback rates $\gamma_1 = 25\ ns^{-1}$ and $\gamma_2 = 20.5\ ns^{-1}$. We have set the coupling time τ_c to zero which is justified by our experiments demonstrating that the observed phenomena are independent of τ_c.

In a series of simulations, we gradually vary the relative optical feedback phase $\Phi_{rel} = \Phi_1 - \Phi_2$ by changing Φ_2 between zero and 2π for fixed Φ_1. We find distinct dynamical regimes: chaos synchronization in combination with coherence of the optical fields, intermittent synchronization, characteristic large-amplitude oscillations, and uncorrelated intensity dynamics. These four dynamical regimes exhibit excellent qualitative agreement with our experimental findings. Figure 4 depicts the conspicuous large-amplitude oscillations of laser 2 occurring for $0.5\pi < \Phi_{rel} < 0.7\pi$. In agreement with the experiment, the oscillations set on, as soon as the transmitter reaches sufficiently high intensities.

We give an overview over the whole scenario and summarize our experimental and numerical results in Figure 5 by plotting the cross correlation coefficient of the intensity time series of the two lasers in dependence on Φ_{rel}. The experimental data are plotted as black squares, whereas the numerically calculated data are represented by triangles. Despite the restrictions of the model to a single mode description, we find remarkable agreement between experiment and numerical modeling. We find high correlation

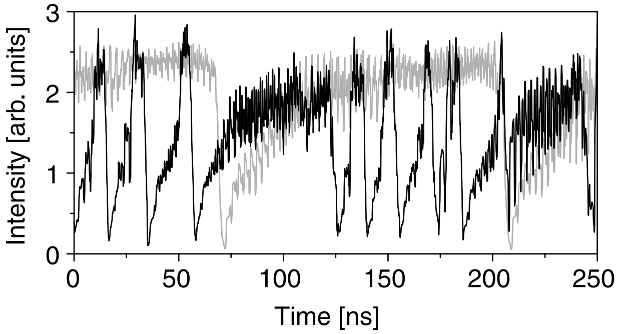

FIG. 4. Numerically obtained intensity time series of two unidirectionally coupled SLs with delayed feedback for $\Phi_{\rm rel} = 0.5\pi$ rad. The parameters correspond to the experiment. (gray line: laser 1, black line: laser 2).

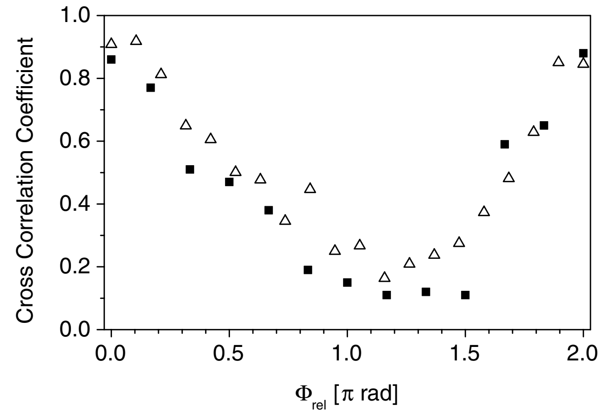

FIG. 5. Cross correlation coefficients of the intensity time series of laser 1 and laser 2 versus $\Phi_{\rm rel}$. The squares depict the values obtained from experimental data, the triangles show the results for numerically obtained time series with corresponding parameters.

coefficients for adjusted phase, where the regime of chaos synchronization is located. With gradually increasing $\Phi_{\rm rel}$, the correlation coefficients are slowly decreasing. We get minimal correlation of the intensity time series around 1.2π, and a steep increase for the correlation coefficients for further increasing $\Phi_{\rm rel}$ to 2π regaining chaos synchronization. Note that both experimental and numerical data show this asymmetry of the correlation coefficients with respect to $\Phi_{\rm rel}$, i.e., $\Sigma_{\rm corr}(\Phi_{\rm rel}) \neq \Sigma_{\rm corr}(2\pi - \Phi_{\rm rel})$ which is plausible, because, due to the unidirectional coupling, the equations are not invariant against substitution of $\Phi_{\rm rel}$ by $-\Phi_{\rm rel}$. The principal mechanisms for the occurrence of the striking intensity oscillations of laser 2 are yet to be resolved in detail.

So far, experimentally no influence of the feedback phase on the dynamics of the single delay system had been observed; neither in the intensity dynamics, nor in the rf and the optical spectra. We find that varying the optical feedback phase of the laser 1 system influences the intensity dynamics of the laser 2 system, whose feedback phase has now been kept constant. Thus, laser 2 system acts as a sensitive detector for changes in the dynamics of the laser 1 delay system, which otherwise would not be visible.

In conclusion, we have proven experimentally and supported by modeling, that the relative optical feedback phase strongly determines the synchronization scenario of two unidirectionally coupled semiconductor lasers subjected to optical feedback. In particular, the scenario evolves along four different regimes of intensity dynamics: chaos synchronization in combination with coherent optical fields, intermittent synchronization, characteristic large-amplitude oscillations, and uncorrelated intensity dynamics, in combination with incoherent fields. Thus, our results show consequences and the decisive importance of vectorial coupling in chaotic systems. Finally, the second coupled system can also be regarded as a sensitive detector, which reveals the influence of the optical feedback phase on the dynamics of the first delayed feedback system, which has been experimentally invisible before.

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