n-$\bar{n}$ Oscillations in Models with Large Extra Dimensions

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We analyze n-$\bar{n}$ oscillations in generic models with large extra dimensions in which standard-model fields propagate and fermion wave functions have strong localization. We find that in these models n-$\bar{n}$ oscillations might occur at levels not too far below the current limit.

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Although current experimental data are fully consistent with a four-dimensional Minkowski spacetime, it is useful to explore the possibility of extra dimensions, both from a purely phenomenological point of view and because the main candidate theory for quantum gravity—string theory—suggests the existence of higher dimensions. Here we shall focus on theories in which the standard-model (SM) fields can propagate in the extra dimensions and the wave functions of the SM fermions have strong localization (with Gaussian profiles) at various points in this extra-dimensional space [1–9]. Such models are of interest partly because they can provide a mechanism for obtaining a hierarchy in fermion masses and quark mixing. In generic models of this type, excessively rapid proton decay can be avoided by arranging that the wave function centers of the u and d quarks are separated far from those of the e and μ [1]. However, this separation does not, by itself, guarantee adequate suppression of another source of baryon number violation, namely n-$\bar{n}$ oscillations. Here we shall analyze these oscillations in generic models of this type. Early studies of n-$\bar{n}$ oscillations in conventional $d = 4$ dimensional spacetime include [10–16]; there is currently renewed experimental and theoretical interest [17,18].

Our theoretical framework is as follows. Usual spacetime coordinates are denoted as x_ν, $\nu = 0, 1, 2, 3$, and the n extra coordinates as y_λ; for definiteness, the latter are taken to be compact. The framework is such that fermion fields have the form $\Psi(x, y) = \psi(x)\chi(y)$. In the extra dimensions the SM fields are assumed to have support in an interval $0 \le y_\lambda \le L$. We define $\Lambda_L \equiv L^{-1}$. The $d = (4 + \ell)$-dimensional fields thus have Kaluza-Klein mode decompositions. We shall work in a low-energy effective field theory approach with an ultraviolet cutoff M_*. These models provide a possible explanation for the hierarchy in the fermion mass matrices via the localization of fermion wave functions with half width $\mu^{-1} \ll L$ at various points in the higher-dimensional space. We denote $\xi = \mu L$; $\xi \sim 30$ is chosen for adequate separation of the various fermion wave functions while still fitting well within the thick brane. As a result of this localization, the y-dependent part of the wave function for a fermion field f has the generic form $\chi_f(y) = Ae^{-\mu^2|y-y_f|^2}$, where y_f denotes the position vector of this fermion in the extra dimensions, $|y_f| = (\sum_{\lambda=1}^{\ell} y_{f,\lambda}^2)^{1/2}$. For $\ell = 1$ and 2, this fermion localization can be accomplished in a low-energy field-theoretic manner by coupling to a scalar with a kink or vortex solution, respectively [19]. The normalization factor $A = (2/\pi)^{\ell/4}\mu^{\ell/2}$ is included so that after the integration over the ℓ extra dimensions the kinetic term $\bar{\psi}(x)i\partial\!\!\!/\psi(x)$ has canonical normalization. Starting from an effective Lagrangian in the d-dimensional spacetime, one obtains the resultant effective theory in four dimensions by integrating over the extra ℓ dimensions. In particular, starting from a Yukawa interaction in the d-dimensional space with coefficients of order unity and integrating over the ℓ extra coordinates, using the fact that the convolution of two of the fermion Gaussian wave functions is another Gaussian, one finds that the coefficients of the resultant four-dimensional Yukawa interaction contain factors $e^{-(\mu^2/2)|y_f - y_{f'}|^2}$, so that moderate spatial separations $|y_f - y_{f'}|$ for localized fermion wave functions can produce strong hierarchies in the 4D Yukawa matrices. We focus here on the case $d = 6$, i.e., $\ell = 2$, for which a fit to the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix was obtained [1,8]. The UV cutoff M_* satisfies $M_* > \mu$ for the validity of the low-energy field theory analysis and $M_*/\Lambda_L \sim 10^3$ for the perturbativity of the top quark Yukawa coupling. Further, $\Lambda_L \gtrsim 100$ TeV for adequate suppression of neutral flavor-changing current (NFCC) processes [3,20,23]; with the ratio $\xi = 30$, this yields $\mu \sim 3 \times 10^3$ TeV.

In field-free vacuum, the 2×2 Hamiltonian $\mathcal{H}$ in the $(n, \bar{n})$ space has matrix elements $\langle n|\mathcal{H}|n\rangle = \langle\bar{n}|\mathcal{H}|\bar{n}\rangle = m_n$ (assuming CPT conservation) and $\langle\bar{n}|\mathcal{H}|n\rangle = \langle\bar{n}|\mathcal{H}_{\rm eff}|n\rangle \equiv \delta m$, where $\mathcal{H}_{\rm eff}$ denotes the interaction responsible for the n-$\bar{n}$ oscillations. This leads to the oscillation probability $|\langle\bar{n}|n(t)\rangle|^2 = \sin^2(t/\tau_{n\bar{n}})$, where $\tau_{n\bar{n}} = 1/|\delta m|$, and it is straightforward to generalize this to the case where the Earth's magnetic field is included. Searches with free neutrons from reactors have yielded the lower limit $\tau_{n\bar{n}} \ge 0.86 \times 10^8$ sec [24]. Further, the n-$\bar{n}$ oscillations cause matter instability since the $\bar{n}$ annihilates with surrounding matter and, related to this via crossing, two neutrons can annihilate to pions. Although the n-$\bar{n}$ mixing in matter is strongly suppressed by the fact that the diagonal matrix elements

$\langle n|\mathcal{H}|n\rangle = m_n + V_n$ and $\langle \bar{n}|\mathcal{H}|\bar{n}\rangle = m_n + \mathrm{Re}(V_{\bar{n}}) + i\,\mathrm{Im}(V_{\bar{n}})$ are different, this is compensated for by the large number of nucleons in modern nucleon decay detectors. Searches using these detectors have yielded the limit for matter instability (m.i.) caused by this process (leading to multipion final states) of $\tau_{\mathrm{m.i.}} \gtrsim 0.6 \times 10^{32}$ yr which, with reasonable inputs for the nuclear potentials V_n and $V_{\bar{n}}$, have yielded a current limit very close to that from searches with free n's: $\tau_{n\bar{n}} \geq 1.2 \times 10^8$ sec, i.e., $|\delta m| \leq 0.6 \times 10^{-32}$ GeV [25].

In 4D, $\mathcal{H}_{\mathrm{eff}}$ consists of a sum of six-quark operators $\mathcal{O}_i$ with coefficients c_i having (mass) dimension -5. If, as is the case here, the effective scale relevant for n-$\bar{n}$ oscillations is large compared with the electroweak symmetry-breaking scale of $\sim$250 GeV, then the $\mathcal{O}_i$ must be invariant under $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. There are four relevant (linearly independent) $\mathcal{O}_i$ of this type, so that $\mathcal{H}_{\mathrm{eff}} = \sum_{i=1}^{4} c_i \mathcal{O}_i$, where the $\mathcal{O}_i$ can be chosen to be

$$\mathcal{O}_1 = [u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \quad (1)$$

$$\mathcal{O}_2 = [u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \quad (2)$$

$$\mathcal{O}_3 = [Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}, \quad (3)$$

$$\mathcal{O}_4 = [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}] \times [d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}, \quad (4)$$

where $Q_L = \binom{u}{d}_L$, Greek and Latin indices are color SU(3) and weak SU(2) indices, respectively, C is the Dirac charge conjugation matrix, and

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}, \quad (5)$$

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}. \quad (6)$$

The matrix elements of $\langle \bar{n}|\mathcal{O}_i|n\rangle$ for $1 \leq i \leq 4$ were calculated in the MIT bag model in [15].

In the higher-dimensional field theory, we denote the effective Hamiltonian responsible for the n-$\bar{n}$ transitions as $\mathcal{H}_{\mathrm{eff},4+n} = \sum_{i=1}^{4} \kappa_i O_i$, where the operators O_i are composed of the $(4+\ell)$-dimensional quark fields corresponding to those in $\mathcal{O}_i$ as Ψ corresponds to ψ. The coefficient of an operator product consisting of N_Ψ fermions has (mass) dimension $\dim[c] = d - (N_\Psi/2)(d-1)$, so that $\dim[c] = -9$ for $d = 6$ and $N_\Psi = 6$. We denote the effective scale of physics responsible for the n-$\bar{n}$ oscillations as M_X and, thus, for the $d = 6$ case of interest here, write $\kappa_i = \eta_i/M_X^9$, where the η_i's are dimensionless constants. It will be convenient, and will incur no loss of generality, to define M_X so that $\eta_4 \equiv 1$.

We next carry out the integrations over the extra dimensions to get, for each i,

$$c_i \mathcal{O}_i(x) = \kappa_i \int d^2y\, O_i(x,y). \quad (7)$$

Letting $\rho_c \equiv 4\mu^4/(3\pi^2 M_X^9)$, we find

$$c_i = \rho_c \eta_i \exp[-(4/3)\mu^2 |y_{u_R} - y_{d_R}|^2], \qquad i = 1,2, \quad (8)$$

$$c_3 = \rho_c \eta_3 \exp[-(1/6)\mu^2(2|y_{Q_L} - y_{u_R}|^2 + 6|y_{Q_L} - y_{d_R}|^2 + 3|y_{u_R} - y_{d_R}|^2)] \quad (9)$$

$$c_4 = \rho_c \exp[-(4/3)\mu^2 |y_{Q_L} - y_{d_R}|^2]. \quad (10)$$

A fit to data for $n = 2$ yielded $|y_{Q_L} - y_{u_R}| = |y_{Q_L} - y_{d_R}| \simeq 5\mu^{-1}$ and $|y_{u_R} - y_{d_R}| \simeq 7\mu^{-1}$ [1]. One can also include corrections due to Coulombic gauge interactions between fermions [7]. Using these inputs, we find that c_j for $j = 1,2,3$ are negligibly small compared with c_4, and we hence focus on c_4. Neglecting small CKM mixings, the quantity $|y_{Q_L} - y_{d_R}|$ is determined by m_d via the relation $m_d = h_d(v/\sqrt{2})$ with $h_d = h_{d,0}\exp[-(1/2)\mu^2|y_{Q_L} - y_{d_R}|^2]$, where $h_{d,0}$ is the Yukawa coupling in the $(4+\ell)$-dimensional space, whence

$$\exp[-(1/2)\mu^2|y_{Q_L} - y_{d_R}|^2] = \frac{2^{1/2}m_d}{h_{d,0}v}. \quad (11)$$

We take $h_{d,0} \sim 1$, and $m_d \simeq 10$ MeV. Hence, the contribution to δm from the $\mathcal{O}_4$ term is

$$\delta m \simeq c_4\langle \bar{n}|\mathcal{O}_4|n\rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_X^9}\right)\left(\frac{2^{1/2}m_d}{v}\right)^{8/3}\langle \bar{n}|\mathcal{O}_4|n\rangle. \quad (12)$$

The matrix element $\langle \bar{n}|\mathcal{O}_4|n\rangle$ ($= 4\langle \mathcal{O}_3\rangle_{LLR}$ in the notation of [15,16]) was calculated to be 1×10^{-4} and 0.8×10^{-4} GeV^6 in the MIT bag model for its two main parameter fits [15]; here we average these. Requiring that the resultant value of $|\delta m|$ be less than the experimental limit, $1/(1.2 \times 10^8 \text{ sec}) = 0.55 \times 10^{-32}$ GeV [24,25], we obtain the bound

$$M_X \gtrsim (45 \text{ TeV})\left(\frac{\tau_{n\bar{n}}}{1.2 \times 10^8 \text{ sec}}\right)^{1/9} \times \left(\frac{\mu}{3 \times 10^3 \text{ TeV}}\right)^{4/9}\left(\frac{|\langle \bar{n}|O_4|n\rangle|}{0.9 \times 10^{-4} \text{ GeV}^6}\right)^{1/9}. \quad (13)$$

The uncertainty in the calculation of the matrix element $\langle \bar{n}|\mathcal{O}_4|n\rangle$ is relatively unimportant for our bound because of the 1/9 power in (13) [26].

The result (13) is very interesting, since it shows that for values of M_X in the range relevant to extradimensional models of the type considered here, n-$\bar{n}$ oscillations might occur at levels that are in accord with the current experiment limit but not too far below this limit. Our finding provides motivation for further searches for n-$\bar{n}$ oscillations, e.g., via an analysis of Super-Kamiokande data or a new free-neutron reactor experiment with increased sensitivity, such as that proposed in [17]. Considering only the analysis of existing data from Super-Kamiokande, a rough estimate is that the search for matter instability via n-$\bar{n}$ oscillations followed by annihilation to multipion final states might be able to set a lower bound on $\tau_{\rm m.i.}$ in the range of 10^{33} yr, similar to bounds that have been set by Super-Kamiokande on various proton decay modes; this could increase the lower bound on $\tau_{n\bar{n}}$ to $\sim 3 \times 10^8$ sec. If a signal were to be seen, other experimental data and analysis would be necessary to determine whether these oscillations are associated with extra dimensions, since, as has been shown in [18], they can also occur with $\tau_{n\bar{n}} \lesssim 10^9$–$10^{10}$ sec in certain four-dimensional supersymmetric models.

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