

Neutrino Cross Sections and Future Observations of Ultrahigh-Energy Cosmic Rays

Alexander Kusenko^{1,2} and Thomas J. Weiler³

¹*Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547*

²*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973*

³*Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235*

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We show that future detectors of ultrahigh-energy cosmic-ray neutrinos will be able to measure neutrino-nucleon cross section, $\sigma_{\nu N}$, at energies as high as 10^{11} GeV or higher. We find that the flux of upgoing charged leptons per unit surface area produced by neutrino interactions below the surface is *inversely* proportional to $\sigma_{\nu N}$. This contrasts with the rate of horizontal air showers (HAS) due to neutrino interactions in the atmosphere, which is proportional to $\sigma_{\nu N}$. Thus, by comparing the HAS and upgoing air shower rates, the neutrino-nucleon cross section can be inferred. Taken together, upgoing and horizontal rates ensure a healthy total event rate, regardless of the value of $\sigma_{\nu N}$.

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Detection of ultrahigh-energy (UHE) neutrinos is one of the important challenges of the next generation of cosmic-ray detectors. Their discovery will mark the advent of UHE neutrino astronomy, allowing the mapping on the sky of the most energetic, and most distant, sources in the Universe. In addition, detection of UHE neutrinos may help resolve the puzzle [1] of cosmic rays with energies beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff [2] by validating Z bursts [3], topological defects [4], superheavy relic particles [5], new strong interactions [6], etc.

Several approved and proposed experiments plan to detect UHE neutrinos by observation of the nearly horizontal air showers (HAS) in Earth's atmosphere resulting from ν -air interactions. The expected rates are proportional to the neutrino-nucleon cross section. Calculations of this cross section at 10^{20} eV necessarily use an extrapolation of parton distribution functions and standard model parameters far beyond the reach of experimental data. The resulting cross section at 10^{20} eV is $\sim 10^{-31}$ cm² [7]. It has recently been argued that the extrapolated neutrino cross section may be too high [8] at energies above about 3×10^{17} eV. If the cross section is lower, then the event rate for neutrino-induced HAS is reduced by the same factor. This reduction would compromise the main detection signal that has been proposed for UHE neutrino experiments. On the other hand, the extrapolated cross section may be too low, for it ignores possible contributions from new physics.

We will show, however, that regardless of possible theoretical uncertainties in the cross section, the future experiments can observe the UHE neutrinos. In fact, a smaller cross section would offer a double advantage for the planned experiments. First, with a new search strategy (described below), the neutrino event rate with a small cross section is actually larger than the HAS rate with a large cross section. Second, the future detectors can also measure the neutrino cross section at energies far beyond those achievable in collider experiments. The first advantage is a boon for neutrino astronomy, while the second

provides important information for particle physics. Here we will take the value of the cross section to be a free parameter with a wide range of values.

This study is motivated in part by a recent analysis of upward events by Feng *et al.* [9]. The emphases in the two papers are quite different.

In addition to HAS, proposed cosmic-ray experiments can also observe upgoing air showers (UAS) initiated by muon and tau leptons produced by neutrinos interacting just below the surface of Earth, and may possibly observe the fluorescence signal from upgoing charged muon and tau leptons (UCL) themselves. Prior estimates for the rate of "Earth-skimming" events have used the extrapolated neutrino cross section [9,10]. A smaller value of this cross section reduces the shadowing of UHE neutrinos by Earth. Therefore, the neutrino angles with respect to horizon need not be so "skimming." More importantly, the expected rate of UCL and UAS may (i) be larger, and (ii) depend on the cross section.

Indeed, a lower cross section *increases* the UCL rate per surface area as $\sigma_{\nu N}^{-1}$ as long as the neutrino absorption mean free path (MFP) in Earth is small in comparison with Earth's radius, R_{\oplus} ; i.e., for $\sigma_{\nu N} \geq 2 \times 10^{-33}$ cm² the UCL event rate is proportional to $F_{\nu}/\sigma_{\nu N}$, as shown in Fig. 1. This inverse dependence of the UCL rate on the cross section is to be contrasted with the rate of HAS events resulting from ν -air interactions, which decreases as $\sigma_{\nu N}$ decreases. Furthermore, we find that, for small but possible values of the cross section, the UAS rate can exceed the HAS rate by several orders of magnitude, as displayed in Fig. 2.

There is a firm prediction for a flux F_{ν} of GZK neutrinos in the energy range 10^{15} to 10^{20} eV, based on the observed flux of UHE cosmic-ray (UHECR) protons at the GZK limit. This flux is expected to peak in the decade 10^{17} to 10^{18} eV for uniformly distributed proton sources, and around 10^{19} eV for "local" sources within ~ 50 Mpc of Earth [11]. If the GZK flux is the dominant source of UHE neutrinos, then one can use the predicted GZK flux

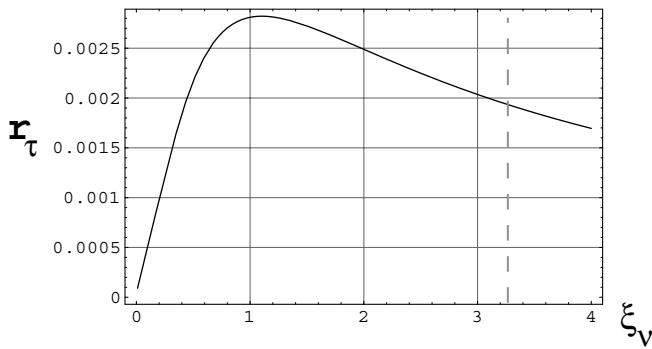


FIG. 1. The ratio r_τ of the upward going τ flux to the incident tau neutrino flux F_{ν_τ} as a function of $\xi_\nu = \lambda_\nu/R_\oplus = 1/(\sigma_{\nu N} n R_\oplus)$, from Eq. (5), with fixed $\lambda_\tau/R_\oplus = 3.5 \times 10^{-3}$, appropriate for events initiated by $\sim 10^{20}$ eV neutrinos. Here n is the mean nucleon number density. Assuming a monotonic cross section dependence on \sqrt{s} , the value of ξ is limited from above by the HERA measurements, as shown by a vertical dashed line.

value and either the inverse relation between the UCL rate and $\sigma_{\nu N}$ (shown in Fig. 1) or the linear relation between the HAS rate and $\sigma_{\nu N}$ to infer from future data the value of the neutrino cross section at ultrahigh energies.

In addition, a comparison of the UAS and HAS rates, or a measurement of the angular distribution of UCL/UAS events, may allow an experiment to determine $\sigma_{\nu N}$ independent of F_ν , as discussed below. More promising for $\sigma_{\nu N}$ determination are possible neutrino fluxes at and above 10^{20} eV given, e.g., in [3,12].

Let us now estimate the rate of upward showers. UHE neutrinos are expected to arise from pion and subsequent muon decay. The initial flavor composition is therefore ν_μ and ν_e with a ratio 2:1. These flavors oscillate and eventually decohere during their Hubble-time journey. The resulting neutrino state includes a ν_τ fraction $\frac{2}{3} \sum_j |U_{\mu j}|^2 |U_{\tau j}|^2 + \frac{1}{3} \sum_j |U_{e j}|^2 |U_{\tau j}|^2$, where $U_{\alpha j}$ are

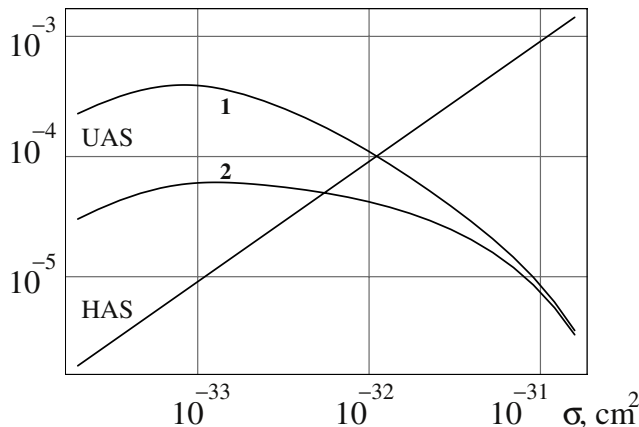


FIG. 2. The air shower probability per incident tau neutrino $R_{\text{UAS}}/F_{\nu_\tau} \pi A$ as a function of the neutrino cross section [Eq. (6)]. The incident neutrino energy is 10^{20} eV and the assumed energy threshold for detection of UAS is $E_{\text{th}} = 10^{18}$ eV for curve 1 and 10^{19} eV for curve 2.

the mixing elements relating the neutrino mass and the flavor bases. If $|U_{\tau 3}| \approx |U_{\mu 3}|$ are large, as inferred from the Super-Kamiokande data, then the oscillations nearly equalize the number of UHE neutrinos of each flavor [13]. In particular, $F_{\nu_\tau} \approx \frac{1}{3} F_\nu$ is expected. The energy-loss MFPs, λ_τ and λ_μ , for taus and muons to lose a decade in energy are 11 and 1.5 km, respectively, in surface rock with density $\rho_{sr} = 2.65$ g/cm³, and 2.65 times longer for lepton trajectories passing through ocean water. Tau and muon decay MFPs are long above 10^{18} eV: $c\tau_\tau = 490(E_\tau/10^{19} \text{ eV})$ km, and $c\tau_\mu \sim 10^8 c\tau_\tau$ for the same lepton energy. Because the energy-loss MFP for a τ produced in rock or water is much longer than that of a muon, the produced taus have a much higher probability to emerge from Earth and to produce an atmospheric shower. Thus, the dominant primary for initiation of UAS events is the tau neutrino. In what follows, we focus on tau neutrinos incident at 10^{20} eV.

Let us consider an incident tau neutrino whose trajectory cuts a chord of length l in Earth. The probability for this neutrino to reach a distance x is $P_\nu(x) = e^{-x/\lambda_\nu}$, where $\lambda_\nu^{-1} = \sigma_{\nu N} \rho$ (the conversion from matter density to number density via N_A/gm is implicit). The probability to produce a tau lepton in the interval dx is $\frac{dx}{\lambda_\nu}$. The produced τ carries typically 80% of the parent neutrino energy; we approximate this as 100%. Then the τ produced at point x emerges from the surface with energy $E_\tau = E_\nu e^{-(l-x)/\lambda_\tau}$. The probability of a τ produced at point x to emerge with sufficient energy E_{th} to produce an observable shower can be approximated as $P_{\tau \rightarrow \text{UAS}} = \Theta(\lambda_\tau + x - l)$, with $\lambda_\tau = \frac{1}{\beta_\tau \rho_{sr}} \ln(E_\nu/E_{\text{th}})$; $\beta_\tau \approx 0.8 \times 10^{-6}$ cm²/g [14] is the exponential energy-attenuation coefficient. For taus propagating through rock, one can take $\lambda_\tau \approx 22$ km for $E_\nu \sim 10^{20}$ eV and $E_{\text{th}} \sim 10^{18}$ eV, while for taus propagating through ocean λ_τ is 2.65 times larger. The rates in Fig. 2 are shown for UAS over land. An UAS over an ocean, where part or even all of the tau path is in water [15], is best addressed with a simulation. The threshold E_{th} depends on details of detector sensitivity and aperture, as well as the assumed neutrino spectrum; these can suitably be incorporated in a threshold parameter.

Taking the product of these conditional probabilities and integrating over the interaction site x , we get the probability for a tau neutrino incident along a chord of length l to produce an UCL:

$$P_{\nu_\tau \rightarrow \tau}(l) = \int_{l-\lambda_\tau}^l \frac{dx}{\lambda_\nu} e^{-x/\lambda_\nu} = (e^{\lambda_\tau/\lambda_\nu} - 1)e^{-l/\lambda_\nu}. \quad (1)$$

The emerging tau decays in the atmosphere with probability $P_d = 1 - \exp(-2R_\oplus H/c\tau_\tau l)$, where $H \approx 10$ km parametrizes the height of the atmosphere. Thus, the probability for a tau neutrino to produce an UAS is

$$P_{\nu_\tau \rightarrow \text{UAS}}(l) = (1 - e^{-2R_\oplus H/c\tau_\tau l}) P_{\nu_\tau \rightarrow \tau}(l). \quad (2)$$

Next we calculate the probability for an incident neutrino trajectory to have chord length l . Because of isotropy of the neutrino flux, it is enough to consider incident neutrinos with parallel trajectories. The length l and impact parameter h with respect to Earth's center of a chord are related by $l^2/4 + h^2 = R_\oplus^2$. The fraction of neutrinos with chord lengths in the interval $\{l, l + dl\}$ is therefore

$$P_{\text{chord}}(l)dl = \frac{2\pi h dh}{\pi R_\oplus^2} = \frac{l}{2R_\oplus^2} dl. \quad (3)$$

To get an event rate probability from the incident neutrino flux, there are two further geometric factors to be included: the solid angle factor π for a planar detector with hemispherical sky coverage, and the tangential surface area A of the detector [16].

Putting all probabilities together, we arrive at the rate of UCL and UAS events:

$$R_{\tau(\text{UAS})} = F_{\nu_\tau} \pi A \int_0^{2R_\oplus} \frac{l dl}{2R_\oplus^2} P_{\nu_\tau \rightarrow \tau(\text{UAS})}(l). \quad (4)$$

The double integral in Eq. (4) is easily done analytically for the UCL case, and for the UAS case when the angle above the horizon satisfies $\theta \gg (E_\tau/10^{19} \text{ eV})^\circ$, such that $2R_\oplus H/c\tau_\tau l \ll 1$.

These results in the limit $\lambda_\nu \gg \lambda_\tau$ are

$$R_\tau = F_{\nu_\tau} \pi A \frac{1}{2} \xi_\tau \xi_\nu [1 - e^{-2/\xi_\nu} (1 + 2/\xi_\nu)], \quad (5)$$

where $\xi_\tau = \lambda_\tau/R_\oplus$ and $\xi_\nu = \lambda_\nu/R_\oplus$, and

$$R_{\text{UAS}} = F_{\nu_\tau} \pi A \frac{H \xi_\tau}{c\tau_\tau} (1 - e^{-2/\xi_\nu}). \quad (6)$$

For neutrino trajectories through Earth's mantle, $\xi_\nu = 0.66/\sigma_{33}$, where σ_{33} is the neutrino cross section in units of 10^{-33} cm^2 .

The tau flux scales as $\lambda_\nu^{-1} \propto \sigma_{\nu N}$ for $\lambda_\nu \gg R_\oplus$ (i.e., small $\sigma_{\nu N}$), because the large neutrino MFP exceeds Earth's diameter, making the interactions rarer for increasing λ_ν . For $\lambda_\tau \ll \lambda_\nu \ll R_\oplus$ (i.e., $\sigma_{\nu N} \geq 2 \times 10^{-33} \text{ cm}^2$), the probability scales as $\lambda_\nu \propto \sigma_{\nu N}^{-1}$. The rise with increasing MFP is attributable to shrinkage of Earth's "shadow" and the consequent increase in the target volume. The ratio $r_\tau = R_\tau/F_\nu \pi A$ is shown in Fig. 1. We note that the neutrino cross section may be determinable from the angular distribution of UCL events alone (if they can be observed), independent of the neutrino flux. One expects the angular distribution of UCL to peak near $\cos\theta_{\text{peak}} \sim \lambda_\nu/2R_\oplus$, which implies $\sigma_{\nu N} \sim (2\langle\rho\rangle R_\oplus \cos\theta_{\text{peak}})^{-1}$.

The UAS rate scales differently from the UCL rate, due to the dependence on path length in the atmosphere.

In Fig. 2 we show the number of expected UAS events per incoming neutrino as a function of the neutrino cross section (using the exact integral expression). The cross section is bounded from below by the value $\sim 2 \times 10^{-34} \text{ cm}^2$ measured at HERA at $\sqrt{s} = 314 \text{ GeV}$, corresponding to a laboratory energy $E_\nu = 5.2 \times 10^{13} \text{ eV}$. For comparison, we also show the

number of expected HAS events per neutrino that crosses a 250 km field of view, up to an altitude of 15 km. It is clear that, for the smaller values of the cross section, UAS events will outnumber HAS events, while the larger values of the cross section favor the HAS events. The ratio of HAS to UAS rates may provide a good measure of the cross section.

We give some examples of the UAS event rates expected from a smaller neutrino cross section at 10^{20} eV . Let us choose $\sigma_{\nu N} = 10^{-33} \text{ cm}^2$, for example. Taking the mantle density of $\rho_m = 4.0 \text{ g/cm}^3$ and $R_\oplus = 6.37 \times 10^8 \text{ cm}$, one gets $\xi_\nu = 0.65$. Reference to Fig. 1 then shows that the $\nu_\tau \rightarrow \tau$ conversion probability is $r_\tau = 0.1\%$ for land events with $E_\tau \geq 10^{18} \text{ eV}$, initiated by a $\sim 10^{20} \text{ eV}$ primary neutrino. Including the probability for a tau to decay in the atmosphere, the $\nu_\tau \rightarrow \text{UAS}$ probability is 4×10^{-4} (7×10^{-5}) for a shower-energy threshold $E_{\text{th}} = 10^{18} \text{ eV}$ (10^{19} eV), according to Fig. 2. The Extreme Universe Space Observatory and the Orbiting Wide-angle Light collector have shower-energy thresholds $\sim 10^{19} \text{ eV}$ corresponding to curve 2 in Fig. 2. They have apertures $\sim 6 \times 10^4 \text{ km}^2$ and $3 \times 10^5 \text{ km}^2$, respectively, for a wide angular range of UAS. These detectors should observe F_{20} and $7F_{20}$ UAS events per year, respectively (not including duty cycle); here F_{20} is the incident neutrino flux at and above 10^{20} eV in units of $\text{km}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$, one-third of which are ν_τ 's. Including showers from taus originating outside the field of view, and direct tau events, increases these rates. The rates may be further increased in space-based detectors by tilting towards the horizon so as to maximize the acceptance for events with smaller chord lengths (where the neutrino attenuation is less and the field of view is greater, but the energy threshold is higher) and to allow more atmospheric path length for tau decay. The rates will also increase if E_{th} can be reduced; reducing E_{th} to 10^{17} eV or less would allow the GZK flux to be observed and measured. Although the spectrum of UHE neutrinos is not known, a variety of assumed spectra can be encompassed by our parametrization, and our conclusions are not sensitive to spectral details.

Finally, we comment on two important inferences. First, the reported bounds on the UHE neutrino flux due to the nonobservation of neutrino-initiated HAS [17] and of radio signals produced by neutrino interactions near the surface of the Moon [18] are weaker if the cross section is smaller. Concerning the lunar radio bound, the lunar radius is 1740 km, about 3.5 times smaller than that of Earth, and the density of the Moon is about the same as Earth's surface density. Thus, $\xi_\nu = \lambda_\nu/R$ for the Moon is $3.6/\sigma_{33}$, which is about 5.5 times the value for earthly neutrinos. Consequently, the range of ξ_ν for the Moon allowed by our ignorance of the true neutrino cross section is very large, and the true neutrino flux limit from lunar radio could be different from that previously reported. The second inference has to do with the predicted angle independence for upgoing τ neutrinos at $\sim 10^{14} \text{ eV}$ [19]. For

a smaller cross section, a harder spectrum of unattenuated ν_τ 's above 10^{14} eV and a larger angle dependence may be expected.

In conclusion, the overall prospects for UHE neutrino astronomy are not diminished by the theoretical uncertainties in value of the neutrino-nucleon cross section. UHECR neutrinos can be observed at a healthy rate for any allowed value of the cross section. Furthermore, future neutrino cosmic-ray experiments can determine the neutrino-nucleon cross section at energies as high as 10^{11} GeV, or higher, by comparing the rates of UAS with those of HAS, or by measuring the angular distribution of UCL events.

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