Experimental and Numerical Studies of Noise-Induced Coherent Patterns in a Subexcitable System

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(Received 27 September 2001; published 19 March 2002)

A subexcitable medium of Belousov-Zhabotinsky (BZ) reaction subjected to external Gaussian white noise is studied in experiments and numerical simulations. We observe that at an optimal level of noise the wave sources of excited traveling waves become synchronous, as though there exists a long distance spatial correlation. The synchronous behavior fades if the noise level becomes larger or smaller. Numerical simulations confirm the experimental findings, and point out that the best synchronous behavior takes place when the signal-to-noise ratio of waves becomes largest.

It is well established that noise can play ordering roles in the dynamics of nonequilibrium systems. Noise induced transition [1] and stochastic resonance (SR) [2] are well-known examples of the beneficial effect of fluctuations in nonlinear dynamical systems. In recent years, the phenomenon of SR is a subject of intense investigations in light of its useful application in physical, technological, and biomedical contexts, because SR points to a possible novel way of enhancing the detection of weak signals by adding an optimized amount of noise in the measured system. The original model of SR includes a symmetrical bistable system, additive external Gaussian white noise, and a weak input periodic signal. Later, a quite wide variety of systems are used to study SR, such as monostable system [3], multistable system [4], excitable system [5], chemical reactions [6], and neural network [7]. Noise can be colored [8] or multiplicative [9], the input signal can be aperiodic [10], and SR can even be shown to occur without an external periodic drive [11–15].

Recently, the effect of noise on spatial extended systems has received lots of attention. Jung and Mayer-Kress [16] first proposed the concept of spatiotemporal stochastic resonance, showing a sharp peak of synchronization of spatiotemporal patterns in an excitable medium at a finite, well-defined noise level. Since then, the study of interactions between noise and spatiotemporal patterns has become an attractive subject. Noise can enhance propagation in arrays of coupled bistable oscillators [17]. In subexcitable media, noise can induce traveling waves [18], drive avalanche behavior [19], and sustain pulsating patterns and global oscillations [20].

In this Letter, we report our study on the influences of external Gaussian white electric noise parametrically on a subexcitable medium of Belousov-Zhabotinsky (BZ) reaction. Our experimental results show an interesting phenomenon of spatiotemporal coherent patterns. We observe that noise continuously causes traveling waves emerging randomly from the boundary of the reaction medium. Moreover, at an optimal level of the noise, these waves are almost simultaneously excited and travel forward together.

That is, noise of an optimal level causes synchronization of wave sources.

Our experiments are carried out with a quasi-two-dimensional spatial open reactor, on which an external electric field can be imposed. Experimental setup and chemical compositions are the same as those described in Ref. [21], except that eight platinum electrodes are added in the reactor. These electrodes are arranged in a symmetrical way in the two compartments which sandwich the reaction medium, a porous glass disk. As shown in Fig. 1, in each side of the reaction medium, four electrodes are curved, without connection, to form a cycle whose diameter is 25 mm, larger than that of the reaction medium (18 mm, the gray part in Fig. 1). The distance between planes of the electrodes and the reaction medium is about 3 mm. Through the electrodes, an electric field parallel to the surface of the reaction medium can be added. Using a signal generator of white noise (UZ-3A type), we subject the reaction medium to an electric field of Gaussian white noise, with zero mean and frequency density distribution of 20 Hz to 20 KHz. Notice that the applied field $E(r, t)$,
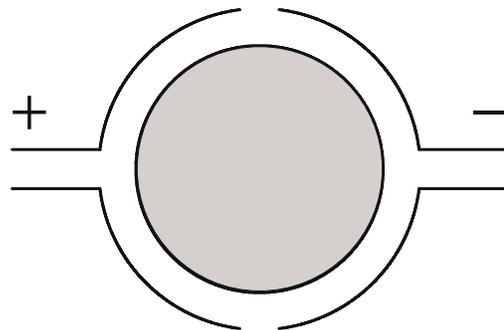
FIG. 1. The side view of the electrode arrangement. In each side of reaction medium, four electrodes (lines) forming a cycle without connection are about 3 mm away from the plane of the medium (the gray circle in the center). One end of each electrode is extended out of the open reactor and is connected with the output of the signal generator. The left four are with the anode, and the right four with the cathode.

although stochastic in time, has a space-fixed orientation determined by the electrode geometry.

The effects of the electrical noise on the subexcitable medium are shown in Fig. 2. The concentrations of components are chosen to maintain the system slightly below the excitability threshold. They are the following: $[CH_2-(COOH)_2]_2 = 0.4$ M; $[KBr]_A = 0.03$ M; $[NaBrO_3]_{A(B)} = 0.4$ M; $[Ferroin]_B = 0.5$ mM; $[H_2SO_4]_B = 0.18$ M. All the concentrations of components are kept fixed, and the noise level is varied from zero in Fig. 2(a), corresponding to the noise-free autonomous system, to the maximum level $D = 2.9$ V in Fig. 2(d), which is determined by the capacity limit of the aqueous solutions under the external electrical field.

Figure 2(a) shows that, under the experimental conditions, the noise-free system cannot support wave propagation. When a wave front emerges on the boundary due to the asymmetry of two areas of the reaction medium [21], it cannot propagate into the inner area of the reaction medium. The situation changes when external electrical noise is imposed onto the system. In this case some small wave fronts grow up randomly from the boundary and traveling forward into the inner area of the reaction medium. Figures 2(b)–2(d) show the propagation of these wave fronts under a different level of noise. At a low noise level [Fig. 2(b)], several traveling waves with different propagating directions are excited simultaneously from the boundary of the reaction medium. As time elapses, these waves travel forward, meet and annihilate one another, and disappear. Then a new group of traveling waves will be excited, repeating the same process. At a high noise level, one wave source dominates the whole system so that regular patterns form, as shown in Figs. 2(c) and 2(d). The wave frequency also increases with the amplitude of noise.

If the noise is shut off, all the wave fronts will slow down, break, and finally disappear, the system comes back to the state of Fig. 2(a). Thus a certain level of external noise is a necessary condition for traveling wave propagation and regular pattern formation.

Figure 3 shows the evolution of traveling waves excited by noise at noise level $D = 1.4$ V, corresponding to Fig. 2(b). A certain number of small wave fronts are excited simultaneously in the boundary of the reactor medium [six wave fronts excited in Fig. 3(a)]. All of them intend to propagate forward. However, some of them enlarge while others shrink [three enlarging in Fig. 3(b)]. The enlarging wave fronts meet one another and disappear in the central part of the system [in Fig. 3(c)]. Then new and different small wave fronts are excited randomly again from the boundary [in Fig. 3(d)], repeating the same process. Since the system has a characteristic periodicity, we define a quantity η and study its periodical variation at a different level of noise. For one image at a time t, η is defined as the following:

$$\eta = \sqrt{\sum_i (A_i - \bar{A})^2 / N}, \qquad (1)$$

where $\bar{A} = \sum_i A_i / N$, A_i is the intensity of any pixel point i in the circular image, N is the total number of pixel points, and the summation includes all the pixel points in the image. Thus the function η is the standard deviation of one image from its mean brightness. For the noise-free image, the quantity η should be equal to zero. If there exists some little wave fronts, the quantity η has small positive value.

Figure 4 shows the variations of η as functions of time t, taken at different noise levels. One observes that all

FIG. 2. The effects of noise on subexcitable medium. (a) The noise-free reaction medium (gray area) shows no wave propagations in the inner area of the system; (b)–(d) coherent patterns are shown at different noise levels $D = 1.4$, 2.6, and 2.9 V. The image in (a) has a size of 18.2×15.0 mm^2 pixels and shows the whole observation window. The others are cut from the center area of the reaction medium in circles whose diameters are 14.0 mm.

FIG. 3. The synchronous excitations induced by noise with the noise level $D = 1.4$ V. The sequential times, respectively, are 0 s; 140 s; 260 s; 480 s. (a) Several randomly excited wave fronts emerge synchronously from the boundary of the reaction medium; (b) some of them enlarge when traveling forward; (c) when the waves meet one another, they disappear; (d) new and different wave fronts are excited again, repeating the same process.
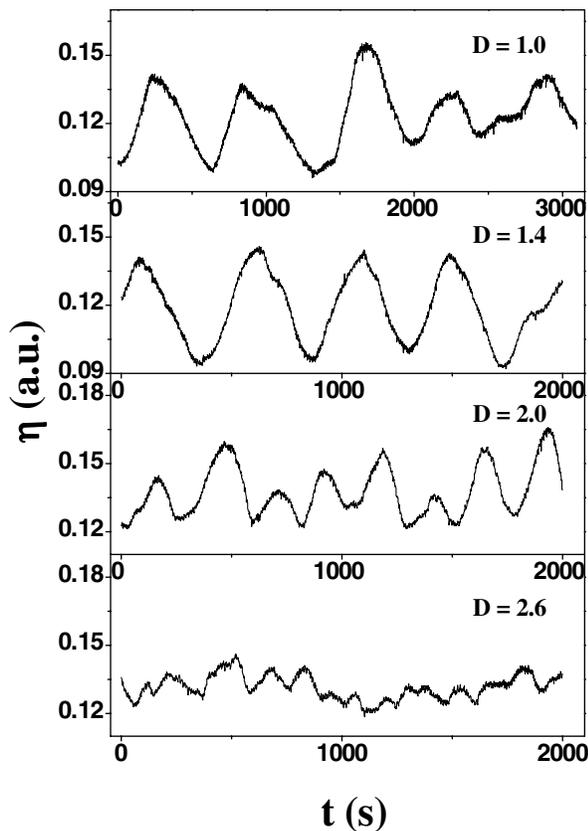
FIG. 4.   The variations of η as functions of time t with different noise levels.

curves except the last one vary more or less periodically as a function of time, especially in the case of the noise level $D = 1.4$ V [Fig. 4(b)], which seems to be quite good periodic and has almost equal amplitudes. Others corresponding to a lower [Fig. 4(a)] or a higher noise level [Fig. 4(c)]. They contain a certain degree of periodicity, but the amplitudes randomly fluctuate. The periodic variation of the function $\eta(t)$, on one hand, means that noise periodically excites wave fronts; on the other hand, it represents phase synchronization of the several wave sources on the boundary of the reaction medium. In other words, at an optimized noise level, the system synchronizes or organizes into a coherent pattern. This behavior is a typical stochastic resonance phenomenon in a spatial distributed system.

For an even strong noise (see the fourth plot of $D = 2.6$ V in Fig. 4), the function $\eta(t)$ has quite small amplitude and seems quite random. This is because the period of excited wave fronts becomes short ($T \approx 400$ s at $D = 1.7$ V), the next wave front emerges before the former disappears. Therefore, several wave fronts can coexist in the whole reaction medium [see Fig. 2(c)]. Furthermore, with strong noise the wave fronts are not excited randomly from the boundary; instead one wave source dominates the whole system. As a result, a spatially regular pattern forms and remains almost unvaried in the whole reac-

tion medium, and the phase synchronization is completely destroyed.

To understand why the most perfect synchronous phenomenon exists at an optimal noise level, we use a two-variable Oregonator model to simulate the experimental observations. In our system, the external electric noise can be simply considered as a spatially uniform electric field $E(t)$, which varies in time t with the form of Gaussian noise. The model is the following:

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon}\left(u - u^2 - fv\frac{u - q}{u + q}\right) + D_u\nabla^2 u + E(t)\nabla u,$$
(2)

$$\frac{\partial v}{\partial t} = u - v + D_v\nabla^2 v + E(t)\nabla v, \qquad (3)$$

where u and v represent, respectively, the concentration of $HBrO_2$ and the catalyst; D_u and D_v are diffusion coefficients; and f, q, and ε are parameters related to the BZ kinetics, which are chosen such that the system is in a subexcitable regime, so that the system does not support sustained wave propagation in the absence of noise. $E(t)$ satisfies $\langle E(t)\rangle = 0$ and $\langle E(t)E(t')\rangle = D_s\delta(t - t')$, a typical temporally varied Gaussian white noise. The effect of electric field is convective-like, as discussed in Ref. [22]. Equations (2) and (3) are numerically integrated using a Euler method with a time step of 1×10^{-3} time unit and a grid size of 0.15 space unit in an array of 350×100 points. Zero flux boundary conditions are considered.

The simulations have been done in two steps: First, at the left boundary we set a part of the grid of 3×90 points in the excited state of reaction, which serves as the wave source. Without noise, waves generated in the small area recede quickly when they travel into the subexcitable area. However, as noise is added, waves can propagate in the subexcitable medium, traveling one by one from the source in the left. As Pikovski introduced [5], we use the standard deviation of the temporal interval of the waves as a noise-signal ratio R_t:

$$R_t = \frac{\sqrt{\langle T^2\rangle - \langle T\rangle^2}}{\langle T\rangle}, \qquad (4)$$

where $\langle T\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}T_i$, N denotes the number of waves, and T_i is the time between the ith and $(i + 1)$th wave front. Figure 5(a) shows the noise-signal ratio R_t as a function of the noise strength D_s. One observes that, at an optimal noise level, the waves sent from the source have the most perfect periodical behavior.

In the next step, in order to observe the synchronization, two wave sources, each having 3×90 grid points, are set at the left and right boundary of the grid, respectively, so that two counterpropagating waves can be sent out. The traveling waves meet in the middle of the system and annihilate. The range of noise level is chosen such that the time intervals of traveling waves are large, so that there is
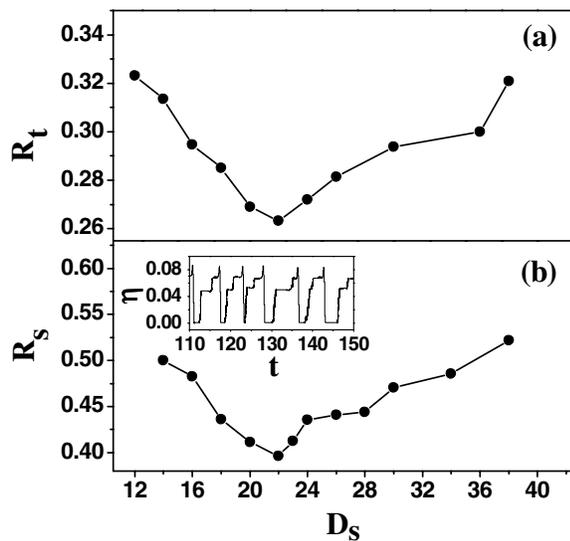
FIG. 5. The noise-signal ratios as functions of noise levels calculated in the simulations. (a) R_t as a function of D_s in the case of one wave source; (b) R_s as a function of D_s in the case of two wave sources. The inset in (b) shows η as a function of time in the second case. The parameters of simulation are $D_u = 1.0$, $D_v = 0.6$, $f = 2.435$, $q = 0.002$, $\varepsilon = 0.1$.

a duration after each wave annihilation while no waves are observed in the system [see the inset of Fig. 5(b)]. We pick the center part of the grid of 160×60 points to evaluate the standard deviation $\eta(t)$, defined as in Eq. (1). In this case, A_i represents the value of each point in the selected grid. The functions of $\eta(t)$ vary periodically. We notice that when the two counter–traveling waves meet each other, the value of η goes up higher, then descends fast to zero, corresponding to a homogeneous state [as shown the insert of Fig. 5(b)]. So that we can get the intervals T_i between two encounters in succession, and obtain a standard deviation R_s using Eq. (4). Figure 5(b) demonstrates the relation between R_s and D_s. Compared with the relation $R_t(D_s)$ in Fig. 5(a), $R_s(D_s)$ is also nonmonotonous, and it is interesting to find that the minimums of the two relations locate at the almost same positions, i.e., near $D_s = 22$. This simulation result means that at an optimal noise level, the excited waves with the most perfect periodicity show the best synchronous behavior.

In conclusion, we observe both in the experiments and in the numerical simulation that noise can induce and sustain wave propagations in a subexcitable medium, and the time interval between waves is more or less regular. Moreover, both investigations reveal that at an optimal noise level, wave fronts from different sources show a synchronous behavior. In numerical simulations, we know that the value of the optimal noise level for excited waves with the most perfect periodicity is coincident with the value for the best

synchronous behavior. This phenomenon leads us to propose that, at a same optimal noise level, the wave fronts excited from different sources show both the most regular and the best synchronous behavior, and the latter probably due to the long distance correlation induced by interaction between the external electric field and the nonlinear system.

We thank H. L. Swinney for providing us porous glass disks. This work is supported by the Chinese Natural Science Foundation.

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