Charged and Superconducting Vortices in Dense Quark Matter

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Quark matter at astrophysical densities may contain stable vortices due to the spontaneous breaking of hypercharge symmetry by kaon condensation. We argue that these vortices could be both charged and electrically superconducting. Current carrying loops (vortons) could be long-lived and play a role in the magnetic and transport properties of this matter. We provide a scenario for vorton formation in protoneutron stars.

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QCD with realistic quark masses possesses an exact U(1) symmetry for each flavor. Ignoring the three heaviest quarks, the relevant symmetry is $U(1)^3 = U(1)_B \times U(1)_Y \times U(1)_{em}$, where $U(1)_B$ is the baryon number, $U(1)_Y$ is hypercharge symmetry, and $U(1)_{em}$ is the electromagnetic gauge symmetry. These symmetries are not explicitly broken when a chemical potential for the baryon number is turned on. However, dense quark matter exhibits color superconductivity at sufficiently high density due to BCS-like pairing of quarks [1–5] (for reviews of recent progress, see [6–9]). The diquark condensate spontaneously breaks the $U(1)_B$ symmetry. Moreover, it has recently been argued that at baryon densities attainable inside compact stars, it is possible that $U(1)_Y$, $U(1)_{em}$, or both are spontaneously broken by kaon condensation [10–13]. Thus topologically stable vortices will exist in dense matter: global vortices due to the breaking of the baryon number, and possibly global vortices, gauged vortices, or both due to kaon condensation. In addition, there will be vortices bounding domain walls from the spontaneous breaking of the approximate $U(1)_A$ symmetry [14,15]. In this Letter we focus on global $U(1)_Y$ vortices resulting from K^0 condensation. While the existence of such vortices has been previously mentioned in [15,16], we are providing the first analysis of the peculiar properties of the cores of such vortices. In this Letter we argue that the K^0 vortices may, in fact, be electrically charged and superconducting, allowing for long-lived current loops (vortons [17,18]). These vortons may affect the electromagnetic and neutrino transport properties of dense matter. In addition, we exploit the properties of the phase diagram derived in [12] to provide a plausible mechanism for the formation of these vortons.

The key to the phenomena described here is the behavior of the strange quark in dense matter. The u and d quarks are very light, but the s quark is significantly heavier, with a mass similar to the nonperturbative QCD scale, Λ_{QCD}. For this reason, the strange quark does not play a big role in ordinary matter. However, as the baryon density of hadronic matter is increased, introducing strange quarks can lower the energy of the system, either in the form of hyperons or through Bose-Einstein condensation of antikaons (K^-) [19,20]. The important property of antikaons is that they are the lightest particles carrying the net strange quark number.

At very high matter density, one expects deconfinement to set in and the hadronic description of dense matter to give way to a quark matter description. For three massless quark flavors at sufficiently high density, the ground state of matter is expected to be SU(3) symmetric with equal numbers of u, d, and s quarks; it is also believed to exhibit chiral symmetry breaking due to a quark bilinear order parameter in the color flavor locked (CFL) configuration [21]. This results in a nonet of massless Goldstone bosons with quantum numbers identical to the π, K, $\bar{K}$, η, and η' mesons found in the vacuum.

When a strange quark mass is turned on, this puts a strain on the system, which would like to *reduce* its strange quark density relative to the SU(3) symmetric CFL ground state. For small enough strange quark mass, the stress cannot overcome the quark pairing energy [22]; however, for sufficiently large strange quark mass, kaons will condense, being the lightest excitation that can reduce the strange quark content of the ground state [11]. In Ref. [12] we used chiral perturbation theory to analyze the resulting phase diagram for charge neutral matter, allowing for the presence of leptons. We found that at realistic densities and lepton chemical potential $\mu_L = 0$, a K^0 condensate results; but as one increases μ_L, one passes through a second order phase transition to a mixed K^0/K^+ condensate, and then eventually through another second order phase transition to a pure K^+ condensate [23]. These phases have less exact symmetry than the QCD Lagrangian: The K^0 condensate breaks $U(1)_Y$ symmetry, the K^+ condensate breaks $U(1)_{em}$ symmetry, and the mixed phase breaks both. It follows that the three phases will contain stable vortices—global, gauged, or both. (The possibility of $U(1)_Y$ vortices and domain walls in the K^0 condensate has recently been considered in Refs. [15,16].)

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We focus on the global vortex found in the K^0 condensed phase at zero lepton chemical potential, as is relevant for mature pulsars. The starting point in the analysis of Ref. [12] is the chiral Lagrangian at fixed charge chemical potential μ_Q describing excitations about the SU(3) symmetric CFL vacuum, perturbing in the nonzero quark masses and electric couplings which explicitly break the chiral SU(3) $\times$ SU(3) symmetry:

$$\mathcal{L} = f_\pi^2\Big[\frac{1}{4}\mathrm{Tr}D_0\Sigma D_0\Sigma^\dagger - \frac{v^2}{4}\mathrm{Tr}\vec{\nabla}\Sigma\cdot\vec{\nabla}\Sigma^\dagger + \frac{a}{2}\mathrm{Tr}\tilde{M}(\Sigma + \Sigma^\dagger) + \frac{b}{2}\mathrm{Tr}Q\Sigma Q\Sigma^\dagger\Big], \quad (1)$$

where v is the medium meson velocity and

$$D_0\Sigma = \partial_0\Sigma + i(\mu_Q Q + X_L)\Sigma - i\Sigma(\mu_Q Q + X_R). \quad (2)$$

(Using the low energy theory about the false vacuum [the SU(3) symmetric CFL phase] to discover the true ground state is justified so long as the new phases are connected to the false vacuum by either second order phase transitions or first order phase transitions with sufficiently small latent heat and barrier height.) Here $\Sigma = \exp 2i\pi_a T_a/f_\pi$ is the SU(3) matrix [T_a are the SU(3) generators] describing the meson octet field π_a; for simplicity we have neglected the η'. The meson decay constant f_π has been computed in Refs. [24,25]. Q is the electric charge matrix diag(2/3, −1/3, −1/3), $\tilde{M} = |M|M^{-1}$ where M is the quark mass matrix diag(m_u, m_d, m_s), and $X_{L,R}$ are the Bedaque-Schäfer terms [11]:

$$X_L = -\frac{MM^\dagger}{2\mu}, \qquad X_R = -\frac{M^\dagger M}{2\mu}. \quad (3)$$

From this Lagrangian one derives the kaon masses and chemical potentials

$$m_{K^+}^2 = a(m_u + m_s)m_d + b, \qquad m_{K^0}^2 = a(m_d + m_s)m_u. \quad (4)$$

$$\tilde{\mu}_{K^+} = \mu_Q + \frac{m_s^2 - m_u^2}{2\mu}, \qquad \tilde{\mu}_{K^0} = \frac{m_s^2 - m_d^2}{2\mu}. \quad (5)$$

One can show that the free energy derived from this Lagrangian is minimized for the classical vacuum configuration $\Sigma(x,t) = e^{-i\tilde{\mu}t}\Sigma(x)e^{i\tilde{\mu}t}$, where

$$\tilde{\mu} = \mu_Q Q + X, \qquad X = X_L = X_R = -\frac{M^2}{2\mu}. \quad (6)$$

The time-independent field $\Sigma(x)$ minimizes the free energy

$$\Omega = \frac{f_\pi^2}{4}[\mathrm{Tr}[\tilde{\mu},\Sigma][\tilde{\mu},\Sigma^\dagger] + v^2\,\mathrm{Tr}\vec{\nabla}\Sigma\cdot\vec{\nabla}\Sigma^\dagger - 2a\mathrm{Tr}\tilde{M}(\Sigma + \Sigma^\dagger - 2) - b\mathrm{Tr}[Q,\Sigma][Q,\Sigma^\dagger]], \quad (7)$$

which is equivalent to requiring $\Sigma(x)$ to obey the equation of motion

$$[\tfrac{1}{2}\vec{\nabla}\cdot(\Sigma^\dagger\vec{\nabla}\Sigma) + \tilde{\mu}\Sigma^\dagger\tilde{\mu}\Sigma - a\tilde{M}\Sigma - bQ\Sigma^\dagger Q\Sigma] - \mathrm{H.c.} = 0. \quad (8)$$

In Ref. [12] we found that for $\mu_Q = 0$ the minimum free energy configuration was described by a spatially constant K^0 condensate with

$$\cos\phi_0 = \frac{M_{K^0}^2}{\mu_{K^0}^2}, \qquad \phi_0 \equiv \frac{\langle K^0\rangle}{\sqrt{2}f_\pi}, \quad (9)$$

for $\mu_{K^0} > M_{K^0}$. This latter condition seems to be met for reasonable parameters one might encounter in the core of a compact star. An assumption behind this conclusion is that instanton effects are suppressed; these would induce a Tr$M\Sigma$ operator [21], enhancing the kaon mass.

Since a nonzero K^0 field spontaneously breaks the exact U(1)$_Y$ symmetry of QCD, it follows that there must be topologically stable vortex solutions as well of the form

$$\frac{K^0(r,\theta)}{\sqrt{2}f_\pi} = \phi(r)e^{i\theta}, \qquad \text{with } \phi(0) = 0, \qquad \phi(\infty) = \phi_0. \quad (10)$$

The profile $\phi(r)$ can be computed numerically from Eq. (8), with the results for several values of vacuum condensate ϕ_0 shown in Fig. 1 as solid curves. In the absence of weak interactions, these vortices arise from the spontaneous breaking of an exact U(1)$_Y$ global symmetry, and so aligned vortex-antivortex pairs exert an attractive force that varies logarithmically with the separation, and an isolated vortex has a logarithmically divergent energy per unit length. Including the weak interactions which explicitly break U(1)$_Y$ symmetry leads to domain wall formation and a linear potential between vortex and antivortex pairs [16]. The scale associated with the transition from logarithmic to linear behavior in the potential has been estimated in Ref. [16] to be $\sim 10^3$ fm.

The K^0 condensate necessarily vanishes in the core of the vortex, and hence locally there is an equal density of u, d, and s strange quarks, in spite of the strange quark being massive. This suggests that the K^0 vortex might be unstable to the condensation of K^+ mesons in the core region, which would serve to reduce the strange quark number.

A crude estimate is useful for determining whether or not K^+ mesons might condense in the vortex core. Noting that the core size of the K^0 vortices shown in Fig. 1 is $r_0 \sim c\frac{v}{\mu_{K^0}}$ where $c = O(1)$, the energy cost of creating a single K^+ localized in the core is $\delta E \sim \sqrt{M_{K^+}^2 + \mu_{K^0}^2/c^2}$. An instability will set in if $\delta E < \mu_{K^+}$. For $\mu_Q = 0$, we have $\mu_K^+ \simeq \mu_{K^0} \simeq m_s^2/(2\mu)$. Furthermore, for densities relevant for compact stars and for a superconducting gap $\Delta \ll m_s$, one finds $M_K^+ \ll \mu_{K^0}$. Therefore, at least for Δ

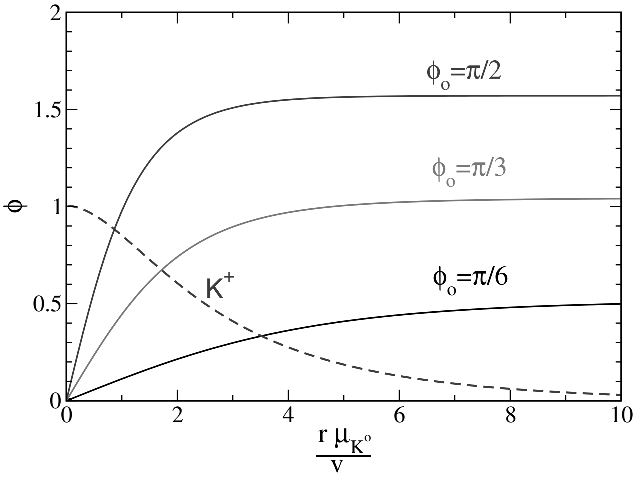

FIG. 1. The profile $\phi(r)$ of the global vortex present in the K^0 condensed phase of dense quark matter plotted versus radial distance rv/μ_K^0 for different values of the asymptotic condensate ϕ_0 (solid lines). The angle ϕ is related to the K^0 condensate amplitude by $\phi = \frac{|\langle K^0 \rangle|}{\sqrt{2} f_\pi}$, and $\cos\phi_0 = M_{K^0}^2/\mu_{K^0}^2$ [12]. Shown by the dashed line is the profile of the unstable K^+ mode which condenses in the vortex core for the case $\phi_0 = \pi/2$.

not too large, it appears that the existence of an instability depends primarily on the core size, parametrized by c.

Ideally one would like to solve the coupled nonlinear differential equation (8) to find such a K^+ condensate. Instead, we have investigated a simpler problem: for the case of $\phi_0 = \pi/2$ (corresponding to $M_{K^0} \ll \mu_{K^0}$) we have linearized the equation of motion for K^+ excitations about the unperturbed K^0 vortex background displayed in Fig. 1. We discovered that there indeed exists an unstable mode, whose spatial profile is indicated in Fig. 1 by the dashed line. The existence of this instability for $\phi_0 = \pi/2$ implies that at least for some range of ϕ_0, a K^0 vortex can lower its free energy by having a K^+ condensate form in its core.

The existence of a K^+ condensate in the core of the K^0 vortices could be of phenomenological interest, as it would mean that vortices would be both electrically charged (neutralized by electrons) and electrically superconducting, capable of carrying persistent currents. (The system described here is similar to the scalar model presented in Ref. [26] as an example of cosmological superconducting strings.) The current would take the form of an angular dependent phase in the K^+ condensate as one circumnavigates the K^0 loop. In the vacuum, a loop of global vortex would be expected to shrink and decay by radiating mesons. However, a loop of vortex in the presence of a Bose-Einstein condensate is kept from shrinking away by the velocity dependent Magnus force [27]; this behavior will persist even with the attractive linear potential due to the electroweak domain walls.

These loops, called vortons [17,18], are further stabilized if they carry electric currents. Since the current density carried by the K^+ core is $K^* i \overleftrightarrow{\nabla} K$, while the kinetic energy density is $|\nabla K|^2$, an estimate of the energy of a vorton of radius R carrying a current J yields $E \sim 2\pi(R\lambda + R^2\sigma + cJ^2/R)$, where λ is the energy per unit length of the K^0 vortex (including the charge screening electrons), σ is the energy per unit area of the electroweak domain wall stretched like a soap bubble across the loop, and c is a dimensionless number expected to be $\gtrsim$O(1). It follows that static vortons carrying classical currents are at least stable against shrinking, although we have not investigated the possibility of shape instabilities, the formation of cusps, and damping due to electromagnetic radiation [28].

There is a plausible mechanism for the formation of vortons during the creation of the protoneutron star. Initially, the protoneutron star is expected to have a sizable lepton number chemical potential μ_L due to neutrino trapping. As discussed in Ref. [12], there is a broad range of values of μ_L for which the ground state of superconducting quark matter spontaneously breaks the $U(1)_Y \times U(1)_{em}$ symmetry of QCD down to nothing via the mixed condensate of K^0 and K^+ mesons. In such a phase, both global K^0 vortices and gauged K^+ vortices are possible, and would be expected to form during the stellar core collapse. In particular, one may expect configurations to form where a K^0 vortex loop encloses net magnetic flux, carried by the K^+ vortices. As the neutrinos diffuse from the core, μ_L decreases, and as implied by the phase diagram in Ref. [12], matter goes through a second order phase transition to the pure K^0 condensed phase. When this occurs, the K^+ vortex will dissolve, and the bulk matter ceases to be electrically superconducting, except in the core of the K^0 vortex. Thus the K^0 vortex loop will persist with a trapped magnetic dipole field, and a concomitant current flowing in its core — a vorton.

The system we have described is quite complex, and more work will be required to show that the scenario described above could lead to a macroscopic density of charge and magnetic field carrying vortons within a young compact star interior. Since such a possibility obviously impacts bulk properties of the young star, such as its neutrino opacity and magnetic properties, we consider further work on this subject warranted.

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[1] B. C. Barrois, Nucl. Phys. **B129**, 390 (1977).
[2] D. Bailin and A. Love, Phys. Rep. **107**, 325 (1984).
[3] M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998).
[4] R. Rapp, T. Schafer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).

[5] D. T. Son, Phys. Rev. D **59**, 094019 (1999).
[6] M. G. Alford, hep-ph/0102047.
[7] K. Rajagopal and F. Wilczek, hep-ph/0011333.
[8] T. Schafer and E. V. Shuryak, nucl-th/0010049.
[9] D. H. Rischke and R. D. Pisarski, nucl-th/0004016.
[10] T. Schafer, Phys. Rev. Lett. **85**, 5531 (2000).
[11] P. F. Bedaque and T. Schafer, Nucl. Phys. **A697**, 802 (2002).
[12] D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042 (2002).
[13] T. Schafer, nucl-th/0109029.
[14] D. T. Son, M. A. Stephanov, and A. R. Zhitnitsky, Phys. Rev. Lett. **86**, 3955 (2001).
[15] M. M. Forbes and A. R. Zhitnitsky, hep-ph/0109173.
[16] D. T. Son, hep-ph/0108260.
[17] R. L. Davis and E. P. S. Shellard, Phys. Lett. B **207**, 404 (1988).
[18] R. L. Davis and E. P. S. Shellard, Nucl. Phys. **B323**, 209 (1989).
[19] D. B. Kaplan and A. E. Nelson, Phys. Lett. B **175**, 57 (1986).
[20] A. E. Nelson and D. B. Kaplan, Phys. Lett. B **192**, 193 (1987).
[21] M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
[22] M. G. Alford, J. Berges, and K. Rajagopal, Nucl. Phys. **B558**, 219 (1999).
[23] In Ref. [12] we considered only the possibility of a homogeneous mixed K^0/K^+ phase; one might expect, however, that a heterogenous phase would be preferred. That possibility is under investigation by the authors. We thank G. Moore for discussions on this subject.
[24] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000).
[25] D. T. Son and M. A. Stephanov, Phys. Rev. D **62**, 059902 (2000).
[26] E. Witten, Nucl. Phys. **B249**, 557 (1985).
[27] We thank D. T. Son for bringing the Magnus force to our attention.
[28] J. J. Blanco-Pillado, K. D. Olum, and A. Vilenkin, Phys. Rev. D **63**, 103513 (2001).