Density Modulations of Bose-Einstein Condensates via Laser-Induced Interactions

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We show that the dipole-dipole interatomic forces induced by an off-resonant running laser beam can lead to a self-bound pencil-shaped Bose condensate, even if the laser beam is a plane wave. For an appropriate laser intensity the ground state has a quasi-one-dimensional density modulation—a Bose-Einstein "supersolid."

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Electrostriction is the tendency of matter to become compressed in the presence of an electric field [1]. In an optically trapped atomic Bose-Einstein condensate (BEC) it is provided by the gradient of the incident electric field (one-body dipole forces). However, it can occur even if the external fields are *homogeneous*: the electrostriction effects of dipole-dipole interatomic forces, induced by certain configurations of far-off-resonant laser beams, are capable of creating a self-bound BEC within a wavelength of the laser (in the near zone) [2]. Thereby, a physical situation can be realized which has analogies to self-gravity [2,3].

Here we show that even in the simplest case of a *single plane-wave,* far-off-resonant laser beam, a BEC can be self-bound by electrostriction, due to the *retarded behavior* of the induced dipole-dipole interaction. By contrast, the purely static ($1/r^3$) dipole-dipole interaction does not allow for such self-binding [4]. The self-bound condensate is predicted here to be elongated and compressed (pencil shaped) along the axis of a circularly polarized laser beam, in principle even for *arbitrarily small* intensity. As the laser intensity increases, the electrostriction can give rise to a remarkable one-dimensional density modulation of the condensate in its ground state. Such a condensate bears a similarity to a "supersolid," i.e., a long-range crystalline-like density modulation imposed upon a superfluid by interparticle forces [5]. The formation of such structures is associated with a strong enhancement of the elastically scattered field, akin to collective ("superradiant") Rayleigh scattering [6,7], and with suppression of the heating due to spontaneous Rayleigh scattering.

We consider a far-off-resonant circularly polarized laser beam propagating along the positive $\hat{z}$ direction with wave vector q (Fig. 1, inset). The dipole-dipole induced interatomic potential [8] then becomes

$$V_{\rm dd}(\mathbf{r}) = \frac{\alpha^2 q^3 I}{8\pi c \varepsilon_0^2}\left[\frac{2z^2 - x^2 - y^2}{q^3 r^5}(\cos qr + qr \sin qr) - \frac{2z^2 + x^2 + y^2}{q r^3}\cos qr\right]\cos(qz), \tag{1}$$

where x, y, z are the components of the interatomic separation $\mathbf{r}$. The induced potential is thus proportional to the laser intensity I and to $\alpha^2(q)$, the squared atomic polarizability at the frequency $\omega = cq$, which scales as the inverse square of the detuning δ^2 from the nearest atomic resonance [2]. The constant prefactor in (1) is conveniently expressed as $\frac{3}{4}\hbar\Gamma_{\rm ray}$, where $\hbar\Gamma_{\rm ray} = \alpha^2 q^3 I/6\pi c\varepsilon_0^2$ is the single-atom rate of Rayleigh scattering. This rate and the corresponding saturation factor $s \propto \alpha^2 I$ must be kept sufficiently small, by choosing an appropriate detuning, as detailed below. The retarded oscillatory behavior of Eq. (1), for $qr \geq 1$, will be shown to cause the self-binding of a BEC, as opposed to its static $1/r^3$ limit, obtained for $qr \ll 1$ [4].

We assume in the following that the condensate contains many atoms per cubic wavelength, so as to ensure the validity of a mean-field description of a zero-temperature BEC with induced dipole-dipole forces [2,4]. Such a description can be accomplished through the Gross-Pitaevskii equation [9] for the condensate order parameter $\Psi(\mathbf{r}, t)$

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{\delta}{\delta\Psi^*}H_{\rm tot}\,. \tag{2}$$

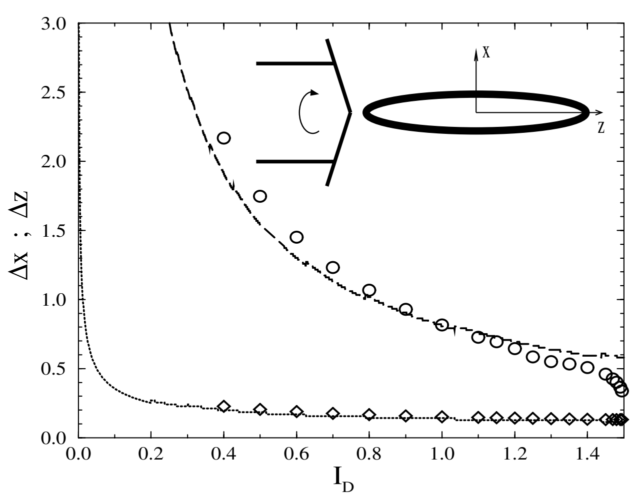

FIG. 1. Inset—the laser beam and condensate geometry. RMS radii Δx and Δz (normalized to λ) of the self-bound condensate versus the parameter I_D (intensity I normalized to $8\pi\hbar^2 c\epsilon_0^2 a/m\alpha^2$) in the TF limit. Circles (diamonds) represent the values of Δz (Δx) calculated from the quasi-1D ansatz Eq. (8). Long-dashed (dotted) lines represent the values of Δz (Δx) obtained from a Gaussian ansatz in all of the coordinates.

Here the total mean-field energy functional

$$H_{\mathrm{tot}} = H_{\mathrm{kin}} + H_{\mathrm{ho}} + H_{\mathrm{dd}} + H_{\mathrm{scat}} \tag{3}$$

consists of the following: (a) the kinetic energy

$$H_{\mathrm{kin}} = \int (\hbar^2/2m)\,|\nabla\Psi|^2\,d\mathbf{r}\,, \tag{4}$$

where m is the atomic mass; (b) the harmonic-trap energy (later shown to be redundant under certain conditions)

$$H_{\mathrm{ho}} = \int V_{\mathrm{ho}}|\Psi|^2\,d\mathbf{r}\,, \tag{5}$$

where $V_{\mathrm{ho}} = m\omega_r^2(x^2 + y^2)/2 + m\omega_z^2 z^2/2$, is a cylindrically symmetric harmonic potential that can be caused by the focusing of the laser beam; (c) the mean-field energy due to the short-range (s-wave) scattering

$$H_{\mathrm{scat}} = (2\pi a\hbar^2/m)\int |\Psi|^4\,d\mathbf{r}\,, \tag{6}$$

where a is a *positive* s-wave scattering length; and (d) the electromagnetically induced dipole-dipole mean-field energy

$$H_{\mathrm{dd}} = (1/2)\int d\mathbf{r}\,d\mathbf{r}'\,V_{\mathrm{dd}}(\mathbf{r} - \mathbf{r}')\,|\Psi(\mathbf{r})|^2|\Psi(\mathbf{r}')|^2, \tag{7}$$

which corresponds to the electrostriction energy of the medium in the plane-wave field, with V_{dd} expressed by Eq. (1).

The static near-zone ($qr \ll 1$) limit of the dipole-dipole potential (1) is positive (repulsive) along the z axis and negative in the radial direction, thereby precluding stable binding. By contrast, the far-zone (retarded) behavior of this potential $\propto -\cos(qz)^2/qz$ is *negative* (attractive) for any z, provided that the atoms are aligned along the z axis. Nonaligned atoms feel less effective attraction, or even repulsion, at far zone separations. Hence, an extended cloud of atoms subject to interaction (1) is expected to have a cigar-shaped ground state. We therefore adopt the following cylindrically symmetric (about the axial $\hat{z}$ direction) variational ansatz for the macroscopic order parameter

$$\Psi(\mathbf{r}) = \psi(z)\exp[-(x^2 + y^2)/2w_r^2]/\pi^{1/2}w_r\,, \tag{8}$$

where w_r is the variational radial width. It is reasonable to approximate the radial profile of Ψ by a Gaussian when its width w_r is less than half the laser wavelength $\lambda = 2\pi/q$; in which case the oscillations of (1) are not manifest along x or y. We then obtain $\psi(z)$, as well as w_r, by numerically minimizing the mean-field energy H_{tot} expressed by Eqs. (3)–(7).

The numerical solutions for the ground state, based on (8), reveal the existence of a *self-bound* condensate, without external confinement ($H_{\mathrm{ho}} = 0$). Such solutions are obtained in the Thomas-Fermi (TF) limit of negligible kinetic energy $H_{\mathrm{kin}} \ll H_{\mathrm{scat}}$, which characterizes condensates with many atoms N, as specified below. Then, upon minimizing $H_{\mathrm{tot}} = H_{\mathrm{dd}} + H_{\mathrm{scat}}$, we obtain a negative value of its expectation value (bound state) (plotted in Fig. 3a) and, concurrently, finite mean quadratic radii $(\Delta x)^2 \equiv \langle x^2\rangle = w_r^2/2$ and $(\Delta z)^2 \equiv \langle z^2\rangle$ (plotted in Fig. 1), in the presence of a plane-wave laser. We have verified that the *static* form of $V_{dd} \propto 1/r^3$ does not allow for variational bound-state solutions.

The control parameter that allows one to play with the ratio of the induced dipole-dipole forces to the s-wave scattering is the dimensionless "intensity"

$$I_D = \frac{I\alpha^2 m}{8\pi\epsilon_0^2 c\hbar^2 a}\,. \tag{9}$$

The laser intensities corresponding to self-binding in Fig. 1 are *below* the threshold of the instability caused by the static $1/r^3$ part of the dipole-dipole potential (1) [4], $I \leq 12\pi\hbar^2 c\epsilon_0^2 a/m\alpha^2 (I_D \leq 3/2)$. Hence, the required intensity can be, in principle, arbitrarily small for any positive scattering length a.

Using a Gaussian ansatz for $\psi(z)$ in (8), we obtain analytical approximations for the radii of the condensate, normalized to the laser wavelength λ, in the limit $I_D \ll 1$: $\Delta x = 0.1125 I_D^{-1/2}$, and $\Delta z = 0.7847 I_D^{-1}$. For $I_D > 0.1$ the condensate is strongly confined in the radial direction, with $\Delta x \sim 0.2$, and less confined in the longitudinal direction, with a typical size larger than or comparable to the wavelength $\Delta z > 0.6$.

The oscillatory long-range behavior of the potential (1) is manifest along the $\hat{z}$ axis, due to the large extension of the condensate in this direction. To gain more insight into this behavior, we introduce the $1D$-reduced form of the electromagnetically induced mean-field energy $H_{dd}^{1D} = (1/2)\int dz\,dz'\,V_{dd}^{1D}(z - z')\,|\psi(z)|^2|\psi(z')|^2$, expressed through the $1D$-reduced dipole-dipole potential

$$V_{dd}^{1D}(z) = \int dx\,dy\,\frac{\exp[-(x^2 + y^2)/2w_r^2]}{2\pi w_r^2}\,V_{\mathrm{dd}}(\mathbf{r})\,. \tag{10}$$

This $1D$-reduced interaction contains an *attractive* singular part $V_{dd}^{1D}(z)|_{\mathrm{sing}} = -(\hbar\Gamma_{\mathrm{ray}}/q^3 w_r^2)\delta(z)$, which arises from the interplay of $\frac{-(x^2+y^2)}{q^3r^5}$ and $\frac{2z^2}{q^3r^5}$ in the static limit of Eq. (1). For $I_D < 1.5$ this attractive singular part is balanced by a similar repulsive term $V_{\mathrm{scat}}^{1D}(z) = (2a\hbar^2/mw_r^2)\delta(z)$ which arises from the (positive) s-wave scattering length, thereby stabilizing the condensate. For $I_D > 1.5$ the system is therefore unstable because of the static part of the $1/r^3$. Figure 2a shows the nonsingular part of V_{dd}^{1D} for three different values of the variational parameter w_r. The near-zone $1/z^3$ on-axis repulsion of the potential (1) is evident in the dashed curve (for small $w_r \leq 0.1$), as opposed to the near-zone attraction (solid line) sufficiently far off axis (for $0.2 \geq w_r \geq 0.5$). For large values of z the $1D$-reduced potential in Fig. 2a oscillates as $V_{dd}^{1D}(z) \approx -(3\hbar\Gamma_{\mathrm{ray}}/2q)\cos(qz)^2/z$, but remains *attractive on average,* indicating a macroscopic electrostrictive force. Its Fourier transform (Fig. 2b) exhibits

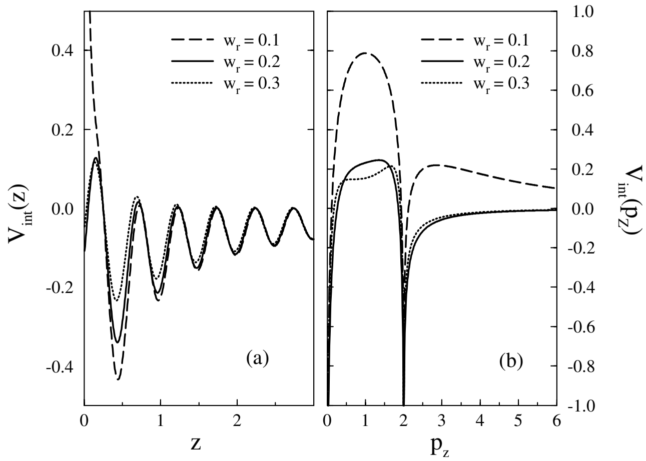

FIG. 2. (a) The 1D-reduced dipole-dipole interaction, in units of $\hbar\Gamma_{\rm ray}$, as a function of z (normalized to λ), and (b) its Fourier transform as a function of p_z (normalized to q). The dashed, solid, and dotted lines correspond to increasing values of the variational radial width (normalized to λ) $w_r = 0.1$, 0.2, and 0.3, respectively.

attractive logarithmic singularities at $p_z = 0$, associated with the average attraction, and at $p_z = \pm 2q$, which can be attributed to the interference of the incident and backscattered laser waves.

A conspicuous feature in the TF limit of negligible $H_{\rm kin}$ is the abrupt modulation of the density along z, namely the formation of *isolated condensate droplets* of size less than $\lambda/4$ at approximately $\lambda/2$ separations. The corresponding variational values for the ground-state energies, $E_{\rm tot}$, are *substantially below* the ones obtained by a Gaussian ansatz for $\psi(z)$ in (8), as displayed in Fig. 3a. This indicates that such longitudinal density modulation further stabilizes the condensate, which may seem counterintuitive. This density modulation is due to the interference of the backscattered and the incident fields, which creates a series of attractive traps at $\lambda/2$ separations (Fig. 2a).

The changes in the density profile as the kinetic energy becomes non-negligible can be discussed in terms of the dimensionless parameter that scales with the number of atoms N and the s-wave scattering length a,

$$\eta = Na/\lambda_L . \qquad (11)$$

This parameter is approximately the ratio between $E_{\rm tot}/N$, the TF ground-state energy per particle for $I_D = 1$, and the recoil energy $E_R = \hbar^2 q^2/2m$. The TF limit corresponds to η large. As soon as the radial part of the kinetic energy $(\hbar^2/2m)\int d\mathbf{r}\,\Psi(\partial^2/\partial x^2 + \partial^2/\partial y^2)\Psi \sim NE_R I_D$ becomes comparable to the TF mean-field energy, for $\eta I_D \sim 1$, both Δx and Δz strongly increase with respect to their TF-limit counterparts. This "bulging" of the cigar-shaped condensate is due to the outward radial pressure associated with the kinetic energy.

This pressure can be compensated, in the limit $\eta I_D \lesssim 1$, by adding external radial confinement, e.g., by an appropriate choice of the focus of the laser beam. Figure 3b shows the longitudinal density profile upon fixing the radial width w_r while decreasing η from its TF limit ($\eta > 100$), corresponding to tightly bound (isolated) droplets (solid curve), down to $\eta \ll 1$ (dashed curve), where kinetic energy effects dominate. Remarkably, in the range $1 \lesssim \eta I_D^2 \lesssim 10$ (dot-dashed curve), the single condensate droplets overlap, creating new long-range ordered density modulation—a Bose "supersolid" [5] is formed. In this novel regime phase coherence is expected to arise between the overlapping droplets that are distributed among the wells. The density oscillations are washed out as soon as $\eta I_D^2 \lesssim 1$ (dashed curve in Fig. 3b), i.e., when the z component of the kinetic energy ($\sim NE_R$), exceeds the energy reduction caused by the TF density modulation (approximately the difference between the diamonds and solid line in Fig. 3a).

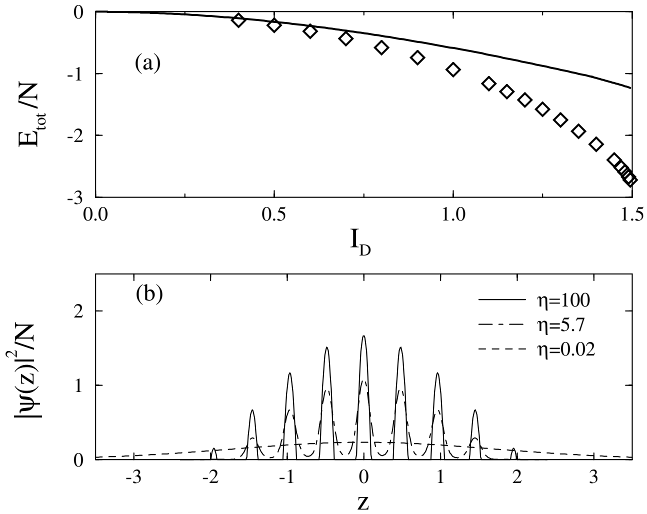

FIG. 3. (a) TF mean-field energies per particle (in units of NaE_R/λ) versus I_D as obtained from the ansatz Eq. (8) (diamonds) and from the Gaussian ansatz (solid line). (b) Kinetic energy effects: longitudinal equilibrium densities (in units of N) as a function of z/λ for different values of η at $I_D = 1$. A radial external confinement is used to keep the radial size of the condensate equal to that obtained for $\eta \gg 1$ without confinement.

Incoherent Rayleigh scattering from the far-off-resonant laser beam leads to heating and depletes the condensate. The rate of incoherent scattering, which is $N\Gamma_{\rm ray}$ for N noninteracting atoms is suppressed by the interatomic interaction [10]. However, the corresponding rate of change of the total energy is still given by the simple universal expression [11] $\frac{d}{dt}(E_{\rm tot}) = 2E_R N\Gamma_{\rm ray}$, which is *independent of the interaction properties* and corresponds to N times the single-atom rate of energy change $2E_R\Gamma_{\rm ray}$. From the rate of change of the ground-state energy we can estimate the effective "evaporation" or "heating" time of the self-bound atomic cloud to be $\tau_{\rm heat} = |E_{\rm tot}|/(dE_{\rm tot}/dt) \sim \eta I_D^2 \Gamma_{\rm ray}^{-1}$, where we used Eq. (11) and our variational ansatz for the TF limit $\eta I_D^2 \gg 1$.

This time scale could be compared with the corresponding values for the radial and longitudinal oscillation frequencies of a self-bound condensate. We estimate the radial frequency to be [12] $\Omega_r \sim (E_R/\hbar) I_D^{3/2} \eta^{1/2}$. The fact that $\tau_{\rm heat}\Omega_r \sim (E_R/\hbar\Gamma_{\rm ray})(\eta I_D^2)^{3/2}$ can be much larger than 1 in the limit $\eta I_D^2 \gg 1$ proves that Rayleigh heating does not preclude the equilibration of the self-bound condensate, which typically requires several oscillation periods.

For the density modulations discussed above, most of the photons are *nearly elastically* and *coherently* scattered by the center of mass of the condensate. Their total scattering cross section is given by $N^2 f_{\rm cm}$ times the single-atom cross section, where the center-of-mass fraction $f_{\rm cm}$ is defined as an appropriate average over all the possible directions of scattering (in the Born approximation)

$$f_{\rm cm} = \frac{3}{2N^2} \int_{-1}^{1} d[\cos(\theta)][1 + \cos(\theta)^2]|n(\mathbf{p})|^2. \quad (12)$$

Here $n(\mathbf{p})$ is the Fourier component of atomic density corresponding to $\mathbf{p} = 2q[\sin(\theta)\hat{x}, 0, \cos(\theta)\hat{z}]$, the momentum transferred by a single photon in the x-z plane and θ is the angle between the incident beam and the scattered direction. Only the (complementary) fraction of the rate of energy change $(1 - f_{\rm cm})\frac{d}{dt}(E_{\rm tot})$, contributes to the inelastic part of the scattering cross section, whence $f_{\rm cm}$ provides a partial suppression of heating.

The center of mass of the condensate is therefore subject to a constant *radiation force* that is enhanced by a factor $Nf_{\rm cm}$ by the density modulation, associated with the coherent back scattering (diffraction) of the electromagnetic field. This radiation force can either shift the equilibrium position of the condensate, if it is located in a longitudinal trap, or else accelerate it uniformly. In the latter case the scattered light will be slightly Doppler shifted. The effects described here are due to the *same* matter-field interactions as those responsible for "superradiant" Rayleigh scattering [6] and collective atomic recoil (CARL) [7]. However, the essence of our effects is the electrostrictive change of the *atomic* energy and density, unaccounted for thus far.

A central prediction of this paper is that induced dipole-dipole forces result in a density modulation for the condensate *ground state*. Density modulations in a condensate can also arise from presence of phonon excitations, as demonstrated by Inouye *et al.* [6] in the case of superradiant Rayleigh scattering. Traveling phonons can be distinguished from the ground-state density modulation proposed here by diffracting a nearly perpendicular probe laser whose z component of the wave vector is $k_z \approx 2q$. Phonons at frequency $\Omega = v_s 2q$, where v_s is the sound velocity, lead to a density modulation of the form $\cos(2qz - \Omega t)$, so that the first diffraction orders (i.e., Brillouin peaks) of the probe beam will be frequency shifted by $\pm\Omega$. By contrast, diffraction from the ground-state density modulation discussed above will be *elastic,* with no frequency shift.

An example of the experimental conditions required for the predicted effects involves $N \sim 10^3$ sodium atoms and a circularly polarized laser beam, red detuned by 1.7 GHz from the $3S_{1/2}$ $(F = 1) \rightarrow 3P_{3/2}$ $(F = 0, 1, 2)$ transition, for which $E_R/h = 25$ kHz. The threshold for $1/r^3$ instability is then at $I \approx 525$ mW/cm^2, correspondingly, $\Gamma_{\rm ray} = 2.9$ kHz. Below this intensity, one can observe the self-binding, the density modulation, and the acceleration of the center of mass $\approx 500 N f_{\rm cm}$ [m/s^2], with $f_{\rm cm} \sim 0.1$ for $I_D \sim 1$ and $f_{\rm cm} \sim 1$ for $I_D \ll 1$.

To conclude, we have demonstrated a new quasi-one-dimensional regime of self-confined and self-organized ground-state density modulations in a BEC illuminated by a single, circularly polarized laser beam in the weak-saturation limit. This regime is inherently possible even for a plane-wave laser, although it is facilitated by the radial focusing of the beam.

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