Revalidation of the Isobaric Multiplet Mass Equation

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We have determined the energy of the $J^\pi = 1/2^+$, $T = 3/2$ resonance in ^{32}S(p,p) to be $E_p = 3374.7 \pm 0.8$ keV. This disagrees with the previously accepted value of $E_p = 3370 \pm 1$ keV by Abbondanno *et al.* [Nuovo Cimento **70A**, 391 (1970)] and solves a problem raised by recent observations of unexpected deviations from the isobaric multiplet mass equation. This resonance is also important in calibrating the β-delayed proton spectra from ^{33}Ar and ^{32}Ar, and our findings may modify previous conclusions.

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Wigner [1] and Weinberg and Treiman [2] noted that the masses of the members of an isospin multiplet should be related by

$$M(T_z) = a + bT_z + cT_z^2, \tag{1}$$

where $T_z = (N - Z)/2$, provided that the charge-dependent part of the nucleon-nucleon interaction can be described at tree level and no isotensor of rank greater than 2 arises from the potential. Equation (1) is known as the isobaric multiplet mass equation (IMME). Deviations from it can be expected in systems where two states of different isospin lie very close to each other, producing large perturbations, or when the binding energy of the proton-rich member is negative, leading to largely differing wave functions among the multiplet [3]. In the 1960–1980s interest arose in looking for potential deviations from the IMME that would indicate the presence of higher-than-rank-two (i.e., non-Coulombic) interactions and several calculations were performed [4,5] to show that, in the absence of these, the next term in Eq. (1), dT_z^3, should be such that $|d| < 1$ keV. Reviews on the subject can be found in Refs. [6,7]. Recently Herfurth *et al.* [8] measured the mass of ^{33}Ar ($\tau \approx 174$ ms) with an uncertainty of $\approx$4 keV and used the known masses of the other members of the $T = 3/2$ quartet [9] to conclude that $d = -2.95 \pm 0.90$ keV. Such a large value for $|d|$, in a system where the assumptions that lead to Eq. (1) are satisfied, was surprising. Although the state in ^{33}Cl is unbound and thus coupled to the continuum, the coupling is very weak because it breaks isospin symmetry.

The masses of the lowest $T = 3/2$ states in ^{33}P and ^{33}S have been measured by more than a single group and show mutual agreement [8]. However, as shown in Table I the measurements of the energy of the lowest $T = 3/2$ resonance in ^{32}S(p,p), from where the mass of the state in ^{33}Cl is deduced, do not exhibit such nice agreement. The average of Refs. [11,12], $E_p = 3374.7 \pm 1.7$ keV, places the resonance at a higher energy than Ref. [10]. Both Ref. [11] and Ref. [12] discuss in detail how the small uncertainty in beam energy was achieved, while we found no discussion in Ref. [10].

Because the $J^\pi = 1/2^+$, $T = 3/2$ state in ^{33}Cl is fed strongly in the decay of ^{33}Ar and the energy uncertainty is the smallest of all known resonances in ^{32}S(p,p) it plays a central role in the calibration of the β-delayed proton spectrum. This line was also used in the calibration of β-delayed proton spectra from ^{31}Ar [13,14] and ^{32}Ar [15–17] because, at ISOL (on-line radioactive isotope separator) facilities, the ^{33}Ar beam is much more intense and can be simply selected by switching the electromagnetic filter. The case of ^{32}Ar is particularly important because the $0^+ \rightarrow 0^+$, $T = 2$ superallowed transition was used to determine limits on possible scalar contributions to the weak interaction. That experiment showed, in addition to the proton group from the $T = 2$ state leaving ^{31}S in its ground state, another proton peak appearing at an energy close to where emission from the $T = 2$ state leaving ^{31}S in its first excited state should lie. Later, using data from a recent experiment at Michigan State University [18], we confirmed that this proton group coincided with γ emission from the first excited state of ^{31}S. However, the proton energy calibration (based on the decays from ^{33}Ar) indicated that the energy difference between the proton groups was $\approx$5 keV lower than expected. This prompted us to look into this issue more carefully. The other lines in ^{33}Cl that are known precisely and used for energy calibrations, at $3.9 < E_x < 4.5$ MeV, have been determined by (p,γ) experiments [19] and are less likely to be affected by systematic errors.

The experiment was performed at the University of Notre Dame's FN-model Tandem accelerator. The $\approx$5 μA

TABLE I. Energy of the lowest $T = 3/2$ state in ^{33}Cl.

E_p (keV)	Ref.
3370 ± 1	[10]
3375 ± 3	[11]
3374.5 ± 2.0	[12]

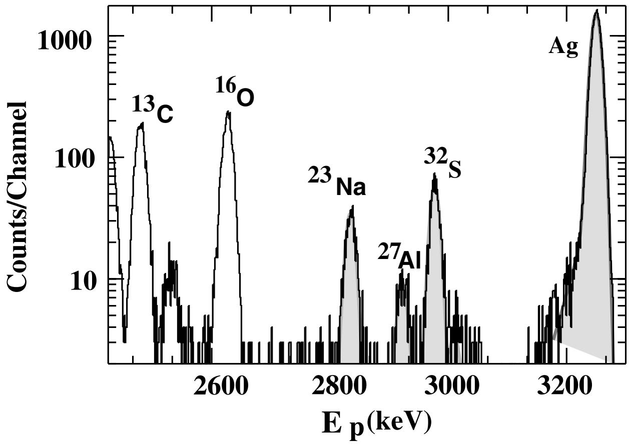

FIG. 1. Spectrum of scattered protons at $\theta \approx 165°$ and fits (shaded).

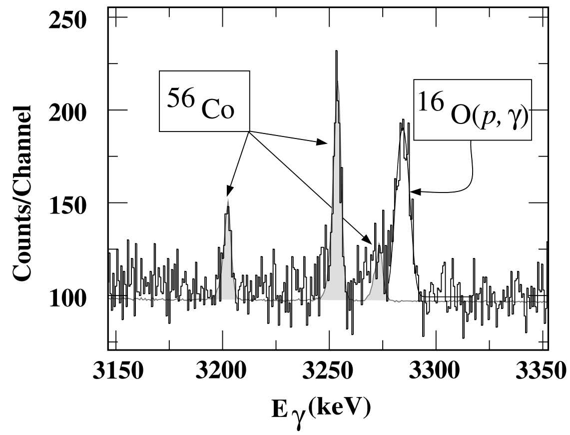

FIG. 2. Ge spectrum. We show the $^{16}O(p,\gamma)$ data and our Monte Carlo calculation. Overlaid is also the spectrum taken with a ^{56}Co source and renormalized (shaded).

proton beam was tuned through the Tandem and 90° analyzing magnet with both object and image horizontal slits set to a total gap of $\approx$1 mm. A switching magnet was then used to send the beam to either a scattering chamber, where we took the excitation data showing the resonances, or a beam line set up to look at γ rays with a Ge detector. The scattering chamber contained six surface barrier detectors located at $\pm 90°$, $\pm 130°$, and $\pm 165°$ with respect to the incident proton beam. The detectors were placed symmetrically to the beam to minimize sensitivity to the exact position of the beam on the target. We produced thick ($\approx$18 $\mu g/cm^2$) and thin ($\approx$2 $\mu g/cm^2$) Ag_2S targets by evaporation on $\approx$20 $\mu g/cm^2$ C foils. Figure 1 shows a spectrum taken at $\theta \approx 165°$. In order to minimize uncertainties due to hysteresis effects in the analyzing magnet, we biased the target ladder from -1 to $+5$ kV to produce our excitation functions, rather than changing the beam energy [20]. Approximately 7 keV higher than the $s\frac{1}{2}$, $T = 3/2$ ($\Gamma \approx 0.1$ keV) resonance lies a $f\frac{7}{2}$, $T = 1/2$ ($\Gamma \approx 1$ keV) resonance [9,10] which is much more noticeable than the resonance of interest given our beam energy resolution of $\approx$1 keV. So we fixed our analyzing magnetic field to yield protons approximately 2 keV higher than the $f\frac{7}{2}$, $T = 1/2$ resonance, found the resonance only using bias in the target ladder, and then switched the beam to measure its energy via $^{16}O(p,\gamma)$.

For the $^{16}O(p,\gamma)$ direct-capture measurement [21] we firmly attached a Ta_2O_5 target to a water-cooled copper target ladder. The target was prepared by anodizing Ta and its thickness was determined using the well-known relationship between target thickness and voltage drop [22] to be $\Delta x = 160 \pm 16$ $\mu g/cm^2$ (beam energy loss of $\Delta E_p = 8.0 \pm 0.8$ keV through the target). A 70% intrinsic Ge detector was placed at 90° with respect to the beam. Immediately before the $^{16}O(p,\gamma)$ measurement, we prepared a source of ^{56}Co ($t_{1/2} \approx 77$ d) by bombarding a Fe foil. Small amounts of ^{56}Co remained in the back of the target ladder, so after removing the main source we could see weak lines from ^{56}Co which were used as a built-in calibration. Figure 2 shows the γ spectrum taken during 11 h of bombardment. We produced a Monte Carlo calculation that took into account the beam energy resolution, the target thickness, the Doppler effect in the Ge detector given its position, and the Ge resolution. We renormalized the Monte Carlo calculation with an arbitrary amplitude, shifted it in energy by an arbitrary amount, and added to it an arbitrary background. These three parameters were then varied to minimize χ^2 with respect to the data. Figure 3 shows the χ^2 plots vs E_p for the different assumptions. The statistical error (taken as the value of the shift that yields $\chi^2 = \chi^2_{min} \pm 1$ is $\approx \pm 0.2$ keV, but the minimum is sensitive to inputs to the Monte Carlo calculation. The result of the best fit using our standard measured conditions is shown in Fig. 2. Table II presents a list of systematic uncertainties. Because the target surface is locally heated by the beam it is known that ions can accumulate on the target creating a layer that would produce additional energy loss. Our fits to the $^{16}O(p,\gamma)$ data yield indeed the best fit

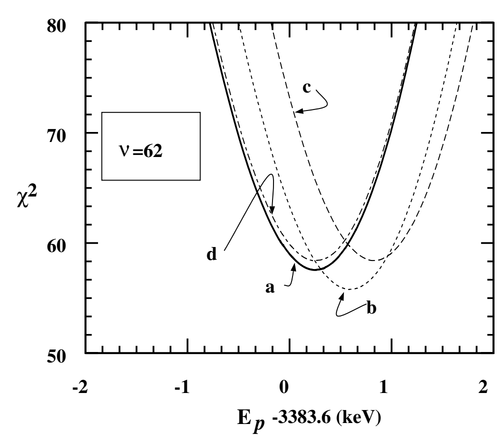

FIG. 3. Plot of χ^2 vs $E_p - 3383.6$ keV under different assumptions. Degrees of freedom, $\nu = 62$. (a) Parameters set to measured values; (b) target thickness increased by 10%; (c) Ge angle changed by $-2°$; (d) Ge distance increased by 1 cm.

TABLE II. Systematic uncertainties associated with determining the beam energy.

Origin	Uncertainty	ΔE_p (keV)
Ge distance	± 1 cm	$\leq \pm 0.1$
Ge angle	$\pm 2°$	∓ 0.6
Ge tilt	$\pm 2°$	$\leq \pm 0.1$
Target thickness	$\pm 10\%$	± 0.38
Carbon buildup	0.25 ± 0.25 keV	± 0.25

for a "target thickness" $\approx 10\%$ larger than the one we determined in the target preparation which could be caused by a continuous buildup during the 11 h of $^{16}O(p,\gamma)$ measurement. However, analyzing five (each ≈ 2 h long) subdivisions of the total run yields no time-dependent shift. Using this argument we put an upper limit of ≈ 0.5 keV to the energy lost in the built-up layer. This yields a beam energy of $E_p = 3384.1 \pm 0.8$ keV.

Once we had determined the beam energy we sent the beam back to the scattering chamber where we measured excitation functions covering both resonances. Because our target bias allowed scanning only 6 keV, we turned the beam energy down by ≈ 5 keV and verified overlap of points taken with different beam energies but identical effective energies. Figure 4 shows the excitation functions measured in the scattering chamber together with our fits. In a complementary experiment [23] we measured cross sections and analyzing powers from $E_p = 3.2$ to 3.7 MeV. Those measurements allowed determination of the background and neighboring resonance parameters so we were able to fix these and use the present data to fit the energies of both resonances. The fits were performed following the formalism of Refs. [24–26] as described in Ref. [27]. In order to disentangle the different contributions to the energy resolution in our excitation functions we took data with two different Ag_2S targets: one 9.3 times thinner than the other one (as measured by the yields in a given detector). We then produced simultaneous fits assuming the same beam energy resolution and target thicknesses with the measured ratios. The best fit yielded a beam energy resolution of ≈ 0.60 keV [28]. We measured the built-up layer by moving the target to a fresh spot after 8 h of bombardment and taking four additional points around the $f\frac{7}{2}$ resonance.

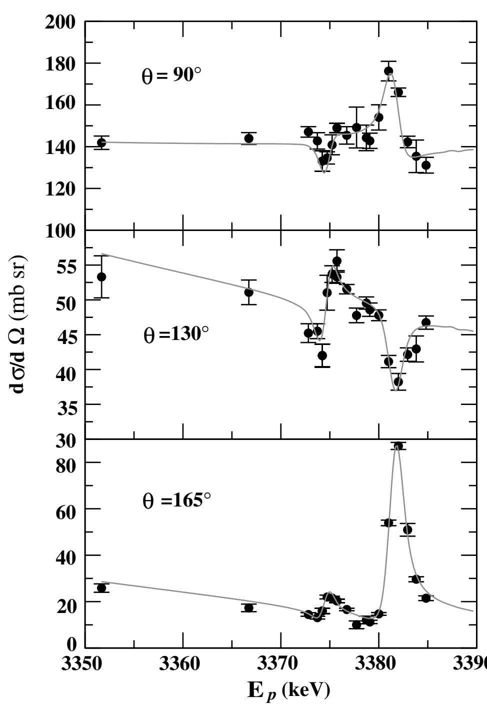

FIG. 4. Excitation functions measured at $\theta = 90°$, 130°, and 165° with our fits.

Our fits yield the energies of the resonances in $^{32}S(p,p)$ to be $E_p(s\frac{1}{2}, T = 3/2) = 3374.7 \pm 0.8$ keV and $E_p(f\frac{7}{2}, T = 1/2) = 3381.5 \pm 0.8$ keV. Although we agree with Ref. [10] on the energy difference between the two resonances, we strongly disagree on the absolute value of the energies. Our result implies a center-of-mass energy of $E_{CM}(s\frac{1}{2}, T = 3/2) = 3271.4 \pm 0.8$ keV and excitation energy of $E_x(s\frac{1}{2}, T = 3/2) = 5547.9 \pm 0.8$ keV, which yields a mass excess of $M(^{33}Cl, T = 3/2) = -15\,455.6 \pm 0.8$ keV. Table III presents the results of fitting the masses of all members of the quartet and shows that the deviation from the IMME observed in Ref. [8] vanishes when using our new measurement.

Our measurement has other consequences. Previous measurements of the β-delayed proton spectrum from ^{33}Ar concluded that there were many states populated in the β decay that did not have corresponding $^{32}S(p,p)$ resonances [15]. For example, a state at $E_x \approx 6247$ keV [labeled as ($\ell = 1$) so not expected to be populated in allowed β decay] was supposed to be fed in β decay and to emit a proton leaving ^{32}S in its ground state while "another" at $E_x \approx 6257$ keV would emit a proton leaving ^{32}S in its first excited state. However the latter had been observed as a resonance in *elastic* scattering [11], so its decays to the ground state of ^{32}S should have been present in the β-delayed proton spectrum but were not. Our present results change the energy calibration in such a way that both excitation energies coincide and the apparent contradictions disappear. A detailed analysis of

TABLE III. Comparison of the measured mass excesses of the lowest $T = 3/2$ quintet in $A = 33$ to predictions of the isospin-multiplet mass equation $[P(\chi^2, \nu) = 0.54]$.

Isobar	T_3	$M_{\rm exp}$ (keV)[a]	$M_{\rm IMME}$ (keV)
^{33}P	$+3/2$	$-26\,337.7 \pm 1.1$	$-26\,337.6 \pm 1.1$
^{33}S	$+1/2$	$-21\,106.54 \pm 0.15$[b]	$-21\,106.55 \pm 0.15$
^{33}Cl	$-1/2$	$-15\,455.6 \pm 0.8$[c]	$-15\,455.41 \pm 0.63$
^{33}Ar	$-3/2$	-9381.9 ± 4.2[d]	-9384.1 ± 2.1

[a]Unless noted otherwise, ground state masses are from Ref. [9]. [b]Excitation energy from Ref. [29]. [c]From $^{32}S(p,p)$ resonance energy (this work). [d]Reference [8].

this and other issues will be presented later. Our result also solves the discrepancy between the energies of the proton groups corresponding to decays from the $T = 2$ state in ^{32}Cl following the β decay of ^{32}Ar: using the energy calibration derived from this work the excitation energies of the two groups agree to within $\lesssim 1$ keV.

The masses of the lowest $T = 2$ quintuplet in the $A = 32$ system showed good agreement with the IMME prediction previous to this work [16,17]. Although the present result implies a value for the mass of the lowest $T = 2$ state in ^{32}Cl ≈ 6 keV higher than previously thought, $M(^{32}\text{Cl}, T = 2) = -8291.5 \pm 1.8$ keV, it still yields excellent agreement with the IMME predictions. The extracted Q_{EC}, however, is quite different and the effects on the conclusions of the experiment [16,17] performed to search for scalar contributions to the weak interactions will be looked at in a later publication. Within the $A = 32$ multiplet the only "weak link" is the mass of the $T = 2$ state in ^{32}S, which was determined by Antony *et al.* [30] but was published only in a conference proceedings without details on how the difficult task of measuring the excitation energy of ≈ 12 MeV to a precision of ± 0.4 keV was carried out. We are presently setting up an experiment to measure with precision the excitation energy of the $T = 2$ state in ^{32}S via $^{31}\text{P}(p, \gamma)$.

In summary, as a result of the measurement of the mass of ^{33}Ar in Ref. [8], it appeared that the IMME was violated. The result obtained in this Letter shows that the mass of another member of the isospin multiplet (the lowest $T = 3/2$ state in ^{33}Cl) was erroneous and restores the validity of the IMME.

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