New Mechanism of Pulsar Radio Emission

M. Gedalin and E. Gruman
Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

D. B. Melrose
School of Physics, University of Sydney, NSW 2006, Sydney, Australia
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It is shown that pulsar radio emission can be generated effectively through a streaming motion in the polar-cap regions of a pulsar magnetosphere causing nonresonant growth of waves that can escape directly. As in other beam models, a relatively low-energy high-density beam is required. The instability generates quasitransverse waves in a beam mode at frequencies that can be well below the resonant frequency. As the waves propagate outward, growth continues until the height at which the wave frequency is equal to the resonant frequency. Beyond this point, the waves escape in a natural plasma mode (LO mode). This one-step mechanism is much more efficient than previously widely considered multistep mechanisms.

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Since the discovery of pulsars in 1967, the mechanism of generation of their highly nonthermal (brightness temperatures up to 10^{29} K) pulsed radio emission (in the range 10^8-10^{11} Hz) remains one of the most intriguing astrophysical puzzles [1]. Compact sizes (radius $R_p \sim 10^6$ cm), fast rotation (period $P \sim 1$ s), and superstrong polar magnetic fields ($B \sim 10^{12}$ G) result in the efficient avalanche production of an ultrarelativistic pair plasma (Lorentz factor $\gamma_p \gg 1$) in the vicinity of the magnetic poles of the neutron star (see, e.g., Ref. [2]). This plasma flows outward along the open magnetic field lines and escapes the pulsar magnetosphere as a relativistic wind beyond the light cylinder, $R = cP/2\pi$. In a standard polar-cap model, a primary beam (Lorentz factor $\gamma_b \gg \gamma_p$) of particles of one species propagates through a secondary pair plasma [1]. The superstrong magnetic field of the pulsar implies a very short lifetime for the electrons and positrons to radiate away all their perpendicular momenta, so that the plasma distribution is one dimensional. The properties of this pulsar plasma determine the natural wave modes, and the problem is to explain how excitation of these modes occurs and how it produces the observed radiation that escapes from the magnetosphere.

It is probable that the radio spectrum forms in the inner magnetosphere [3], where the infinite magnetic field approximation is appropriate. The properties of low-frequency (well below the cyclotron frequency) waves in a one-dimensional, relativistic pair plasma have been extensively studied (e.g., Ref. [4] and references therein). It has been found [5,6] that for a rather wide class of plasma distributions the natural modes are the electromagnetic t mode, $\omega = kc$, and two mixed (with transverse and longitudinal components of the electric field vector) modes, the almost nondispersive Alfvén mode, $\omega = k_\| v_A$ (here and hereafter subscripts $\|$ and $\perp$ refer to the direction with respect to the external magnetic field), and the LO mode which has a long wavelength cutoff. The LO mode is superluminal, $\omega/k_\| > c$, near the cutoff but can be subluminal for sufficiently small angles, θ, of propagation at high frequencies. The critical features of any plausible mechanism for the radio wave generation can be summarized as follows [5]. Only t waves (which cannot be generated through a beam instability) and waves which eventually appear on the LO branch can freely leave the pulsar magnetosphere. Beam instabilities are widely favored, and in the pulsar magnetosphere are of hydrodynamical type, where the whole beam excites the modes, in contrast to kinetic instabilities which are driven only by a group of resonant particles [7].

One of the most widely favored scenarios for the radio emission mechanism involves a resonant instability in which an energetic beam causes quasilongitudinal subluminal waves to grow. This mechanism encounters certain difficulties. The first difficulty is that the emission mechanism is *indirect*: The postulated growing waves need to be converted into quasitransverse waves (in the t or LO modes) via some secondary (e.g., nonlinear) mechanism before they can produce escaping radiation. A *direct* mechanism in which the escaping waves are generated directly by the instability would be preferable, and the mechanism proposed here has this desirable feature. The second difficulty concerns the beam: When the generation is attributed to the primary, highly relativistic, $\gamma_b \sim 10^7$, the growth is too slow to be effective. Thus, efficient beam instability requires a denser, lower energy beam. One suggestion is that such a beam results from nonstationary avalanche pair generation [8,9]. Alternatively, even in the steady avalanche regime, the high-energy tail of the pair plasma distribution [10] transforms into a dense beam due to the inverse Compton scattering [11]. The resulting distribution typically consists of a plasma body with $\gamma_p \sim 10^1-10^2$ and a beam with $\gamma_b \sim 10^3$. In the steady case such a beam forms at about $10R_p$. Although existence of such a beam seems plausible, it is still a model dependent assumption. The third difficulty is related to the nature of the wave generation. Most authors have concentrated on resonant excitation of parallel (to the magnetic field) propagating modes (Langmuir waves) [4,11–13], where it is widely believed that there is a sharp maximum of the growth rate. Growth for slightly oblique propagation has been considered [5,14] and also found to favor quasilongitudinal LO waves. Only subluminal waves can be resonantly excited by a beam. The LO mode becomes subluminal only at frequencies $\sim\gamma_p \times$ plasma frequency [5]. Growth is possible at the resonant frequency, which is just above the frequency where the waves become subluminal. These LO waves can eventually escape directly. However, the frequencies are too high to account for the broadband pulsar emission extending to much lower frequencies (so that a conversion mechanism to lower frequencies would be required). Moreover, growth is restricted to a narrow range of heights in the magnetosphere where the resonance condition is satisfied, restricting the growth factor to too small a value to allow effective growth [5]. These constraints led to our relatively pessimist view in [5] of the effectiveness of direct growth of escaping quasitransverse waves.

In this Letter, we show that a nonresonant version of the instability causes a beam mode to grow over a broad range of lower frequencies. These waves can grow over a large range of heights. As they propagate outward, that ratio of their frequency to the resonant frequency decreases and, at the height at which this ratio become unity, they evolve into LO mode waves, and escape, without any secondary conversion process being required.

We use the following notation: $p = mu$ is the (one-dimensional) particle momentum, with m the electron mass, and $v = u/\gamma$, $\gamma = (1 - v^2)^{-1/2} = (1 + u^2)^{1/2}$ is the particle velocity in units with $c = 1$. We assume that the distribution function consists of a pair (p) and a beam (b) component, $f(u) = n_p f_p(u) + n_b f_b(u)$, with $\int f_p\,du = \int f_b\,du = 1$. We adopt the infinite magnetic field limit, which is justified in the inner part of the plasma magnetosphere where the beam-plasma system is formed [11], and then it is not necessary to distinguish between electrons and positrons, which contribute in an identical manner. The dispersion relation for this beam-plasma system takes the following form [4–6]:

$$\epsilon_\parallel = \frac{\tan^2\theta}{z^2 - 1}, \tag{1}$$

with $z = \omega/k_\parallel$, $k_\parallel = k\cos\theta$, $k_\perp = k\sin\theta$, and where $\epsilon_\parallel = 1 - (\omega_p^2/k_\parallel^2)W(z) + \epsilon_b$ is the parallel dielectric constant. Here $\omega_p = (8\pi n_p e^2/m)^{1/2}$ (equal electron and positron densities) is the plasma frequency for the pairs, $W = \int_{-\infty}^{\infty}(v - z - i\tau)^{-1}(df_p/du)\,du$, $\tau \to +0$, is the dispersion function for the pairs; the contribution of the beam is $\epsilon_b = -\omega_b^2\gamma_b^{-3}(\omega - k_\parallel v_b)^{-2}$. We consider a cold beam for simplicity; the cold approximation is justified when the instabilities are known to be hydrodynamic. The dispersion relation (1) takes the form

$$\frac{\omega_b^2}{\gamma_b^3(\omega - k_\parallel v_b)^2} = \frac{z^2 - \cos^{-2}\theta}{z^2 - 1} - \frac{\omega_p^2 W(z)}{k_\parallel^2} \equiv K(z). \tag{2}$$

In the absence of the beam, the dispersion relation for the natural modes of the pair plasma is $K(z) = 0$, which can be written in the parametric form (recall $c = 1$)

$$k_\parallel^2 = \frac{\omega_p^2 W(z^2 - 1)}{z^2 - \cos^{-2}\theta}, \qquad \omega^2 = \frac{\omega_p^2 W z^2(z^2 - 1)}{z^2 - \cos^{-2}\theta}. \tag{3}$$

The inclusion of the beam introduces additional solutions, called beam modes, and hydrodynamic instabilities may be attributed to a beam mode becoming intrinsically growing. The instability is said to be nonresonant when the beam mode does not coincide with a natural mode of the pair plasma, and resonant when it does. The contribution of the beam is significant only when the denominator in the left-hand side of (2) is small, that is, near $z = v_b$. Writing $\omega = k_\parallel v_b + \delta\omega$, $|\delta\omega| \ll |\omega|$, one finds

$$\delta\omega = \omega_b\gamma_b^{-3/2}K(v_b)^{-1/2} = \omega_b\gamma_b^{-3/2}\left[\frac{v_b^2 - \cos^{-2}\theta}{v_b^2 - 1} - \frac{\omega_p^2 W(v_b)v_b^2}{\omega^2}\right]^{-1/2}. \tag{4}$$

The unstable solution $\delta\omega = i\Gamma$, $\Gamma > 0$, exists for $K(v_b) < 0$, that is, $\omega^2 < \omega_p^2 W(v_b)v_b^2(v_b^2 - 1)/(v_b^2 - \cos^2\theta)$, which requires $W(v_b) > 0$. This is the nonresonant beam instability which sets on the beam mode $\omega \approx k_\parallel v_b$.

The expression (4) becomes invalid for $K(v_b) \to 0$, that is, where the beam mode $\omega = k_\parallel v_b$ resonates with the LO mode [which is the solution of $K(v_b) = 0$]. In this case, the right-hand side of (2) should be Taylor expanded up to the first nonzero term, which immediately gives (see, e.g., Ref. [4]) $\Gamma_r = \mathrm{Im}\,\delta\omega = (\sqrt{3}/2)(\omega_b^2/\gamma_b^3 K')^{1/3}$, with $K' \equiv (\partial K/\partial\omega)_{\rm res} = 2v_b^2\gamma_b^4\tan^2\theta/\omega - \omega_p^2 v_b^3 W'(v_b)/\omega^3$, and $W'(z) = dW/dz$. For a wide class of distributions the approximation $W'(v_b) \sim \gamma_p^2 W(v_b)$ holds. This implies that the resonant growth rate is insensitive to $\theta \lesssim \gamma_p/\gamma_b^2$, and decreases slowly with $\theta \gtrsim \gamma_p/\gamma_b^2$. This point is important in the following discussion: If the growth rate were very sensitive to θ, then a small change in θ as the waves propagate outward along the curved field lines would restrict the distance over which growth can occur, and, hence, severely limit the possible growth factor.

The polarization of the unstable mode is given by $(E_\perp/E_\parallel) = \tan\theta(1 - z^2)^{-1} = \gamma_b^2\tan\theta$. Thus, the unstable mode is quasilongitudinal ($\mathbf{E} \parallel \mathbf{B}_0$) for $\theta \lesssim \gamma_b^{-2}$ and quasitransverse ($\mathbf{E} \perp \mathbf{B}_0$) otherwise. The polarization of the growing waves is relatively unimportant in practice: The polarization evolves as the waves propagate outward, and the observed polarization may be quite different from the polarization at the point of emission [15].

The derived expressions give the growth rate for arbitrary propagation angle and beam parameters. For small

$\theta \ll 1$ and a highly relativistic beam $v_b \approx 1 - 1/2\gamma_b^2$, $\gamma_b \gg 1$, the growth rate for the nonresonant instability (4) reduces to

$$\Gamma_n = \omega_b \gamma_b^{-1/2}\left[\frac{\omega_p^2 W(v_b)}{\omega^2} - 1 - \gamma_b^2\theta^2\right]^{-1/2}, \qquad (5)$$

which is a slightly increasing function of θ. In the range $\gamma_b^{-2} \lesssim \theta \lesssim \gamma_b^{-1}$, where the waves are quasitransverse, the dependence on θ is negligible. The maximum unstable frequency, $\omega_m^2 = \omega_p^2 W(v_b)/(1 + \gamma_b^2\theta^2)$, is also the frequency at which the resonant instability occurs, and it is also almost independent of θ in this range. We conclude that under quite general conditions the beam instability generates weakly oblique, quasitransverse waves with a similar efficiency to that for quasilongitudinal waves.

To illustrate the results in the simplest possible way, we use the cold plasma approximation, $f_p = \delta(u - u_p)$. In this case, one has $W(z) = \gamma_p^{-3}(z - v_p)^{-2}$. The parametric equations (3) become $k_\parallel^2 = \omega_p^2(z^2 - 1)/\gamma_p^3(z^2 - \cos^{-2}\theta)(z - v_p)^2$, $\omega = k_\parallel z$. For small θ and $\gamma_b \gg \gamma_p \gg 1$, the growth rate of the nonresonant instability becomes

$$\Gamma_n = \omega_b \gamma_b^{-3/2}[4\omega_p^2\gamma_p/\omega^2 - 1 - \gamma_b^2\theta^2]^{-1/2}. \qquad (6)$$

Thus, the growth rate for the resonant instability also decreases monotonically with increasing θ. Although these results are derived using the cold plasma expressions, as we plan to show in detail elsewhere, they are illustrative of a rather wide class of distributions due to the fact that the hydrodynamic instability is insensitive to the details of the beam distribution.

For small propagation angles $\theta \lesssim 1/\gamma_b$, the growth rates are almost independent of θ, and we can use the following approximations. In the ultrarelativistic limit $\gamma_b \gg \gamma_p \gg 1$, one has $\Gamma_n = \omega_b\gamma_b^{-3/2}(4\omega_p^2\gamma_p/\omega^2 - 1)^{-1/2}$. For low frequencies, $\omega \ll 2\omega_p\gamma_p^{1/2}$, this expression simplifies to $\Gamma_n = \omega\omega_b/2\omega_p\gamma_b^{3/2}\gamma_p^{1/2}$. The resonant frequency is $\omega_r = 2\omega_p\gamma_p^{1/2}$, and the resonant growth rate is $\Gamma_r = 3^{1/2}2^{-4/3}(\omega_p\omega_b^2)^{1/3}/\gamma_b\gamma_p^{1/2}$. Note that $\Gamma_n(\omega = \omega_r)/\Gamma_r \sim (\gamma_p/\gamma_b)^{1/2}(n_b/n_p)^{1/3}$ implies that for moderate $n_p\gamma_p/n_b\gamma_b$ the ratio of the two growth rates is of the order of unity. One can approximate the growth rate in the whole range by $\Gamma = (\omega/2\gamma_b^{3/2}\gamma_p^{1/2})H(\omega_r - \omega)$, where $H(x \geq 0) = 1$ and $H(x < 0) = 0$. This approximation is also valid if the above ratio is small, except in a narrow frequency range around that resonant frequency. We exploit this approximation in our estimates below. Note that the condition $\gamma_b \gg \gamma_p$ is made for simplicity, and it is not an essential condition for the instability to operate.

The direct excitation of quasitransverse waves is a fast process, faster than any nonlinear conversion mechanism. Let the growth rate be $\Gamma(\omega)$, which is a function of the plasma parameters, n_p, n_b, γ_p, γ_b, and n_p, through which it depends on the radius, R from the center of the star. The wave amplitude evolves according to $(da_\omega/dt) = \Gamma(\omega, R)a_\omega$. With the plasma streaming outward at close to the speed of light, the solution implies $a_\omega(R) = a_\omega(R_0)\exp[\int_{R_0}^R \Gamma(\omega, R)\, dR]$, where R_0 is the radius where the instability sets in. The power spectrum of the escaping radiation is proportional to the square of this amplitude.

In a homogeneous static plasma, the fastest growing mode is the resonant one, and one might expect that the resonant frequency ultimately dominates the spectrum. In the inhomogeneous plasma of the pulsar magnetosphere, the conditions change with the radius. A wave which is excited at the frequency ω at radius R_0 propagates outward into the lower density plasma. A wave initially at resonance does not remain resonant as it then propagates. The frequency width of the resonance is $\sim\Gamma_r \ll \omega_r$, where $\Gamma_r \propto \omega_r \propto n^{1/2} \propto R^{-3/2}$. For a given ω, the resonant condition $|\omega - \omega_r(R)| \lesssim \Gamma_r(R)$ is satisfied only for a small $\Delta R/R \sim \Gamma_r/\omega_r$. As a consequence, the resonant growth condition can be met only for a short time, and effectively only at a single height in the magnetosphere, which places a severe restriction on the gain factor $G = \exp(2\int \Gamma\, dR)$. It is the gain factor G which determines the efficiency of the wave generation and not the local growth rate Γ. It was for this reason that a pessimistic view of the effectiveness of the resonant instability was taken in [5]. However, a wave at a given ω can grow nonresonantly for $\omega \lesssim \omega_r$, and the slightly lower growth rate for the nonresonant instability, compared with the resonant instability, is relatively unimportant compared with the much greater distance over which nonresonant growth occurs. The much longer growth path through the magnetosphere results in a much larger gain factor. We note that during the propagation the wave vector, in general, deviates from the initial propagation direction, so that the propagation angle θ changes, and, in principle, this would limit the growth if the wave moves out of resonance as θ increases. This issue will be studied elsewhere. A wave that starts growing nonresonantly with $\omega \ll \omega_r$ at some radius, R_0, keeps growing while propagating outward until $\omega_r \propto R^{-3/2}$ decreases to $\omega_r = \omega$. At this resonant point, the beam mode joins onto the LO mode [16]. Beyond this point amplification ceases and the wave escapes as in the LO mode.

Let us estimate the gain factor using the approximation for Γ_n for the cold plasma case, assuming γ_p and γ_b do not change during the outflow, with $\omega_r \propto n_p^{1/2} \propto R^{-3/2}$ [1]. The gain factor at a given ω is $G = \exp[2\int_{R_0}^{\infty}\Gamma(\omega, R)\, dR] = \exp[2(\omega_{b0}R_0/\gamma_b^{3/2})x(x^{-2/3} - 1)]$, where ω_{p0} is the plasma frequency at $R = R_0$, and $x = \omega/2\omega_{p0}\gamma_p^{1/2}$. The factor G is a maximum at $x \approx 0.2$ and, reverting to ordinary units, $G_{\max} \approx \exp(0.25\omega_{b0}R_0/c\gamma_b^{3/2})$. For numerical estimates we use the following parameters: $P = 1$ s, $B = 10^{12}$ G, $n_p = Mn_{\rm GJ} = 10^2 \times 6 \cdot 10^{10}$ cm^{-3} (near the pulsar surface), $\gamma_p = 10$ [10,17], $\gamma_b = 10^3$, $n_b/n_p \sim 1$, and $R_0 = 10R_p$ [11]. This immediately shows that the maximum is achieved at $\omega/2\pi \approx 500$ MHz. The corresponding gain factor $\exp(G) \approx \exp(30) \approx 10^{12}$, which implies efficient growth. Lower multiplicities would result in lower maximum gain frequencies. Lower γ_b, on the other hand, would make the instability more efficient. Better knowledge of the pulsar plasma parameters is required for direct comparison of our predictions with observations. The above estimate is invalid for low frequencies, corresponding to wave excitation at very large radii $R > 10^2 R_0$. At such radii, one has $\omega_r \sim \Omega/\gamma_p$ [6], where Ω is the gyrofrequency, and then the infinite magnetic field approximation is no longer valid. Thus, there should be a significant decrease of efficiency of wave growth at low frequencies. Specifically, using parameters chosen by Ref. [6], the spectrum should be cut off for $\omega/\omega_r < 10^{-3}$.

Let us summarize the predictions of the proposed model. Quasitransverse waves are generated efficiently in a wide frequency range below $\omega_{p0}\gamma_p^{1/2}$ for small angles of propagation. The power spectrum of the escaping radiation is assumed $\propto G^2$, where G is the gain factor, and there is a maximum in G^2 at the frequency $\omega_{\max} \sim 0.1\omega_{p0}\gamma_p^{1/2}$. The efficiency of wave growth decreases at both $\omega < \omega_{\max}$ and $\omega > \omega_{\max}$, and the rate of decrease [steepness of the curve $G(\omega)$] increases with the distance from $\omega_{\max}$. The implied form of spectrum is consistent with observations (e.g., Ref. [1]).

We conclude that the nonresonant beam instability efficiently generates quasitransverse waves in the radio range in a one-step process. The growing waves are in a beam mode well below the resonant frequency. As these waves propagate outward through the magnetosphere, they evolve into the LO mode when their frequency matches the resonant frequency. It is also the place where the growth ceases and, therefore, the spectrum forms. This emission mechanism plausibly reproduces the basic features of the observed radio spectra of pulsar, notably the existence of a maximum frequency with the spectrum steepening towards both higher and lower frequencies. However, a detailed interpretation of the observed power spectra requires that the emission mechanism proposed here be complemented with a statistical model for the emission. The observed spectra are obtained by integration over many pulses, and each pulse probably involves emission from a statistically large number of individual beams. Moreover, there remains a potential problem in explaining the lowest frequencies inferred for some pulsar emission: Although the nonresonant model allows emission considerably below the resonant frequency, a more detailed investigation is required to determine whether the frequency range can extend low enough. A more detailed model for pulsar radio emission based on the mechanism proposed here will be presented elsewhere.

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