Shot Noise through a Quantum Dot in the Kondo Regime

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The shot noise in the current through a quantum dot is calculated as a function of voltage from the high-voltage Coulomb-blockaded regime to the low-voltage Kondo regime. Using several complementary approaches, it is shown that the zero-frequency shot noise (scaled by the voltage) exhibits a nonmonotonic dependence on voltage, with a peak around the Kondo temperature. Beyond giving a good estimate of the Kondo temperature, it is shown that the shot noise yields additional information on the effects of electronic correlations on the local density of states in the Kondo regime, unaccessible in traditional transport measurements.

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The Kondo effect has become one of the main paradigms of condensed matter physics as it is one of the simplest models that exhibit many-body correlations [1]. The original model was devised to explain the nonmonotonic resistivity of metal due to enhanced scattering by magnetic impurities below the Kondo temperature. This effect, however, was also predicted to play a dramatic role in transport through quantum dots [2–4], due to the enhancement of the local density of states at the Fermi energy. Indeed, the recent observation of the Kondo effect in transport through a quantum dot [5,6] has paved the way for a new class of experimental investigations of strongly correlated electrons in general and the Kondo effect in particular. These and later experiments [7–12] demonstrated the ability to exploit the tunable physical characteristics of the quantum dot in order to yield important information on Kondo systems, information unavailable from experiments in bulk systems. Such studies, for example, included the full crossover between the Kondo limit, the mixed valence regime and the nonKondo limit [7], the emergence of the unitarity limit [8], the determination of the phase of the transmission coefficient through such an Anderson impurity [9], the study of the Kondo effect under external irradiation [10], and even surprises such as the observation of the Kondo effect for an integer-spin dot [11] and the enhancement of the Kondo effect by a magnetic field [12]. These new probes enhanced our understanding of the Kondo system and provided critical tests of various theoretical approximations, an imperative step towards better understanding of strongly correlated electron systems. Nevertheless, detailed experimental information on how electronic correlations affect the density of states is still lacking.

In this Letter we propose and explore theoretically another experimental tool to probe the Kondo regime–shot noise measurements. Such measurements proved very successful in mesoscopic structures formed in other strongly correlated electron systems, such as a fractional Hall liquid [13] or a superconductor [14]. Here we demonstrate that the noise measurements yield additional information on the structure of the local density of states, information unavailable by the usual transport measurements. In addition shot noise yields a direct estimate of the Kondo temperature.

Current noise, defined as

$$S(t) \equiv \langle I(t)I(0)\rangle - \langle I\rangle^2, \tag{1}$$

or, alternatively, its Fourier transform, $S(\omega)$, has been studied extensively in the context of mesoscopic systems in the past couple of decades [15] (I above is the current operator). While the equilibrium zero-frequency noise $S_0 \equiv S(\omega = 0)$ can be related to the conductance via the fluctuation-dissipation theorem, and does not carry additional information, the zero-frequency noise out of equilibrium (shot noise) can yield information on charge fluctuations in the mesoscopic system. Since for voltage bias much larger than temperature, finite temperature can be ignored, we concentrate in the following on $T = 0$, where the thermal noise vanishes and the only contribution to the current noise is shot noise. Because of the lack of a single accurate method that can describe the Kondo system out of equilibrium in all regimes, we employed five different methods that span all physically relevant regimes.

The current I through a quantum dot in the presence of a dc voltage bias V can be directly related to the transmission coefficient through the dot, $\mathcal{T}(\epsilon, V)$, which, in turn, is proportional to the local density of states $\rho(\epsilon, V)$ [16,17], $I = 2e/h \int_0^{eV} \mathcal{T}(\epsilon, V)\, d\epsilon$, where the factor 2 is due to spin-degeneracy. For noninteracting electrons, $\mathcal{T}(\epsilon, V)$ is voltage independent, and thus the differential conductance, dI/dV, yields directly $T(\epsilon = V)$, and hence the full local density of states. In the present case, however, the Kondo peak at the Fermi energy is dramatically affected by voltage [3,4]. Thus, while its structure is of major importance, current measurements cannot yield the energy-dependent transmission coefficient $\mathcal{T}(\epsilon, V)$, but rather its average over a scale $0 \le \epsilon \le eV$. Since $\rho(\epsilon, V)$ is of major interest, due to the Kondo resonance, an experimental probe of, e.g., its higher moments is highly desirable.

For noninteracting electrons the shot noise can be expressed in terms of the transmission coefficients [18],

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$$S_0(V) = \frac{4e^2|V|}{h} \int_0^{eV} \mathcal{T}(\epsilon)[1 - \mathcal{T}(\epsilon)]\, d\epsilon\,. \quad (2)$$

One might expect anomalous shot noise dependence on voltage or temperature due to the Kondo effect because of the following argument. In the absence of the Kondo effect (e.g., when temperature or bias are much larger than the Kondo temperature) the conductance through a quantum dot, or the effective transmission coefficient, is suppressed due to the Coulomb blockade (except at the Coulomb blockade peaks), and $\mathcal{T}(1 - \mathcal{T})$ is very small. With lowering of temperature or voltage, the conductance is enhanced (for an odd number of electrons on the dot), leading to an increase in the shot noise. However, at the unitarity limit, at zero temperature and linear response, $\mathcal{T} = 1$ (for the special case of symmetric barriers), and the noise again vanishes. Thus one may expect a nonmonotonic dependence of the noise (scaled by V) on voltage, for example. As we will show below, such nonmonotonicity should indeed be observed.

Shot noise through a quantum dot has been studied in the past, with an emphasis on the Coulomb blockade regime [15,19]. These studies revealed indeed that the shot noise is quite small at the conductance valleys at high voltages (or temperatures). Several attempts to look at the Kondo regime have been made. Perturbative results were reported [20], while an exact solution is available for a particular limiting case (not quite relevant to the present case) [21]. Here we report calculations made in the noncrossing approximation (NCA) [22], augmented by high-voltage perturbation theory, renormalization group (RG) calculations, perturbation theory around the zero-voltage Fermi-liquid point, and by slave-boson mean field theory (SBMFT). While NCA is valid for a wide range of voltages, including $eV, T < T_K$ (except a small region for small eV and T), the RG calculation is valid at high voltages, while the SBMFT is valid at small voltages. Thus our methods complement each other and give a consistent view of the dependence of shot noise on voltage.

Our starting point is the infinite- U single-impurity Anderson Hamiltonian,

$$H = \sum_{\sigma;k\epsilon L,R} \epsilon_{k\sigma} \mathbf{c}^+_{k\sigma}\mathbf{c}_{k\sigma} + \sum_{\sigma} \epsilon_0 \mathbf{d}^+_{\sigma}\mathbf{d}^+_{\sigma} + \sum_{\sigma;k\epsilon L,R} (V_{k\sigma}\mathbf{c}^+_{k\sigma}\mathbf{d}_{\sigma} + \text{H.c.})\,, \quad (3)$$

where $\mathbf{c}^+_{k\sigma}$ ($\mathbf{c}_{k\sigma}$) creates (destroys) an electron with momentum k and spin σ in one of the two leads, and $\mathbf{d}^+_{\sigma}$ ($\mathbf{d}_{\sigma}$) creates (destroys) a spin-σ electron on the quantum dot. Coulomb interactions among electrons, in the limit of $U \to \infty$, forbid double occupancy of the quantum dot. The last term describes the hopping between the leads and the dot, and determines this coupling $\Gamma = \Gamma^L_\sigma + \Gamma^R_\sigma$, via $\Gamma^{L(R)}_\sigma(\omega) = 2\pi \sum_{k\epsilon L(R)} |V_{k\sigma}|^2 \delta(\omega - \epsilon_{k\sigma})$.

We start by studying the limiting cases, $eV \gg T_K$ and $eV \ll T_K$. In these regimes it is more convenient to study the Kondo Hamiltonian, obtained from the Anderson Hamiltonian (3) by a Schrieffer-Wolf transformation. For convenience, one performs first a unitary transformation where the dependence on external voltages is shifted to the couplings $\Gamma^{L(R)}_\sigma$ [23]. The resulting Kondo Hamiltonian is

$$H_K = \sum_{k\sigma j=L,R} \xi_k \psi^\dagger_{kj\sigma}\psi_{kj\sigma} + \sum_{j,j'=L,R} J_{jj'}(t)\psi^\dagger_j(0)\hat{P}\psi_{j'}(0)\,, \quad (4)$$

with $\hat{P} = \frac{1}{4}\hat{I} + \vec{S}\cdot\vec{s}$ and where $\hat{I}$, $\vec{s}$, and $\vec{S}$ are identity, electron spin operator of the leads, and and of the electron on the impurity, respectively. The coupling parameters $J_{jj'}$ are related to those of the Anderson Hamiltonian and due to the unitary transformation depend on time,

$$J_{LR} = J^*_{RL} = J_0 \exp\left[\frac{iteV}{\hbar}\right], \qquad J_0 = \frac{\sqrt{\Gamma_L\Gamma_R}}{\pi\nu\epsilon_0}\,, \qquad J_{jj} = \frac{\Gamma_j}{\pi\nu\epsilon_0}\,, \quad (5)$$

where ν is the density of states in the leads. Using the Keldysh formalism [24] we can evaluate the noise S_0 to third order in the coupling,

$$S_0(V) = \frac{2e^3|V|}{2h}(\pi\nu J_0)^2 + \frac{3e^3|V|}{h}(\pi\nu J_0)^2\left[1 + 2\nu(J_{LL} + J_{RR})\log\frac{D}{eV}\right]. \quad (6)$$

Here D is the effective bandwidth, and the leading term is separated into two terms, expecting the RG procedure. Indeed, to get an expression that is also valid for lower voltages, one can use the RG to sum up the diverging logarithms [second line in (6)], leading to

$$S_0(V) = \frac{3e^3\gamma|V|}{4h}\left[\frac{\pi}{\log(eV/T_K)}\right]^2 \quad (7)$$

with $\gamma \equiv 4\Gamma_L\Gamma_R/(\Gamma_L + \Gamma_R)^2$, and where the relation between the renormalized coupling $J_0(V)$ and T_K, $J_0(V) = \sqrt{\gamma}/2\nu\log(eV/T_K)$ [23], was used.

In the other limit, of small voltage, $eV \ll T_K$, one can expand around the strong coupling Fermi-liquid fixed point [25]. Identifying the current operator around this point [23], a straightforward, lengthy calculation yields

$$S_0(V) = \frac{2e^3}{h}|V|\frac{(\Gamma_L - \Gamma_R)^2}{(\Gamma_L + \Gamma_R)^2} + \frac{4e^3\gamma}{3h}|V|\left(\frac{eV}{T_K}\right)^2. \quad (8)$$

In the case of symmetric barriers $\Gamma^L_\sigma = \Gamma^R_\sigma$, where the effective transmission coefficient is unity, we find that $S_0/V \to 0$ as $V \to 0$, in agreement with the expectation from the noninteracting noise formula.

To expand the treatment beyond second order in voltage, we transform the Anderson Hamiltonian (3) into a new Hamiltonian, expressed in terms of new local operators [26]. These operators create the three possible states of the site: a boson operator $\mathbf{b}^+$, which creates an empty site, and two fermion operators, $\mathbf{f}_\sigma^+$, which create the singly occupied states. The ordinary electron operators on the site, which transform the empty site into a singly occupied site or vice versa, are decomposed into a boson operator and a fermion operator, $\mathbf{d}_\sigma(t) = \mathbf{b}^+(t)\mathbf{f}_\sigma(t)$. The additional constraint, that the number of fermions and bosons is equal to one, prevents double occupancy on the site, as required by the $U \to \infty$ limit.

In the slave-boson representation, the Hamiltonian for the infinite-U Anderson model becomes

$$H = \sum_{\sigma;k\epsilon L,R} \epsilon_{k\sigma}\mathbf{c}_{k\sigma}^+\mathbf{c}_{k\sigma} + \sum_\sigma \epsilon_\sigma \mathbf{f}_\sigma^+\mathbf{f}_\sigma + \sum_{\sigma;k\epsilon L,R}(V_{k\sigma}\mathbf{c}_{k\sigma}^+\mathbf{b}^+\mathbf{f}_\sigma + \text{H.c.}), \tag{9}$$

where the Hamiltonian operates only in the subspace where the total number of fermions and bosons is one. The advantage of the slave-boson representation is that the hopping term, which is usually the smallest physical term, can be treated perturbatively using standard diagrammatic techniques.

We first apply the SBMFT, a theory that is known to give the correct qualitative behavior at low temperatures and voltages ($eV, T \leq T_K$). In this theory, motivated by a large-N expansion, where N is the degeneracy of the level, one replaces the boson operator by a classical, nonfluctuating value, giving rise to an effective resonant-tunneling model, whose parameters are obtained self-consistently [26]. These equations were numerically solved for a set of parameters leading to the same Kondo temperature used for the above calculations, and indeed reduced to the results of the Fermi-liquid perturbation theory for small voltages.

To bridge the small-V treatment with the large-V treatment, we next employ the NCA, which has been used successfully to treat the infinite-U Anderson model in [22] and out [4] of equilibrium. At lowest order in perturbation theory the boson self-energy involves the fermion propagator while the fermion self-energy involves the boson propagator. By using the two relations self-consistently, one obtains a set of coupled integral equations, which can be solved numerically. Solving these self-consistent equations corresponds to summing a subset of diagrams to all orders in the hopping matrix element. It can be shown [22] that all diagrams of leading order in $1/N$, where N is the number of spin degrees of freedom, are included in this subset. Therefore, the noncrossing approximation is expected to be a quantitative approach in the limit of large N. For the case $N = 2$, of interest for quantum dots, Cox [22] has shown that the calculated equilibrium occupancy and susceptibility agree with the exact Bethe ansatz results to within the 0.5% convergence accuracy of the NCA. For the electrical current calculations [4], which use an extension of the above equations to include Keldysh Green functions [24], at worse an overestimate of 15% on the linear response conductance has been observed [4].

Here we employ a two-step approximation. The general noise diagram involves two single-particle Green functions, which, in the first step, are decoupled, i.e., vertex diagrams are neglected. In the second step, these single-particle Green functions are replaced by their NCA values [4]. This procedure ensures that our expression for the noise obeys exactly the zero-voltage fluctuation-dissipation theorem. All the calculations were done for $T = 10^{-4}\Gamma$, well below the Kondo temperature.

Figure 1 summarizes our main findings, depicting S_0/V as a function of $\log(eV/T_K)$, the natural parameter in the Kondo regime. In (a) we plot results from three different approaches. The RG calculation agrees with the NCA result at large voltages, but becomes nonphysical once eV is about $10T_K$, while the SBMFT, which agrees with perturbation theory around the Kondo fixed point at low voltages, complements our NCA calculations for $eV \ll T_k$, where the NCA approximation stops to be valid. Thus we have a quantitative picture of the noise on the whole range of voltages. Here and in (b) the NCA results are for $\epsilon_0 = -2\Gamma$, resulting in $T_K \simeq 0.005\Gamma$. Figure 1(b) demonstrates that S_0/V exhibits a maximum of a value about $1/2$, as one might expect from the heuristic arguments above. Indeed, one can notice the similarity of S_0/V to Eq. (2) (long-dashed line), where $T(\epsilon, V)$ was calculated from the NCA for the same bias. Thus the noise gives a direct measurement of the second moment of the density of states. In order to check how sensitive

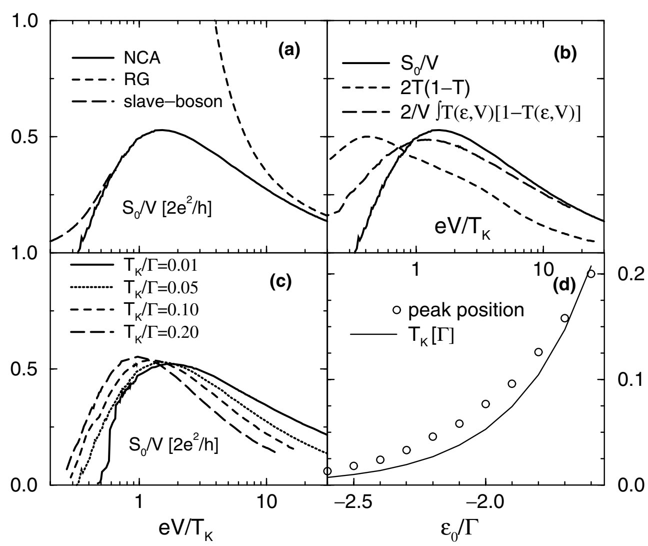

FIG. 1. The main results of this work (see text). (a) Three of the methods used to estimate the noise in the different regimes. (b) Comparison of the results of the NCA approximation to the naive noninteracting formula [Eq. (2)] without and with the voltage-dependent transmission coefficient, obtained from the differential conductance. (c) S_0/V for different level energies (and different Kondo temperatures). (d) Comparison of the maxima of S_0/V to the Kondo temperature.

the noise is to the effects of electronic correlations, we also plot in the same figure (short-dashed line) the expected noise for a noninteracting system, with the same differential conductance. The significant difference between the two curves is due to the effect that, because of the interactions, the density of states is strongly dependent on the bias voltage. This also demonstrates that the information available in noise measurements cannot be obtained in the usual transport experiments. In Fig. 1(c) we plot S_0/V for different values of ϵ_0, the electron level energy, leading to values of T_K differing by a factor of 20. Nevertheless, the value of V/T_K where S_0/V is maximal changes by less than a factor of 2, demonstrating that the peak position gives a reliable estimate of the value of the Kondo temperature. This point is further demonstrated in Fig. 1(d), where we plot the dependence of the position of the peak on voltage and compare it to the known NCA values of $T_K = (\Gamma D^2/2\pi\epsilon_0)^{1/2}\exp[-\pi\epsilon_0/\Gamma]$. Thus shot noise measurements lead to a straightforward determination of the all important Kondo temperature, which in usual current measurements can only be determined by further nontrivial analysis.

To conclude, we have used several methods which give a consistent determination of the current noise through a quantum dot in the Kondo regime, demonstrating the importance of the electronic correlations. It has been shown that the noise yields additional information about the Kondo state, and we hope that this paper will indeed motivate experimental effort in this direction.

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