Hall Effect Induced by a Spin-Polarized Current in Superconductors

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We propose a novel anomalous Hall effect caused by the spin-polarized current in superconductors (SC). The spin-polarized quasiparticles flowing in SC are deflected by spin-orbit scattering to yield a quasiparticle charge imbalance in the transverse direction. Overall charge neutrality gives rise to a compensating change in the number of Cooper pairs. A transverse electric field builds up as opposed to an acceleration of the Cooper pairs, producing the Hall voltage. It is found that the Hall voltages due to the side jump and skew scattering mechanisms have different temperature dependence in the superconducting state. A spin-injection Hall device to generate the ac Josephson effect is proposed.

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Spin-polarized transport plays an important role in the spin-dependent phenomena of magnetic nanostructures. When spin-polarized electrons are injected from a ferromagnet (FM) into a nonmagnetic metal (NM) such as normal metal, semiconductor, and superconductor (SC), the nonequilibrium spin polarization is created in NM [1–5]. Recently, various spin injection devices have been proposed and demonstrated; for example, a strong suppression of superconductivity in thin SCs [6,7] and the magnetization reversal in a FM film by injection of spin-polarized electrons into the film [8].

A double tunnel junction FM/SC/FM containing a superconducting layer is one of the spin injection devices which exhibits unique spin-dependent effects, depending on whether the magnetizations of FMs are parallel or antiparallel [9,10]. In the antiparallel alignment, the injected spin populations are accumulated in SC and suppress the superconductivity. In the parallel alignment, the injected spin current is conserved to flow across SC without suppressing the superconductivity.

It is well known that the spin-orbit interaction in metals causes a spin asymmetry in the scattering of conduction electrons by impurities; up-spin electrons are preferentially scattered in one azimuthal direction and down-spin electrons in the opposite direction [11–14]. As a consequence, the spin-polarized electrons flowing in a NM metal are deflected in the transverse direction to accumulate an excess charge on the sides of the sample, yielding the anomalous Hall effect [15,16].

The spin current flowing in SC is particularly interesting because SC is considered as a coupled system of the condensate (Cooper pairs) and the quasiparticles (QP), where spin and charge imbalance plays a central role in the spin and charge transport [17–20]. If the spin current induces a QP charge imbalance in SC, the imbalance is compensated by the Cooper pair charge to maintain overall charge neutrality [17], and thus the Cooper pairs are strongly involved in the Hall effect.

In this Letter, we propose an anomalous Hall effect for the Cooper pairs induced by a spin current flowing in SC. We calculate the Hall voltage $V_{\rm H}$ due to the mechanisms of side jump (SJ) and skew scattering (SS), and show that $V_{\rm H}^{\rm SJ}$ is proportional to the spin current in SC and $V_{\rm H}^{\rm SS}$ is proportional to the gradient in the spin imbalance in SC. It is found that $V_{\rm H}^{\rm SJ}$ and $V_{\rm H}^{\rm SS}$ have different temperature dependence by the onset of superconductivity. This provides an opportunity to investigate the mechanism of the anomalous Hall effect.

We consider a spin-injection Hall device shown in Fig. 1. The left and right FMs are made of the same FM, and their magnetizations are aligned parallel and point to the z direction. When the thickness d of SC is smaller than the spin diffusion length λ_S ($d < \lambda_S$), the spin current j_S in SC is conserved across SC and its magnitude is given by $j_S \approx P j_{\rm inj}$ [20], where $j_{\rm inj}$ is the injection (bias) current density and P the tunneling spin polarization [1].

We start with the one-electron Hamiltonian of SC in the presence of impurities in the normal state ($T > T_c$) above the superconducting critical temperature T_c;

$$\mathcal{H} = \sum_{\mathbf{k},\sigma} \xi^0_{\mathbf{k}} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} \sum_{\sigma,\sigma'} U^{\sigma'\sigma}_{\mathbf{k}'\mathbf{k}} a^\dagger_{\mathbf{k}'\sigma} a_{\mathbf{k}\sigma} . \quad (1)$$

Here, $a^\dagger_{\mathbf{k}\sigma}$ is the creation operator of a conduction electron with the kinetic energy $\xi^0_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \varepsilon^0_{\rm F}$ relative to the Fermi energy $\varepsilon^0_{\rm F}$ and spin σ, and $U^{\sigma'\sigma}_{\mathbf{k}'\mathbf{k}}$ the scattering

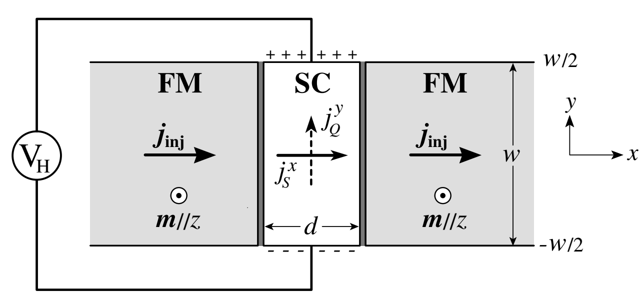

FIG. 1. Schematic diagram of the spin injection device FM/SC/FM which yields a nonequilibrium Hall voltage $V_{\rm H}$ in the transverse y direction.

amplitude for the δ-function potentials

$$U_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma} = V_{\rm i}[\delta_{\sigma'\sigma} + i\lambda_{\rm so}\boldsymbol{\sigma}_{\sigma'\sigma}\cdot(\mathbf{k}\times\mathbf{k}')]\sum_i e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i}, \tag{2}$$

where $\lambda_{\rm so}$ is the spin-orbit coupling parameter, $\boldsymbol{\sigma}$ the Pauli matrix, and $\mathbf{r}_i$ the impurity position.

Following Lyo and Holstein [21], we introduce the velocity operator of an electron $\hat{\boldsymbol{v}} = d\hat{\boldsymbol{r}}/dt = (1/i\hbar)\times[\hat{\boldsymbol{r}},\mathcal{H}]$, whose matrix element between the scattering state $|\mathbf{k}^+\sigma\rangle$ in the presence of impurities is given by $\boldsymbol{v}_{\mathbf{k}}^{\sigma} = \langle\mathbf{k}^+\sigma|\hat{\boldsymbol{v}}|\mathbf{k}^+\sigma\rangle = \hbar\mathbf{k}/m + \boldsymbol{\omega}_{\mathbf{k}}^{\sigma}$, where the anomalous velocity is calculated in the Born approximation

$$\boldsymbol{\omega}_{\mathbf{k}}^{\sigma} = (\lambda_{\rm so}/\tau_{\rm imp})[\boldsymbol{\sigma}_{\sigma\sigma}\times\mathbf{k}], \tag{3}$$

with $1/\tau_{\rm imp} \approx (2\pi/\hbar)n_{\rm i}N(0)V_{\rm i}^2$ the scattering rate due to impurities, $n_{\rm i}$ the impurity concentration, and $N(0)$ the density of states at the Fermi level. The current operator for electrons with velocity $\boldsymbol{v}_{\mathbf{k}}^{\sigma}$ may be written in the form

$$\hat{\mathbf{j}}_\sigma = e\sum_{\mathbf{k}}[\hbar\mathbf{k}/m + \boldsymbol{\omega}_{\mathbf{k}}^{\sigma}]a_{\mathbf{k}\sigma}^{\dagger}a_{\mathbf{k}\sigma}, \tag{4}$$

where $e = -|e|$ is the electronic charge.

When SC is superconducting below T_c, it is convenient to rewrite the electron operators $a_{\mathbf{k}\sigma}$ in terms of QP operators $\gamma_{\mathbf{k}\sigma}$ using the Bogoliubov transformation: $a_{\mathbf{k}\sigma} = u_{\mathbf{k}}\gamma_{\mathbf{k}\sigma} + \sigma v_{\mathbf{k}}\gamma_{-\mathbf{k}-\sigma}^{\dagger}$, where $u_{\mathbf{k}}^2 = 1 - v_{\mathbf{k}}^2 = \frac{1}{2}(1 + \xi_{\mathbf{k}}/E_{\mathbf{k}})$ and $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$ is the QP excitation energy, $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu_{\rm p}$ being the kinetic energy relative to the chemical potential $\mu_{\rm p}$ of the condensate and Δ being the superconducting gap parameter. In nonequilibrium situations, $\mu_{\rm p}$ differs from its equilibrium value $\varepsilon_{\rm F}^0$.

By expressing the current in Eq. (4) in terms of the QP operators $\gamma_{\mathbf{k}\sigma}$, the total charge current $\mathbf{j}_Q^{\rm tot} = \mathbf{j}_\uparrow + \mathbf{j}_\downarrow$ and the total spin current $\mathbf{j}_S^{\rm tot} = \mathbf{j}_\uparrow - \mathbf{j}_\downarrow$ in SC are written as

$$\mathbf{j}_Q^{\rm tot} = \mathbf{j}_{\rm pair} + \mathbf{j}_Q + \eta_{\rm SJ}[\hat{\mathbf{z}}\times\mathbf{j}_S], \tag{5}$$

$$\mathbf{j}_S^{\rm tot} = \mathbf{j}_S + \eta_{\rm SJ}[\hat{\mathbf{z}}\times\mathbf{j}_Q]. \tag{6}$$

Here, $\eta_{\rm SJ} = m\lambda_{\rm so}/\hbar\tau_{\rm imp}$, $\mathbf{j}_{\rm pair}$ is the pair current [17], and $\mathbf{j}_Q$ and $\mathbf{j}_S$ are the charge and spin currents:

$$\mathbf{j}_Q = e\sum_{\mathbf{k},\sigma}(\hbar\mathbf{k}/m)f_{\mathbf{k}\sigma}, \qquad \mathbf{j}_S = e\sum_{\mathbf{k}}\boldsymbol{v}_{\mathbf{k}}^{g}[f_{\mathbf{k}\uparrow} - f_{\mathbf{k}\downarrow}], \tag{7}$$

where $f_{\mathbf{k}\sigma} = \langle\gamma_{\mathbf{k}\sigma}^{\dagger}\gamma_{\mathbf{k}\sigma}\rangle$ is the distribution function of QPs with energy $E_{\mathbf{k}}$ and spin σ, and $\boldsymbol{v}_{\mathbf{k}}^{g}$ is the group velocity of QPs: $\boldsymbol{v}_{\mathbf{k}}^{g} = (1/\hbar)(dE_{\mathbf{k}}/d\mathbf{k}) = (\hbar\mathbf{k}/m)(\xi_{\mathbf{k}}/E_{\mathbf{k}})$. The last terms in Eqs. (5) and (6) are the Hall currents due to *side jump* (SJ).

The spin number density in SC is given by $S = \sum_{\mathbf{k}\sigma}\langle a_{\mathbf{k}\sigma}^{\dagger}a_{\mathbf{k}\sigma}\rangle = \sum_{\mathbf{k}}[f_{\mathbf{k}\uparrow} - f_{\mathbf{k}\downarrow}]$. The total charge number density $Q_{\rm tot} = \sum_{\mathbf{k}\sigma}\langle a_{\mathbf{k}\sigma}^{\dagger}a_{\mathbf{k}\sigma}\rangle$ in SC is separated into a part associated with the superfluid component (condensate) $Q_{\rm p} = 2\sum_{\mathbf{k}}v_{\mathbf{k}}^2$, and a part associated with the normal component (QPs) $Q_{\rm n} = \sum_{\mathbf{k}}q_{\mathbf{k}}[f_{\mathbf{k}\uparrow} + f_{\mathbf{k}\downarrow}]$, where $q_{\mathbf{k}} = u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2 = \xi_{\mathbf{k}}/E_{\mathbf{k}}$ is the effective charge of QPs [17]. Consequently, the injection of an electron into SC adds a QP of charge $eq_{\mathbf{k}}$ and spin 1/2, and a Cooper pair of charge $2ev_{\mathbf{k}}^2$ and spin 0 in SC.

In addition to the side jump contribution to the Hall effect, the spin-orbit scattering gives rise to the skew scattering contribution [11–14]. The scattering probability $P_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma}$ from the state $|\mathbf{k}\sigma\rangle$ to the state $|\mathbf{k}'\sigma'\rangle$ by impurity scattering is calculated from the formula $P_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma} = (2\pi/\hbar)n_{\rm i}|\langle\mathbf{k}'\sigma'|\hat{T}|\mathbf{k}\sigma\rangle|^2\delta(E_{\mathbf{k}} - E_{\mathbf{k}'})$, where $\hat{T}$ is the T matrix. In the second-order Born approximation [22], $P_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma}$ has the symmetric part

$$P_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma(1)} \approx \frac{2\pi}{\hbar}n_{\rm i}V_{\rm i}^2\frac{|\xi_{\mathbf{k}}|}{E_{\mathbf{k}}}\delta_{\sigma\sigma'}\delta(\xi_{\mathbf{k}'} - \xi_{\mathbf{k}}), \tag{8}$$

and the asymmetric part

$$P_{\mathbf{k}'\mathbf{k}}^{\sigma'\sigma(2)} \approx \lambda_{\rm so}\frac{2\pi V_{\rm i}}{\tau_{\rm imp}}\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(\mathbf{k}'\times\mathbf{k})\cdot\boldsymbol{\sigma}_{\sigma\sigma} \times\delta_{\sigma\sigma'}\delta(\xi_{\mathbf{k}'} - \xi_{\mathbf{k}}). \tag{9}$$

Therefore the change of the distribution function $f_{\mathbf{k}\sigma}(\mathbf{r})$ due to impurity scattering is evaluated from

$$(\partial f_{\mathbf{k}\sigma}/\partial t)_{\rm scatt} = \sum_{\mathbf{k}'}P_{\mathbf{k}\mathbf{k}'}^{\sigma\sigma(1)}[f_{\mathbf{k}'\sigma} - f_{\mathbf{k}\sigma}] + \sum_{\mathbf{k}'}P_{\mathbf{k}\mathbf{k}'}^{\sigma\sigma(2)}[f_{\mathbf{k}'\sigma} + f_{\mathbf{k}\sigma}]. \tag{10}$$

The distribution function $f_{\mathbf{k}\sigma}$ is separated into three parts [22], $f_{\mathbf{k}\sigma} = f_{\mathbf{k}\sigma}^0 + g_{\mathbf{k}\sigma}^{(1)} + g_{\mathbf{k}\sigma}^{(2)}$, where $f_{\mathbf{k}\sigma}^0$ is a nondirectional one defined by the average of $f_{\mathbf{k}\sigma}$ with respect to the solid angle $\Omega_{\mathbf{k}}$ of $\mathbf{k}$: $f_{\mathbf{k}\sigma}^0 = \int f_{\mathbf{k}\sigma}\frac{d\Omega_{\mathbf{k}}}{4\pi}$, whereas $g_{\mathbf{k}\sigma}^{(i)}$ is a directional one: $\int g_{\mathbf{k}\sigma}^{(i)}d\Omega_{\mathbf{k}} = 0$, and $g_{\mathbf{k}\sigma}^{(1)}$ and $g_{\mathbf{k}\sigma}^{(2)}$ are of the zeroth and first order in $\lambda_{\rm so}$, respectively. By solving the Boltzmann equation having the scattering term of Eq. (10), we find

$$f_{\mathbf{k}\sigma} \approx f_{\mathbf{k}\sigma}^0 - \tau_{\rm imp}^{(\rm s)}\boldsymbol{v}_{\mathbf{k}}^{g}\cdot\nabla f_{\mathbf{k}\sigma}^0 + \eta_{\rm SS}\tau_{\rm imp}(\boldsymbol{\sigma}_{\sigma\sigma}\times\hbar\mathbf{k}/m)\cdot\nabla f_{\mathbf{k}\sigma}^0, \tag{11}$$

where $\eta_{\rm SS} = (2\pi/3)\tilde{\lambda}_{\rm so}N(0)V_{\rm i}$, $\tilde{\lambda}_{\rm so} = k_{\rm F}^2\lambda_{\rm so}$ is the nondimensional spin-orbit parameter, and $\tau_{\rm imp}^{(\rm s)} = (E_{\mathbf{k}}/|\xi_{\mathbf{k}}|)\times\tau_{\rm imp}$ is the scattering time in the superconducting state [23]. Using Eq. (11) in Eq. (7), the total charge and spin currents become up to the first order in $\tilde{\lambda}_{\rm so}$:

$$\mathbf{j}_Q^{\rm tot} = \mathbf{j}_{\rm pair} + \mathbf{j}_Q^0 + \eta_{\rm SJ}[\hat{\mathbf{z}}\times\mathbf{j}_S^0] - \eta_{\rm SS}eD[\hat{\mathbf{z}}\times\nabla S], \tag{12}$$

$$\mathbf{j}_S^{\rm tot} = \mathbf{j}_S^0 + \eta_{\rm SJ}[\hat{\mathbf{z}}\times\mathbf{j}_Q^0] - \eta_{\rm SS}eD[\hat{\mathbf{z}}\times Q_{\rm n}], \tag{13}$$

where the last terms are the Hall currents due to *skew scattering* (SS), $D = \frac{1}{3}\tau_{\rm imp}v_{\rm F}^2$ is the diffusion constant in the normal state, $Q_{\rm n} = \sum_{\mathbf{k}\sigma}q_{\mathbf{k}}f_{\mathbf{k}\sigma}^0$, $S = \sum_{\mathbf{k}\sigma}\sigma f_{\mathbf{k}\sigma}^0$, and $\mathbf{j}_Q^0$ and $\mathbf{j}_S^0$ are the longitudinal currents given by

$$\mathbf{j}_Q^0 = -eDN(0)\int_{-\infty}^{\infty}(\xi_{\mathbf{k}}/|\xi_{\mathbf{k}}|)\nabla[f_{\mathbf{k}\uparrow}^0 + f_{\mathbf{k}\downarrow}^0]d\xi_{\mathbf{k}}, \tag{14}$$

$$\mathbf{j}_S^0 = -eDN(0)\int_{-\infty}^{\infty}(|\xi_{\mathbf{k}}|/E_{\mathbf{k}})\nabla[f_{\mathbf{k}\uparrow}^0 - f_{\mathbf{k}\downarrow}^0]\,d\xi_{\mathbf{k}}\,. \quad (15)$$

Equation (12) indicates that $\mathbf{j}_S^0$ and ∇S generate the transverse charge currents due to SJ and SS, respectively, while Eq. (13) indicates that $\mathbf{j}_Q^0$ and ∇Q_{n} generate the transverse spin currents due to SJ and SS, respectively. In our Hall device in Fig. 1, the last two terms of Eq. (13) have no contribution to the Hall voltage, so we retain only the longitudinal spin current $\mathbf{j}_S^0$ by setting $\mathbf{j}_S^{\mathrm{tot}} = \mathbf{j}_S^0 = (j_S^0, 0, 0)$ and $j_S^0 = Pj_{\mathrm{inj}}$ in the following.

Nonequilibrium spin and charge imbalance is described by the distribution function $f_{\mathbf{k}\sigma}^0$. In the FM/SC/FM tunnel junction, the tunneling time τ_{t} of an electron passing through SC can be longer than the energy relaxation time τ_E [24], so that electrons that enter SC relax to the Fermi distribution before leaving SC. Then, except at low temperatures, $f_{\mathbf{k}\sigma}$ is described by f_0 with the shifted chemical potentials as $f_{\mathbf{k}\sigma}^0 = f_0(E_{\mathbf{k}}^\sigma)$, where $E_{\mathbf{k}}^\sigma = \sqrt{(\epsilon_{\mathbf{k}} - \mu_{\mathrm{n}})^2 + \Delta^2} - \sigma\delta\mu_S \approx E_{\mathbf{k}} - q_{\mathbf{k}}\delta\mu_Q - \sigma\delta\mu_S$ [18,20], μ_{n} is the chemical potential of the normal component, and $\delta\mu_Q = \mu_{\mathrm{n}} - \mu_{\mathrm{p}}$ (see Fig. 2). If $f_{\mathbf{k}\sigma}^0$ is expanded with respect to $\delta\mu_S$ and $\delta\mu_Q$ as $f_{\mathbf{k}\sigma}^0 \approx f_0(E_{\mathbf{k}}) - (\sigma\delta\mu_S + q_{\mathbf{k}}\delta\mu_Q)\partial f_0(E_{\mathbf{k}})/\partial E_{\mathbf{k}}$, the spin and charge densities become $S \approx 2N(0)\chi_S^0\delta\mu_S$ and $Q_{\mathrm{n}} \approx 2N(0)\chi_Q^0\delta\mu_Q$, where the susceptibilities χ_S^0 and χ_Q^0 are given by [18]

$$\chi_S^0 = 2\int_\Delta^\infty dE\,\frac{E}{\sqrt{E^2 - \Delta^2}}\left(-\frac{\partial f_0}{\partial E}\right), \quad (16)$$

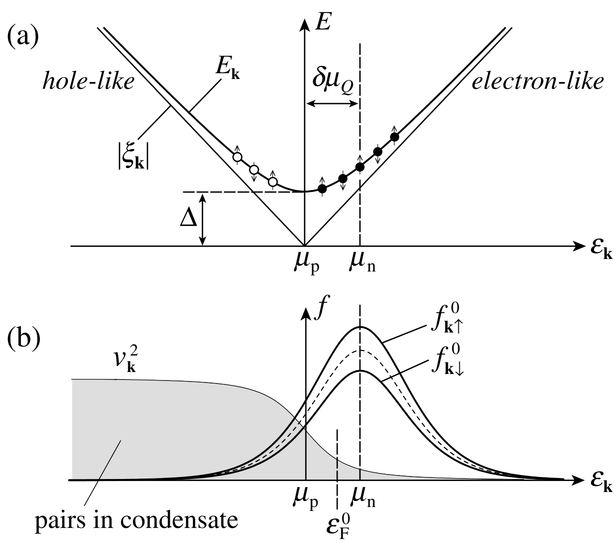

FIG. 2. (a) Distribution of quasiparticles (QPs) on the excitation spectrum $E_{\mathbf{k}}$ of SC in a nonequilibrium situation, where the chemical potentials μ_{n} and μ_{p} are shifted relative to the equilibrium value of $\varepsilon_{\mathrm{F}}^0$, and differ by $\delta\mu_Q$. (b) Distribution functions of $f_{\mathbf{k}\uparrow}^0$ and $f_{\mathbf{k}\downarrow}^0$ for up and down spin QPs. The QPs are distributed around μ_{n}, yielding charge imbalance; the QPs with up spin are more populated than those of down spin, yielding spin imbalance.

$$\chi_Q^0 = 2\int_\Delta^\infty dE\,\frac{\sqrt{E^2 - \Delta^2}}{E}\left(-\frac{\partial f_0}{\partial E}\right), \quad (17)$$

whose asymptotic values are given in Ref. [25]. Likewise the spin and charge currents become $\mathbf{j}_S^0 \approx -eD_S\nabla S$ and $\mathbf{j}_Q^0 \approx -eD_Q\nabla Q_{\mathrm{n}}$, where $D_S = [2f_0(\Delta)/\chi_S^0]D$ is the spin diffusion constant which decreases with decreasing temperature T below T_c, and $D_Q = [2f_0(\Delta)/\chi_Q^0]D$ is the QP charge diffusion constant which increases below T_c. Thus, the total QP charge current is written in the form

$$\mathbf{j}_Q^{\mathrm{tot}} = \mathbf{j}_{\mathrm{pair}} + \mathbf{j}_Q^0 + [\eta_{\mathrm{SJ}} + (D/D_S)\eta_{\mathrm{SS}}](\hat{\mathbf{z}} \times \mathbf{j}_S^0)\,. \quad (18)$$

We notice that the charge imbalance effect induced by the Hall current is analogous to that induced by a temperature gradient in SC [26]. In the case of the Hall effect, the transverse component of Eq. (18) vanishes, i.e., $j_{\mathrm{pair}}^y + (j_Q^0)_y + [\eta_{\mathrm{SJ}} + (D/D_S)\eta_{\mathrm{SS}}]j_S^0 = 0$. Using the solution of $\nabla^2 Q_{\mathrm{n}} = Q_{\mathrm{n}}/\lambda_Q^2$ (λ_Q being the charge diffusion length) [17] with the boundary condition $j_{\mathrm{pair}}^y(\pm w/2) = 0$, we obtain the charge imbalance along the y direction

$$Q_{\mathrm{n}} = 2eN(0)j_S^0\rho_{\mathrm{N}}\lambda_Q(\tilde{\eta}_{\mathrm{SJ}} + \tilde{\eta}_{\mathrm{SS}})\frac{\sinh(y/\lambda_Q)}{\cosh(w/2\lambda_Q)}, \quad (19)$$

where $\tilde{\eta}_{\mathrm{SJ}} = (D/D_Q)\eta_{\mathrm{SJ}}$, $\tilde{\eta}_{\mathrm{SS}} = (D^2/D_QD_S)\eta_{\mathrm{SS}}$, and $\rho_{\mathrm{N}} = 1/[2e^2N(0)D]$ is the normal-state resistivity. Note that for $w \gg \lambda_Q$, Q_{n} and $(j_Q^0)_y$ are confined in the range of λ_Q from the side of SC, and vanish in the interior of SC where the Hall current induced by the spin current is canceled out by the counterflowing pair current. In the opposite limit ($w \ll \lambda_Q$), $Q_{\mathrm{n}} \propto y$, $j_{\mathrm{pair}}^y \approx 0$, and $(j_Q^0)_y$ is balanced with the Hall current.

The induced charge Q_{n} is compensated by the change in the pair charge Q_{p} to maintain overall charge neutrality in SC, implying that μ_{n} and μ_{p} shift in opposite directions from their equilibrium value of $\varepsilon_{\mathrm{F}}^0$ as shown in Fig. 2. The change of Q_{p} from its equilibrium value is $\delta Q_{\mathrm{p}} = 2N(0)\delta\mu_{\mathrm{p}}$ ($\delta\mu_{\mathrm{p}} = \mu_{\mathrm{p}} - \varepsilon_{\mathrm{F}}^0$), and thus the charge neutrality ($Q_{\mathrm{n}} + \delta Q_{\mathrm{p}} = 0$) leads to the relation: $\chi_Q^0\delta\mu_Q + \delta\mu_{\mathrm{p}} = 0$ [18].

In a stationary state, the electrochemical potential $\Phi_{\mathrm{p}} = \mu_{\mathrm{p}} + e\phi$ for the condensate must be constant throughout SC, where ϕ is the electric potential [17,18]. Otherwise, the Cooper pairs are accelerated by the force $-\nabla_y\Phi_{\mathrm{p}}$. Consequently, ϕ is induced in the transverse direction according to $\delta\mu_{\mathrm{p}} + e\phi = 0$, which yields the Hall voltage $V_{\mathrm{H}} = \phi(w/2) - \phi(-w/2)$ across the sides of SC:

$$V_{\mathrm{H}} = (\tilde{\eta}_{\mathrm{SJ}} + \tilde{\eta}_{\mathrm{SS}})P\rho_{\mathrm{N}}wj_{\mathrm{inj}}\mathcal{G}(w)\,, \quad (20)$$

where $\mathcal{G}(w) = (2\lambda_Q/w)\tanh(w/2\lambda_Q)$: $\mathcal{G} = 1$ for $w \ll \lambda_Q$ and $\mathcal{G} = 2\lambda_Q/w$ for $w \gg \lambda_Q$. In the limit of $\Delta \to 0$ and $\lambda_Q \propto 1/\sqrt{\Delta} \to \infty$ [27], Eq. (20) reduces to that in the normal state [16]. If one introduces the Hall resistivity $\rho_{\mathrm{H}} = V_{\mathrm{H}}/(wj_{\mathrm{inj}})$, then

$$\rho_{\mathrm{H}} = \alpha_{\mathrm{SS}}\rho_{\mathrm{N}} + \beta_{\mathrm{SJ}}\rho_{\mathrm{N}}^2\,, \quad (21)$$

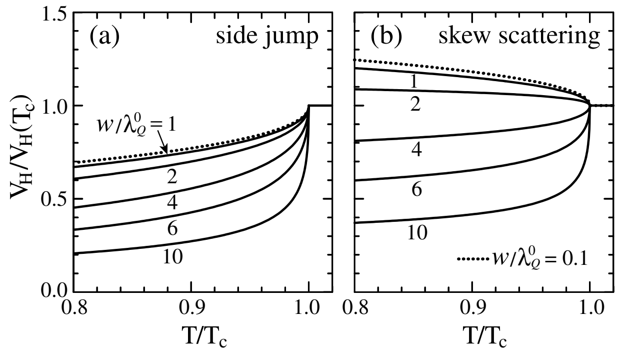

FIG. 3. Hall voltage vs temperature for different values of w/λ_Q^0. The dotted curves indicate the values for $w/\lambda_Q^0 \ll 1$.

where $\alpha_{SS} = \frac{2\pi}{3} P \tilde{\lambda}_{so} N(0) V_i (D^2/D_Q D_S) \mathcal{G}(w)$ and $\beta_{SJ} = \frac{2}{3\pi} P \tilde{\lambda}_{so} (e^2/h) k_F (D/D_Q) \mathcal{G}(w)$.

Figure 3 shows the temperature dependence of the Hall voltage of the side jump contribution V_H^{SJ} and of the skew scattering contribution V_H^{SS}. We use the form $\lambda_Q = \lambda_Q^0 (1 - T/T_c)^{-1/4}$ ($\lambda_Q^0 \sim 5$ μm for Al and 2 μm for Sn) [17,26]. The normalized V_H^{SJ} and V_H^{SS} exhibit strong T dependence below T_c, and their values differ by the factor D_S/D, because $V_H^{SJ} \propto j_S^0$ and $V_H^{SS} \propto \nabla_x S \propto (D/D_S) j_S^0$. This difference stems from the fact that the SJ is governed by the group velocity $\boldsymbol{v}_{\mathbf{k}}^g$ through $\mathbf{j}_S$, while the SS is governed by the phase velocity $\hbar \mathbf{k}/m$ through $f_{\mathbf{k}\sigma}$ [cf. Eq. (11)]. In addition, V_H depends sensitively on the width of SC; for $w \lesssim \lambda_Q^0$, V_H^{SJ} decreases while V_H^{SS} increases below T_c. For $w \gg \lambda_Q^0$, both V_H^{SJ} and V_H^{SS} rapidly decrease just below T_c, reflecting the strong T dependence of λ_Q. However, when the ratio $V_H/\mathcal{G}$ is plotted, all the solid curves in each panel collapse to the dotted curve which represents the intrinsic part of the Hall effect described by $\tilde{\eta}_{SJ} + \tilde{\eta}_{SS}$ [see Eq. (20)]. Thus, if one measures the T dependence of V_H for $w \lesssim \lambda_Q^0$ or that of $V_H/\mathcal{G}$, one can determine which mechanism (SJ or SS) is dominant for the Hall effect. This provides a method for distinguishing the mechanisms without invoking the power dependence of ρ_H on ρ_N ($\rho_H \propto \rho_N$ or ρ_N^2) [11,13].

The Hall voltage generated by the present device is estimated as follows. If the values of $\rho_H/\rho_N \sim 10^{-2}$, $\rho_N \sim 10^{-6}$ Ω cm, $j_{inj} \sim 10^5$ A/cm^2, $w \sim 10$ μm, and $P \sim 0.5$ are used, we have $V_H \sim 1$ μV, which is measurable by experiments. It follows from $P j_{inj} \sim -[2 f_0(\Delta)/e\rho_N] \times \nabla \delta\mu_S$ that $\delta\mu_S \ll \Delta$ ($\Delta \sim 1$ meV for Nb) for the parameter values and $d \sim 10$ nm except for temperatures well below T_c, resulting in little suppression of Δ.

We propose a spin-injection Hall device with a circuit of a Josephson junction (JJ). The leads of JJ are connected to SC in the Hall geometry. When the injection current flows through the device, the Hall voltage V_H is generated across JJ, so that the supercurrent oscillates across JJ at a frequency $\omega = 2eV_H/\hbar$, thereby emitting and absorbing quanta of microwave of this frequency. Thus, the device is used not only to probe the spin current by the ac Josephson effect but also provides a new Josephson device utilizing the Hall effect.

In summary, we have studied the Hall effect caused by the spin current across SC in the spin injection device, and shown that the Hall effect results from the strong coupling between the QP charge imbalance and the Cooper pair charge. We find that the Hall voltages due to side jump and skew scattering have different temperature dependence in the superconducting state. The results provide a method for identifying the mechanisms of the anomalous Hall effect.

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