Collective Deceleration of Ultrarelativistic Nuclei and Creation of Quark-Gluon Plasma

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We propose a unified space-time picture of baryon stopping and quark-gluon plasma creation in ultrarelativistic heavy-ion collisions. It is assumed that the highly Lorentz contracted nuclei are decelerated by the coherent color field which is formed between them after they pass through each other. This process continues until the field is neutralized by the Schwinger mechanism. Conservation of energy and momentum allow us to calculate the energy losses of the nuclear slabs and the initial energy density of the quark-gluon plasma.

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Ultrarelativistic heavy-ion collisions open the unique possibility to study collective dynamical effects at the partonic level. Before a collision the partons are confined in individual hadron configurations and the Lorentz contracted nuclei can propagate in the physical vacuum without distortion. After the nuclei collide, thousands of partons are quickly liberated as a result of multiple soft-gluon exchange. The nuclei are transformed into two partonic sheets which recede from each other, leaving behind strong gluon fields [1]. Since color charges on the sheets are distributed stochastically, they generate chromoelectric fields which are nonuniform in the transverse plane. This field configuration may be envisaged as a collection of densely packed color flux tubes or strings stretched between the sheets; see Fig. 1(a). The energy accumulated in the coherent field is taken from the kinetic energy of the partonic sheets that causes their deceleration. At later times the coherent fields are neutralized via the Schwinger pair-production mechanism [2].

In this Letter we formulate a simple model to describe the collective deceleration of ultrarelativistic nuclei by the coherent field together with the creation of quark-gluon plasma. Previous applications of the coherent field picture for nuclear collisions [3] were focused on the central rapidity region and ignored the back reaction of the field on the motion of nuclei, which is essentially an infinite-energy approximation. Also, in contrast to microscopic string-based models such as FRITIOF [4] and QGSM [5], which are formulated in momentum space and deal only with hadronic secondaries, we describe the space-time evolution of the coherent field and include the possibility of the quark-gluon plasma formation. Our model is similar in spirit to that proposed recently in Ref. [6]. Some geometric and kinematic aspects of the model are also similar to those introduced in Ref. [7]. All calculations here are performed in the center-of-rapidity frame where the nuclei have initial rapidities $\pm y_0$.

We consider only beam energies so great that the nuclei can be thought of as very thin, Lorentz contracted, sheets. Each sheet is divided into many small elements, or slabs, of unit transverse area labeled by an index a where $a = p$ for the projectile nucleus and $a = t$ for the target nucleus. Each slab is characterized by a baryon number N_a which is assumed to be strictly conserved. This number can be

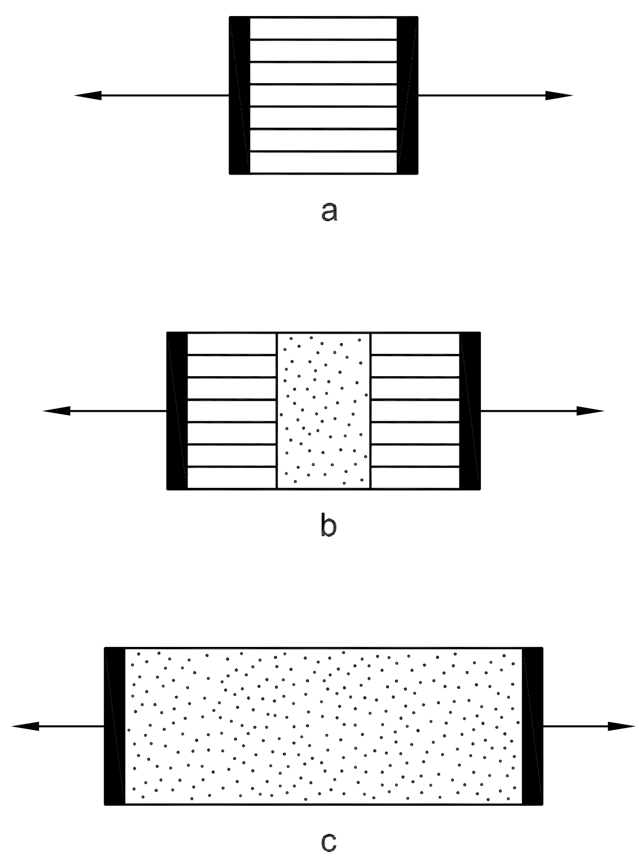

FIG. 1. Spatial view of a slab-slab collision at different times specified in the text: (a) $0 < t < \tau_0$, (b) $\tau_0 < t < t^*$, and (c) $\tau \geq t^*$. The black boxes represent the receding projectile and target slabs. The horizontal straight lines indicate strings. The dotted areas show the regions occupied by the quark-gluon plasma.

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expressed through the slab thickness, l_a, defined in the rest frame of the respective nucleus,

$$N_a(\mathbf{b}) = \rho_0 l_a(\mathbf{b}) = \int dz\, \rho_a(\mathbf{b}, z)\,, \qquad (1)$$

where $\rho_a(\mathbf{b}, z)$ is the density distribution in nucleus a, $\mathbf{b}$ is the position of the slab in the impact parameter plane, and $\rho_0 \approx 0.15\ \mathrm{fm}^{-3}$ is the equilibrium nuclear density. We assume that before and after nuclear overlap, which takes place at $t = 0$, each slab propagates as a rigid body along the beam axes z. The energy and momentum of slab a is parametrized in terms of its proper energy per baryon ε_a and longitudinal rapidity y_a (for details, see Ref. [7]),

$$E_a = N_a \varepsilon_a \cosh y_a\,, \qquad P_a = N_a \varepsilon_a \sinh y_a\,. \qquad (2)$$

Because of hard parton interactions at $t \approx 0$ ε_a may increase as compared to its value in normal nuclei, $\varepsilon_0 \approx m_N$, the nucleon mass.

For simplicity we disregard here a short time delay which is needed for the formation of the coherent field. At $t > 0$ the trajectories of the projectile and target slabs, $z_p(t)$ and $z_t(t)$, are affected by the energy and momentum losses for stretching the strings. If the string tension (energy per unit length) for individual strings is σ, and the number of strings per unit transverse area is n, then the potential energy stored in strings is $V(z) = n\sigma|z_p - z_t|$ (for definiteness we assume that $z_p > z_t$). Accordingly the force or, in the present context, the longitudinal pressure exerted by strings on slab a is $-\partial V/\partial z_a = \mp n\sigma$, and therefore Newton's equation of motion has the form

$$\frac{dP_a}{dt} = \mp n\sigma\,. \qquad (3)$$

From here on the upper and lower signs correspond to $a = p$ and $a = t$, respectively. The net force acting on both slabs is, of course, zero and the total energy of the slabs-plus-field system is conserved. Realizing that the energy lost by a slab while traversing distance dz_a is $\mp n\sigma dz_a$, one can write the energy conservation equation as

$$\frac{dE_a}{dz_a} = \mp n\sigma\,. \qquad (4)$$

These equations are identical to those describing the motion of massive capacitor plates due to the action of a uniform (chromo)electric field of a capacitor. This is not surprising since the action of many collinear flux tubes, or strings, is equivalent to the action of a uniform field $\mathbf{E}$ with the same energy density $\epsilon_f = \mathbf{E}^2/2 = n\sigma$. It is important to note that, in the absence of a chromomagnetic field, ϵ_f is a Lorentz invariant quantity.

Using Eqs. (3) and (4) and the definition of slab velocity $dz_a/dt = P_a/E_a$, one finds that $P_a dP_a = E_a dE_a$. This means that the internal energies of the slabs do not change in the course of deceleration, that is $E_a^2 - P_a^2 = N_a^2 \varepsilon_a^2 =$ const. The solution to Eqs. (3) and (4) for the initial condition $y_a(0) = \pm y_0$, $z_a(0) = 0$ are easily found to be

$$\sinh y_a = \pm\left(\sinh y_0 - \frac{t}{\lambda_a}\right), \qquad (5)$$

$$\cosh y_a = \cosh y_0 \mp \frac{z_a}{\lambda_a}\,, \qquad (6)$$

where $\lambda_a = (\varepsilon_a \rho_0/\epsilon_f) l_a$ is the characteristic deceleration length. Eliminating y_a from these equations one obtains the slab trajectories.

$$\left(\cosh y_0 \mp \frac{z_a}{\lambda_a}\right)^2 - \left(\sinh y_0 - \frac{t}{\lambda_a}\right)^2 = 1\,. \qquad (7)$$

These are parts of hyperbolae shown in Fig. 2. Initially the slab trajectories are very close to the light cone, but later on they increasingly deviate from it. If nothing else were to happen the slabs would reach their respective turning points at $t = \lambda_a \sinh y_0$, $z = \pm\lambda_a(\cosh y_0 - 1)$, and then reverse direction [see an example in Fig. 2(b)]. This is the yo-yo type of motion well known in string models. However, such dynamics is very unlikely in nuclear collisions because of irreversible processes associated with the string decay. When the strings become long enough

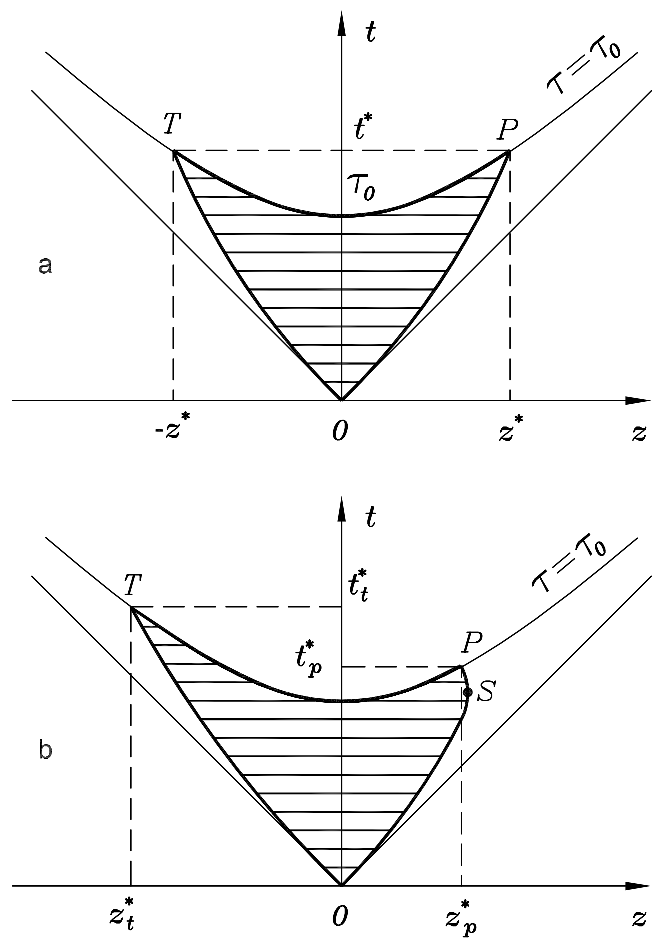

FIG. 2. Schematic space-time representation of a symmetric (a) and asymmetric (b) slab-slab collision in the z-t plane. The projectile and target slab trajectories are shown by thick solid lines which start at the origin and terminate at points P and T, respectively. In (b) the projectile slab is so much smaller than the target one that it stops at the turning point S and reverses its velocity. The quark-gluon plasma is produced at the portion of the hyperbola $\tau = \tau_0$ between points P and T (thick solid line). Horizontal solid lines represent strings.

quark-antiquark and gluon pairs can be produced from vacuum via the Schwinger mechanism [2]. Their color charges will screen the chromoelectric fields and eventually neutralize them. Here we want to avoid numerical simulations of the complicated plasma-field dynamics [3] which is not very well understood yet. Instead we adopt a simplified picture assuming that the strings decay suddenly at a certain proper time $\tau = \tau_0 \sim 1$ fm$/c$, where $\tau = \sqrt{t^2 - z^2}$. As a result, the partonic plasma is created on the hyperbola $\tau = \tau_0$; see Fig. 2.

As shown in Fig. 1(b), the structure of the system changes dramatically at $t > \tau_0$. Now the chromoelectric fields are absent in the middle but still remain in the regions adjacent to the projectile and target slabs. The gap between these two regions is filled by the baryon-free partonic plasma. As time progresses the boundaries between the plasma and coherent fields move closer to the slabs, and the gap expands. Let us write the energy-momentum tensor of the plasma in the standard form

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}, \tag{8}$$

where ϵ and p are the proper energy density and pressure, u^{μ} is the collective four-velocity of the plasma, and $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. At its creation the plasma is assumed to have negligible transverse velocity; collective transverse expansion develops at later times. Thus the four-velocity can be chosen as $u^{\mu} = \gamma(1, 0, 0, v)$, where v is the longitudinal velocity and $\gamma = 1/\sqrt{1 - v^2}$. Now we can write equations expressing the conservation of energy and momentum across the boundary between the plasma and the chromoelectric field. In time dt, when the boundary moves a distance dz, the energy per unit area subtracted from the field is $\epsilon_f dz$. This change must be equal to the energy of a newly produced plasma slice dz: $T^{00}dz = \epsilon_f dz$. Using Eq. (8) this gives

$$(\epsilon + p)\gamma^2 - p = \epsilon_f. \tag{9}$$

Since the field exerts a force $n\sigma = \epsilon_f$ on the plasma charges, the change of momentum in the plasma slice dz is $T^{03}dz = \epsilon_f dt$. This leads to

$$(\epsilon + p)\gamma^2 v = \epsilon_f \frac{dt}{dz} = \epsilon_f \frac{z}{t}. \tag{10}$$

In the last equality we have used the condition that the boundary moves along the hyperbola $\sqrt{t^2 - z^2} = \tau_0$ so that $tdt = zdz$. The two equations (9) and (10) allow us to find two quantities, ϵ and v, characterizing the plasma at $\tau = \tau_0$. Especially simple results are obtained in the case of free streaming, $p = 0$, which seems most appropriate for the early stages of the plasma evolution. By dividing Eq. (10) by Eq. (9) we get

$$v(\sqrt{t^2 - z^2} = \tau_0) = \frac{z}{t} = \tanh\eta, \tag{11}$$

$$\epsilon(\sqrt{t^2 - z^2} = \tau_0) = \frac{\epsilon_f}{\gamma^2} = \frac{\epsilon_f}{\cosh^2\eta}, \tag{12}$$

where the space-time (pseudo)rapidity $\eta = \frac{1}{2}\ln(\frac{t+z}{t-z})$ has been introduced. The first formula gives exactly the velocity field postulated in Bjorken's scaling hydrodynamics [8], but here it follows from the conservation laws at the plasma boundary. The second equation shows that the global energy density $T^{00} = \epsilon\gamma^2$, not the rest-frame energy density ϵ, should be constant at the hypersurface $\tau = \tau_0$ where the plasma is created. These predictions should be used as the initial conditions for further kinetic or fluid-dynamic simulations of the plasma evolution. Thus, the introduction of the coherent field allows us to fill in the missing time interval τ_0 in the Bjorken's picture, as well as the description of the baryon stopping.

The trajectories of the projectile and target slabs are not affected by the string decay until they intersect the hyperbola $\tau = \tau_0$. It is clear that the region occupied by the plasma expands faster than the receding slabs. Thus the plasma eventually eats up the strings at time t_a^*, as shown in Fig. 2. At this time the chromoelectric fields are fully neutralized and no new plasma is produced anymore; see Fig. 1(c). Substituting $z_a(t_a^*)$ in Eq. (6) we obtain the final gamma factors and rapidities of the slabs,

$$\gamma_a^* = \cosh y_a^* = \gamma_0\left[1 - \frac{\tau_0}{\lambda_a}\left(v_0\sqrt{1 + \frac{\tau_0^2}{4\lambda_a^2}} - \frac{\tau_0}{2\lambda_a}\right)\right], \tag{13}$$

where $\gamma_0 = \cosh y_0$ and $v_0 = \tanh y_0$. We see that γ_a^* is entirely determined by a single combination of parameters, $\tau_0/\lambda_a = \epsilon_f\tau_0/\varepsilon_a\rho_0 l_a$. Since $\varepsilon_a \simeq m_N$ is not expected to vary much in the course of the reaction and l_a is given by the geometry, the combination $\epsilon_f\tau_0$ is the only essential parameter of the model which determines the rapidity loss by the nuclei. It is quite natural that this is the same quantity which gives the initial transverse energy of the plasma at central pseudorapidity $\eta = 0$. As follows from Eq. (12), the energy density of the plasma at $\eta = 0$ and $\tau = \tau_0$ is equal to ϵ_f. The total energy of the plasma in a slice $dz = \tau_0\, d\eta$ around $\eta = 0$ is obtained by integrating $\epsilon_f(\mathbf{b})$ over transverse area of the reaction zone. This gives for central collisions

$$\frac{dE_T}{d\eta}(\eta = 0) = \pi R^2\langle\epsilon_f\rangle\tau_0, \tag{14}$$

where $\langle\epsilon_f\rangle$ is the energy density of the chromoelectric field averaged over the transverse area.

From Eq. (13) the condition of complete stopping ($\gamma^* = 1$) is $\tau_0/\lambda_a = \sqrt{2(\gamma_0 - 1)}$. As our estimates show, actual values of τ_0/λ_a are not far from this limit. Thus the proposed scenario provides an efficient mechanism for baryon stopping in ultrarelativistic heavy-ion collisions. The baryon rapidity distribution is obtained by summing up contributions of all pairs of slabs and integrating over impact parameters. Our prediction is that, in the first approximation, this will be a superposition of two peaks centered at rapidities y_p^* and y_t^* given by Eq. (13). These peaks will be smeared out due to the fluctuations in ϵ_f.

The interaction with produced plasma will cause additional drift and diffusion of the baryon charge in rapidity space.

A direct calculation of the energy density ϵ_f accumulated in the chromoelectric field at the early stage of a heavy-ion collision is problematic at present. Therefore, for our estimates we use a simple parametrization which is motivated by several model calculations,

$$\epsilon_f = \epsilon_0 \left(\frac{s}{s_0} \right)^{\alpha/2} \left(\frac{N_p N_t}{N_0^2} \right)^{\beta}. \tag{15}$$

It is assumed that this parametrization applies above a certain c.m. energy squared, $s \geq s_0$. The exponent $\alpha \simeq 0.3$ follows from the low-x behavior of the nuclear structure function or parton density [9]. The third factor relates the number of strings produced to the number of binary parton-parton collisions which is proportional to $N_p N_t$. For convenience we have normalized N_p and N_t by the mean areal baryon density in the proton $N_0 \approx 0.4\ \mathrm{fm}^{-2}$. In the case of uncorrelated strings, as in elementary pp or $p\bar{p}$ collisions, $\beta \approx 1$, while in the case of strong string overlap or percolation $\beta \approx 0.5$ [10]. We believe the values $\beta = 0.5$–0.7 to be appropriate for heavy-ion collisions at RHIC and LHC energies.

We now discuss phenomenological implications of our model in the light of recent RHIC data for central Au + Au collisions at $\sqrt{s} = 130$ GeV ($y_0 = 4.94$). As found by the PHENIX Collaboration [11], the transverse energy in the central pseudorapidity window is $dE_T/d\eta|_{\eta=0} \approx 600$ GeV. For the free-streaming plasma ($p = 0$) $\epsilon(\tau)\tau = \epsilon(\tau_0)\tau_0$, and $dE_T/d\eta|_{\eta=0}$ is independent of τ. Then from Eq. (14) with $\pi R^2 = 145\ \mathrm{fm}^2$ we immediately obtain $\langle \epsilon_f \rangle \tau_0 \approx 4\ \mathrm{GeV/fm}^2$. If the pressure effects are fully included, $p = \epsilon/3$, the transverse energy drops as $\tau^{-1/3}$, and in order to obtain the observed $dE_T/d\eta$ one should increase the initial value by a factor $(\tau_f/\tau_0)^{1/3} \simeq 2$, where τ_0 and τ_f are the initial and final time of hydrodynamic expansion. For estimates we take $\langle \epsilon_f \rangle \tau_0 = 6\ \mathrm{GeV/fm}^2$ which is in between these two extremes. This value is in qualitative agreement with results of microscopic simulations within the QGSM [5].

Now we can estimate the final rapidities of Au nuclei by considering the collision of representative slabs with lengths $l_p = l_t = 4R/3$ averaged over the transverse plane. Here we omit index a for simplicity. The proper energy per baryon in partonic slabs is approximated by $m_T = \sqrt{m_N^2 + \langle p_T \rangle^2}$, where $\langle p_T \rangle \approx 0.5\ \mathrm{GeV}/c$ as measured by the STAR Collaboration [12]. To ensure energy conservation at $t = 0$ we have reduced γ_0 by the factor m_N/m_T. With these inputs $\tau_0/\lambda \simeq 4.1$ and Eq. (13) gives the final projectile and target rapidities $y^* \simeq \pm 1.9$. This corresponds to the final c.m. energy of about 3.5 GeV per baryon compared to the initial energy of 65 GeV per nucleon. The difference is transferred into the quark-gluon plasma. This is a tremendous energy loss which can explain the rather high degree of baryon stopping observed at RHIC. The measurement of the net-baryon rapidity distributions would provide a crucial test of our model. Preliminary BRAHMS data show that the net-proton rapidity distribution has a dip at $y = 0$ and two peaks at $y = \pm(2$–$3)$ [13]. Extrapolating by Eq. (15) to the full RHIC energy $\sqrt{s} = 200$ GeV we obtain $y^* \simeq 2.1$.

In conclusion, the collective dynamics of partonic slabs has been studied within a simple model incorporating strong chromoelectric fields generated early in the reaction. Applying conservation of energy and momentum, we have established a direct correspondence between the rapidity shifts of the projectile and target nuclei and the transverse energy of the produced quark-gluon plasma. Using data from recent RHIC experiments we have estimated the mean energy density accumulated in the chromoelectric field and the resulting rapidity shift in the net baryon distribution. The predicted strong deceleration of the baryon as well as the electric charge makes it very promising to search for photon bremsstrahlung at collider energies (see Ref. [14], and references therein).

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[1] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 3352 (1994); **50**, 2225 (1994).
[2] A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979); T. S. Biro, H. B. Nielson, and J. Knoll, Nucl. Phys. **B245**, 449 (1984); K. Kajantie and T. Matsui, Phys. Lett. **164B**, 373 (1985); M. Gyulassy and A. Iwazaki, Phys. Lett. **165B**, 157 (1985).
[3] G. Gatoff, A. K. Kerman, and T. Matsui, Phys. Rev. D **36**, 114 (1987); K. J. Eskola and M. Gyulassy, Phys. Rev. C **47**, 2329 (1993); F. Cooper, J. M. Eisenberg, Y. Kluger, E. Mottola, and B. Svetitsky, Phys. Rev. D **48**, 190 (1993); R. S. Bhalerao and G. C. Nayak, Phys. Rev. C **61**, 054907 (2000); D. V. Vinnik *et al.,* Eur. Phys. J. **C22**, 341 (2001).
[4] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. **97**, 31 (1983).
[5] N. S. Amelin, N. Armesto, C. Pajares, and D. Sousa, Eur. Phys. J. **C22**, 149 (2001).
[6] V. K. Magas, L. P. Csernai, and D. D. Strottman, Phys. Rev. C **64**, 014901 (2001).
[7] Yu. B. Ivanov, I. N. Mishustin, and L. M. Satarov, Nucl. Phys. **A433**, 713 (1985).
[8] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[9] D. Kharzeev and E. Levin, nucl-th/0108006.
[10] M. A. Braun, F. del Moral, and C. Pajares, Phys. Rev. C **65**, 024907 (2002).
[11] PHENIX Collaboration, K. Adcox *et al.,* Phys. Rev. Lett. **87**, 052301 (2001).
[12] STAR Collaboration, C. Adler *et al.,* Phys. Rev. Lett. **87**, 112303 (2001).
[13] J.-J. Gaardhoje (private communication).
[14] J. I. Kapusta and S. M. H. Wong, Phys. Rev. C **59**, 3317 (1999); S. M. H. Wong, M. Belkacem, J. I. Kapusta, S. A. Bass, M. Bleicher, and H. Stöcker, Phys. Rev. C **63**, 014903 (2000).