Predictions for Higgs and Supersymmetry Spectra from SO(10) Yukawa Unification with $\mu > 0$

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We use t, b, τ Yukawa unification to constrain supersymmetry parameter space. We find a narrow region survives for $\mu > 0$ (suggested by $b \rightarrow s\gamma$ and the anomalous magnetic moment of the muon) with $A_0 \sim -1.9m_{16}$, $m_{10} \sim 1.4m_{16}$, $m_{16} \sim 1200-3000$ GeV and $\mu, M_{1/2} \sim 100-500$ GeV. Demanding Yukawa unification thus makes definite predictions for Higgs and sparticle masses.

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Minimal supersymmetric (SUSY) SO(10) grand unified theories (GUTs) have many profound features [1]: all fermions in one family sit in one **16** dimensional spinor representation; the two Higgs doublets of the minimal supersymmetric standard model sit in one **10** dimensional fundamental representation, and gauge coupling unification at a GUT scale $M_G \sim 3 \times 10^{16}$ GeV fits well with the low-energy data [2,3]. In addition in the simplest version of SO(10) the third generation Yukawa couplings are given by a single term in the superpotential $W = \lambda$ **16 10 16** resulting in Yukawa unification $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} \equiv \lambda$ and a prediction for M_t with large $\tan\beta \sim 50$ [4]. (See Ref. [5].) This beautiful result is however marred by potentially large weak scale threshold corrections [8,9]

$$m_b(M_Z) = \lambda_b(M_Z)\frac{v}{\sqrt{2}}\cos\beta(1 + \Delta m_b^{\tilde{g}} + \Delta m_b^{\tilde{\chi}^+} + \Delta m_b^{\tilde{\chi}^0} + \Delta m_b^{\log}).$$

For $\mu > 0$ the gluino term is positive and in most regions of SUSY parameter space it is the dominant contribution to Δm_b. Reasonable fits prefer $\Delta m_b < 0$; hence Yukawa unification is easy to satisfy with $\mu < 0$.

The decay $b \rightarrow s\gamma$ and the muon anomalous magnetic moment also get significant corrections proportional to $\tan\beta$ [8]. The SUSY contribution to $b \rightarrow s\gamma$ comes from one loop diagrams similar to those contributing to the bottom mass. The chargino term typically dominates and has opposite sign to the standard model (SM) and charged Higgs contributions, thus reducing the branching ratio for $\mu > 0$. This is necessary to fit the data since the SM contribution is somewhat too big. $\mu < 0$ would on the other hand constructively add to the branching ratio and is problematic. In addition, the recent measurement of the anomalous magnetic moment of the muon $a_\mu^{\text{new}} = (g - 2)/2 = 43(16) \times 10^{-10}$ also favors $\mu > 0$ [10]. Thus it is important to confirm that Yukawa unification can work consistently with $\mu > 0$.

In this Letter we assume exact Yukawa unification and search, using a χ^2 analysis, for regions of SUSY parameter space with $\mu > 0$ providing good fits to the low-energy data. We show that Yukawa unification dramatically constrains the Higgs and SUSY spectra. These results are sensitive to the SUSY breaking mechanism.

It is much easier to obtain electroweak symmetry breaking (EWSB) with large $\tan\beta$ when the Higgs up/down masses are split ($m_{H_u}^2 < m_{H_d}^2$) [11]. In our analysis we consider two particular schemes we refer to as Just so and D term splitting. In the first case the third generation squark and slepton soft masses are given by the universal mass parameter m_{16} and only Higgs masses are split: $m_{(H_u,H_d)}^2 = m_{10}^2(1 \mp \Delta m_H^2)$. In the second case we assume D term splitting, i.e., that the D term for $\mathrm{U}(1)_X$ is nonzero, where $\mathrm{U}(1)_X$ is obtained in the decomposition of $\mathrm{SO}(10) \rightarrow \mathrm{SU}(5) \times \mathrm{U}(1)_X$. In this second case, we have $m_{(H_u,H_d)}^2 = m_{10}^2 \mp 2D_X$, $m_{(Q,\bar{u},\bar{e})}^2 = m_{16}^2 + D_X$, $m_{(\bar{d},L)}^2 = m_{16}^2 - 3D_X$. The Just so case does not at first sight appear to be similarly well motivated. It is quite clear, however, that in any SUSY model the Higgs bosons are very special. R parity is used to distinguish Higgs superfields from quarks and leptons. In addition, a supersymmetric mass term μ with value of order the weak scale is needed for an acceptable low-energy phenomenology. If the Higgs are special, it is reasonable to assume splitting of Higgs, while maintaining universal squark and slepton masses. This may be achieved by GUT scale threshold corrections to the soft SUSY breaking scalar masses [11]. Here we present the most compelling mechanism [12]. In SO(10), neutrinos necessarily have a Yukawa term coupling active neutrinos to the "sterile" neutrinos present in the **16**, i.e., we have $\lambda_{\nu_\tau}\ \bar{\nu}_\tau\ L\ H_u$ with $\lambda_{\nu_\tau} \equiv \lambda$. In order to obtain a tau neutrino with mass $m_{\nu_\tau} \sim 0.05$ eV (consistent with atmospheric neutrino oscillations), the sterile $\bar{\nu}_\tau$ must obtain a Majorana mass $M_{\bar{\nu}_\tau} \geq 10^{13}$ GeV. Moreover, since neutrinos couple to H_u (and not to H_d) with a fairly large Yukawa coupling (of order 0.7), they naturally distinguish the two Higgs multiplets. With $\lambda = 0.7$ and $M_{\bar{\nu}_\tau} = 10^{13}$ GeV, we obtain a significant GUT scale threshold correction with $\Delta m_H^2 \approx 10\%$, remarkably close to the value needed to fit the data. At the same time, we obtain a small threshold correction to Yukawa unification $\approx$2.5% (for more details, see [12]).

Our analysis is a top-down approach with 11 input parameters, defined at M_G, varied to minimize a χ^2 function composed of nine low-energy observables. The 11 input parameters are $M_G, \alpha_G(M_G)$, ϵ_3, the Yukawa coupling λ, and the seven soft SUSY breaking parameters μ, $M_{1/2}$, A_0, $\tan\beta$, m_{16}^2, m_{10}^2, $\Delta m_H^2(D_X)$ for the Just so (D term) case. We use two (one) loop renormalization group (RG) running for dimensionless (dimensionful) parameters from M_G to M_Z and complete one loop threshold corrections at M_Z [9]. We require electroweak symmetry breaking using an improved Higgs potential, including m_t^4 and m_b^4 corrections in an effective 2-Higgs doublet model below M_{stop} [13]. The χ^2 function includes the nine observables: six precision electroweak data $\alpha_{EM}, G_\mu, \alpha_s(M_Z), M_Z, M_W, \rho_{\text{new}}$ and the three fermion masses $M_{\text{top}}, m_b(m_b), M_\tau$. The experimental values used for the low-energy observables are given in Table I.

Figure 1 shows the constant χ^2 contours for $m_{16} = 1500$ and 2000 GeV in the case of universal squark and slepton masses. We find acceptable fits ($\chi^2 < 3$) for $A_0 \sim -1.9 m_{16}$, $m_{10} \sim 1.4 m_{16}$, and $m_{16} \geq 1.2$ TeV. The best fit is for $m_{16} \geq 2000$ GeV with $\chi^2 < 1$. Note, electroweak symmetry breaking in this region of parameter space requires splitting Higgs up/down masses, $\Delta m_H^2 \sim$ O(13%). This range of soft SUSY parameters is consistent with solution (B) of Olechowski and Pokorski [11]. In Table I we present the input parameters, the fits, and the predicted Higgs and SUSY spectra for two representative points with universal squark and slepton masses and the best fit value for D term splitting. We have not presented the contour plots for D term splitting since as can be seen from the best fit point in Table I, the bottom quark mass is poorly fit in this case and $\chi^2 > 5$. Recall, since we have 11 input parameters and only 9 observables, we consider such poor fits unacceptable.

Figure 2 gives the constant $m_b(m_b)$ and Δm_b contours for $m_{16} = 2000$ GeV. We see that the best fits, near its central value, are found with $\Delta m_b \leq -2\%$. Why does Yukawa unification work only in this narrow region of SUSY parameter space? The log corrections $\Delta m_b^{\log} \sim 4\%$–$6\%$ (total contribution from gluino, neutralino, chargino, and electroweak loops) are positive and they must be canceled in order to obtain $\Delta m_b \leq -2\%$. The leading mass insertion corrections proportional to $\tan\beta$ are approximately given by [8]

$$\Delta m_b^{\tilde{g}} \approx \frac{2\alpha_3}{3\pi}\frac{\mu m_{\tilde{g}}}{m_{\tilde{b}}^2}\tan\beta \quad \text{and}$$

$$\Delta m_b^{\tilde{\chi}^+} \approx \frac{\lambda_t^2}{16\pi^2}\frac{\mu A_t}{m_{\tilde{t}}^2}\tan\beta\,.$$

They can naturally be as large as 40%. The chargino contribution is typically opposite in sign to the gluino, since A_t runs to an infrared fixed point $\propto -M_{1/2}$ (see, for example, Carena *et al.* [8]). Hence in order to cancel the positive contribution of both the log and gluino terms, a large negative chargino contribution is needed. This can be accomplished for $-A_t > m_{\tilde{g}}$ and $m_{\tilde{t}_1} \ll m_{\tilde{b}_1}$. The first condition can be satisfied for A_0 large and negative, which helps pull A_t away from its infrared fixed point. The second condition is also aided by large A_t. However, in order to obtain a large enough splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{b}_1}$, large values of m_{16} are needed. Note that for universal scalar masses, the lightest stop is typically lighter than the sbottom. We typically find $m_{\tilde{b}_1} \sim 3 m_{\tilde{t}_1}$. On the other hand, D term splitting with $D_X > 0$ gives $m_{\tilde{b}_1} \leq m_{\tilde{t}_1}$. As a result, in the case of Just so boundary conditions excellent fits are obtained for top, bottom, and tau masses, while for D term splitting the best fits give $m_b(m_b) \geq 4.59$ GeV.

TABLE I. Three representative points of the fits. We fit the central values $M_Z = 91.188, M_W = 80.419, G_\mu \times 10^5 = 1.1664, \alpha_{EM}^{-1} = 137.04, M_\tau = 1.7770$ with 0.1% numerical uncertainties and the following with the experimental uncertainty in parentheses: $\alpha_s(M_Z) = 0.1180$ (0.0020), $\rho_{\text{new}} \times 10^3 = -0.200$ (1.1), $M_t = 174.3$ (5.1), $m_b(m_b) = 4.20$ (0.20). The neutral Higgs masses h, H, and A^0 are pole masses, while all other sparticle masses are running masses.

Data points	1	2	3
Input parameters			
α_G^{-1}	24.46	24.66	24.73
$M_G \times 10^{-16}$	3.36	3.07	3.13
ϵ_3	−0.042	−0.0397	−0.046
λ	0.70	0.67	0.80
m_{16}	1500	2000	2000
m_{10}	2027	2706	2400
Δm_H^2	0.13	0.13	0.07
$M_{1/2}$	250	350	350
μ	150	200	115
$\tan\beta$	51.2	50.5	54.3
A_0	−2748	−3748	−731
χ^2 observables			
M_Z	91.13	91.14	91.15
M_W	80.45	80.45	80.44
$G_\mu \times 10^5$	1.166	1.166	1.166
α_{EM}^{-1}	137.0	137.0	137.0
$\alpha_s(M_Z)$	0.1175	0.1176	0.1161
$\rho_{\text{new}} \times 10^3$	0.696	0.460	0.035
M_t	175.5	174.6	177.9
$m_b(m_b)$	4.28	4.27	4.59
M_τ	1.777	1.777	1.777
Total χ^2	1.50	0.87	5.42
h	116	116	115
H	120	121	117
A^0	110	110	110
H^+	148	148	146
$\tilde{\chi}_1^0$	86	130	86
$\tilde{\chi}_2^0$	135	190	126
$\tilde{\chi}_1^+$	123	178	105
$\tilde{g}$	661	913	902
$\tilde{t}_1$	135	222	1020
$\tilde{b}_1$	433	588	879
$\tilde{\tau}_1$	288	420	1173
$a_\mu^{\text{SUSY}} \times 10^{10}$	9.7	5.5	6.1

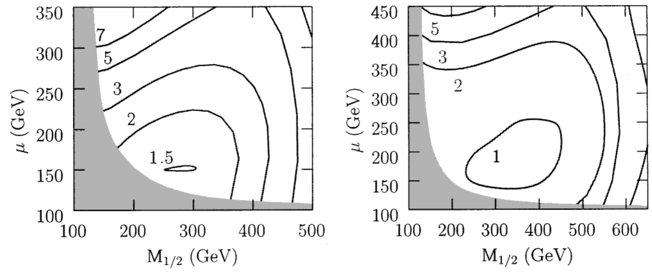

FIG. 1. χ^2 contours for $m_{16} = 1500$ GeV (left panel) and $m_{16} = 2000$ GeV (right panel). The shaded region is excluded by the chargino mass limit $m_{\tilde{\chi}^+} > 103$ GeV.

Finally in Fig. 3 we show the constant light Higgs mass contours for $m_{16} = 1500$ and 2000 GeV (solid lines) with the constant χ^2 contours overlayed (dotted lines). Yukawa unification for $\chi^2 \leq 1$ clearly prefers light Higgs mass in a narrow range, 112–118 GeV. In this region the *CP* odd, the heavy *CP* even Higgs, and the charged Higgs bosons are also quite light (see fit 2 in the Table I) [14]. In addition we find the mass of $\tilde{t}_1 \sim (150-250)$ GeV, $\tilde{b}_1 \sim (450-650)$ GeV, $\tilde{\tau}_1 \sim (200-500)$ GeV, $\tilde{g} \sim (600-1200)$ GeV, $\tilde{\chi}^+ \sim (100-250)$ GeV, and $\tilde{\chi}^0 \sim (80-170)$ GeV. All first and second generation squarks and sleptons have mass of order m_{16}. The light stop and chargino may be visible at the Tevatron. With this spectrum we expect $\tilde{t}_1 \rightarrow \tilde{\chi}^+ b$ with $\tilde{\chi}^+ \rightarrow \tilde{\chi}_1^0 \bar{l} \nu$ to be dominant. Last, $\tilde{\chi}_1^0$ is the lightest SUSY particle and possibly a good dark matter candidate [15].

The region of SUSY parameter space preferred by Yukawa unification may be consistent with a supergravity mechanism for SUSY breaking at M_{Pl} with RG running from M_{Pl} to M_G (see, for example, Murayama *et al.* [11]). It however cannot be obtained with gauge mediated or gaugino mediated SUSY breaking mechanisms where $A_0 = 0$ at zeroth order. It may also be obtained in anomaly mediated schemes, but in this case one still has to worry about slepton masses squared and also the fact that in this case, since the gluino and chargino masses have opposite sign, it is difficult to fit both $b \rightarrow s\gamma$ and a_μ.

In [12] we present the sparticle spectrum in more detail and consequences for Tevatron searches. We discuss the sensitivity of our results to small GUT scale threshold corrections to Yukawa unification with both Just so and D term Higgs up/down splitting. We also check the robustness of the Higgs spectrum by artificially adjusting the *CP* odd Higgs mass using a penalty in χ^2. We find that χ^2 increases by at most 40% for any m_{A^0} less than $\approx$350 GeV. The light Higgs mass m_h is rather insensitive to the value of m_{A^0}, whereas m_H, m_{H^+} are linearly dependent on m_{A^0}. We also consider constraints resulting from the processes $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+\mu^-$, a_μ^{new} and the proton lifetime in a semimodel independent way. We make only a few short comments here. In order to fit $b \rightarrow s\gamma$ we find that the coefficient $C_7^{\text{MSSM}} \sim -C_7^{\text{SM}}$ [see, for example, Eq. (9) in Ref. [16]] with the chargino term dominating by a factor of order 5 over all other contributions. This is due to the light stop $\tilde{t}_1$. In fact, $b \rightarrow s\gamma$ is more sensitive to $m_{\tilde{t}_1}$ than $m_b(m_b)$. Fitting the central value $B(b \rightarrow s\gamma) = 2.96 \times 10^{-4}$ [17] requires a heavier $\tilde{t}_1$ with $(m_{\tilde{t}_1})_{\text{min}} \sim 500$ GeV, significantly larger than the range which provides the best fits to m_b. We now find $m_b(m_b)_{\text{min}} \sim 4.3$. Moreover, no other sparticle masses are affected. The process $B_s \rightarrow \mu^+\mu^-$ provides a lower bound on $m_{A^0} \geq 200$ GeV (see recent work of [18]) [19]. However, this has only a minor impact on χ^2 as discussed above. We recall that proton decay experiments prefer values of $m_{16} > 2000$ GeV and $m_{16} \gg M_{1/2}$ (see Ref. [20]). This is in accord with the range of SUSY parameters found consistent with third generation Yukawa unification. There is, however, one experimental result which is not consistent with either Yukawa unification or proton decay and that is the anomalous magnetic moment of the muon. Large values of $m_{16} \geq 1200$ GeV lead to very small values for $a_\mu^{\text{new}} \leq 16 \times 10^{-10}$. Hence a necessary condition for Yukawa unification is that forthcoming BNL data [10] and/or a reanalysis of the strong interaction contributions to a_μ^{SM} will significantly decrease the discrepancy between the data and the standard model value of a_μ.

In summary, most of the results of our analysis including only third generation fermions remain intact when incorporating flavor mixing. The light Higgs mass and most sparticle masses receive only small corrections. The lightest stop mass increases, due to $b \rightarrow s\gamma$. Nevertheless, there is still a significant $\tilde{t}_1 - \tilde{t}_2$ splitting and $m_{\tilde{t}_1} \ll m_{\tilde{b}_1}$. The A^0, H, H^+ masses are necessarily larger in order to be consistent with $B_s \rightarrow \mu^+\mu^-$ [12], which suggests that this process should be observed soon, possibly at Run II of the

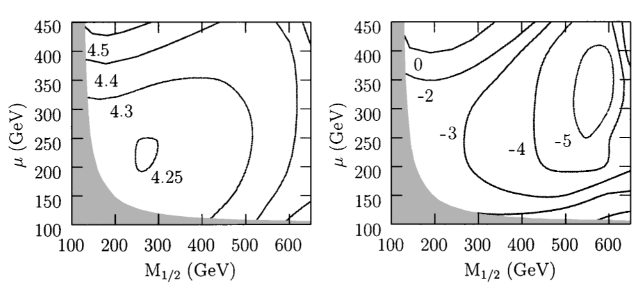

FIG. 2. Contours of constant $m_b(m_b)$ (GeV) (left panel) and Δm_b in % (right panel) for $m_{16} = 2000$ GeV.

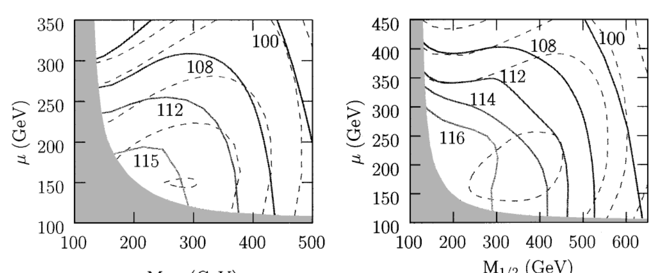

FIG. 3. Contours of constant m_h (GeV) (solid lines) with χ^2 contours from Fig. 1 (dotted lines) for $m_{16} = 1500$ GeV (left panel) and $m_{16} = 2000$ GeV (right panel).

Tevatron. Finally, the central value for a_μ^{new} must significantly decrease. The "smoking guns" of SO(10) Yukawa unification, presented in this Letter, should be observable at Run III of the Tevatron or at CERN Large Hadron Collider. Also, in less than a year we should have more information on a_μ^{new}.

In previous works Yukawa unification with $\mu > 0$ was not possible [21]. Pierce *et al.* [9] assume $\Delta m_H^2 = 0$ and, as a result, they are not able to enter the region of SUSY parameter space consistent with both EWSB and Yukawa unification. Baer *et al.* [22] also cannot obtain Yukawa unification with $\mu > 0$. This is because they use D term splitting for Higgs up/down which as discussed typically leads to sbottom lighter than stop.

While completing this Letter, the paper by Baer and Ferrandis [23] appeared which confirmed our results [24] on the existence of a preferred region of SUSY parameter space consistent with Yukawa unification and $\mu > 0$. Their results, however, require significant GUT threshold corrections to $\lambda_t = \lambda_b = \lambda_\tau$, greater than 28%, which helps them obtain $m_{\tilde{t}_1} < m_{\tilde{b}_1}$. They also claim that better fits are obtained with D term splitting than with the Just so splitting case. We have no explanation for this discrepancy.

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