Effects of Circulating Energetic Ions on Sawtooth Oscillations

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In contrast to the well-known result that the effects of the trapped energetic ions (TEI) on the internal kink mode are due to the toroidal precession of the TEI, it is found that the effects of the circulating energetic ions (CEI) on sawtooth are due to the toroidal circulation of the CEI. The effects of the CEI on sawtooth oscillations are found to be different from the well-known purely stabilizing effects of the TEI on sawtooth oscillations; the toroidal circulation of the co-CEI provides an additional sink of free energy and stabilizes the mode; the toroidal circulation of the counter-CEI provides an additional source of free energy and destabilizes the mode.

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Recent tokamak experiments have shown that sawtooth oscillations can be stabilized by the cocirculating energetic ions (co-CEI) produced by the negative-ion-based neutral beam injection (NNBI) [1,2]. Here, "cocirculating" ("countercirculating") means that the toroidal circulation direction is the same as (opposite to) the direction of the plasma toroidal current in tokamaks. It is widely accepted that the sawtooth oscillations are related to the internal kink mode instabilities [3–7]. The interaction between the energetic ions and the internal kink mode has been extensively investigated [8–14]. It is well known that the trapped energetic ions (TEI) can stabilize the internal kink mode [4,10–12]. The physical mechanism of sawtooth stabilization by the TEI can be summarized as follows. The work done on the TEI by the perturbation can be divided into two parts: the adiabatic part which is similar to the usual magnetohydrodynamic (MHD) fluid response and the nonadiabatic part (δW_k) which is the kinetic (non-MHD) response due to the toroidal precession of the TEI. Since the real part of $\delta W_k(\omega \to 0)$ (ω is the frequency of the mode) is positive finite, the toroidal precession of the TEI provides an additional sink of the free energy, and thus the mode is stabilized. The physical mechanism of sawtooth stabilization by the co-CEI has not been made clear. Previous theories [8–12] on the interaction between energetic ions and the internal kink mode have ignored the nonadiabatic response of the circulating energetic ions (CEI). Recently, the nonadiabatic response (δW_k) of the CEI has been investigated [13,14] in connection with the fishbone modes. δW_k due to the crossing resonance given in Ref. [13] does not provide any stabilization effects of the CEI on sawtooth oscillations, since its real part goes to zero when $\omega \to 0$. δW_k due to the toroidal circulation resonance given in Ref. [14] is valid only when $\omega \sim \omega_{\zeta 0}$, with $\omega_{\zeta 0}$ the toroidal circulation frequency of the CEI evaluated at their birth energy.

Since sawtooth oscillations can degrade the core confinement of tokamaks, it is important to investigate the stabilization of sawtooth oscillations. In a future tokamak fusion reactor, neutral beam current drive (NBCD) is expected to be the main technique to drive the plasma current in the core region, and a population of co-CEI exists in the core region when NBCD is applied. Clearly, it is important to understand the physical mechanism of sawtooth stabilization by the co-CEI. The counterinjection of neutral beams may also be used in controlling the plasma rotation; therefore, it is of interest to investigate whether the counter-CEI can stabilize the sawtooth oscillations as the co-CEI and the TEI do.

Therefore, it is of significant interest to investigate the nonadiabatic response of the CEI to the internal kink mode, in the limit $\omega \to 0$. In this Letter, we show that $\delta W_k(\omega \to 0)$ due to the toroidal circulation of the CEI is positive finite for the co-CEI; thus the toroidal circulation of the co-CEI provides an additional sink of free energy and stabilizes the mode. We show that this is in qualitative agreement with the recent experiments [1,2]. In contrast to the previous theories on sawtooth stabilization by the TEI, an interesting result found in this Letter is that the counter-CEI can destabilize the sawtooth oscillations, since $\delta W_k(\omega \to 0)$ is negative finite for the counter-CEI; the toroidal circulation of the counter-CEI provides an additional source of free energy. The physical mechanism of the effects of the CEI on sawtooth oscillations is also different from the physical mechanism of sawtooth stabilization by the TEI; the effects of the CEI on the mode are due to the toroidal circulation of the CEI, while the previous TEI stabilization is due to the toroidal precession of the TEI.

Consider a large-aspect-ratio tokamak plasma consisting of core and hot components. The inverse of the aspect ratio is $\varepsilon \equiv r/R \ll 1$, with r (R) the minor (major) radius. We make the usual ordering, $\omega/\omega_A \sim \mathcal{O}(\varepsilon^2)$; $\omega_A = \sqrt{3}\, sR/V_A$ is the shear Alfvén frequency; $V_A = B/\sqrt{\mu_0 \rho}$ is the usual Alfvén speed, with B the equilibrium magnetic field and ρ the mass density. s is the magnetic shear $(r/q)dq/dr$ evaluated at the singular surface where $r = r_s$; q is the MHD safety factor; $q(r_s) = 1$. The core plasma beta value is ordered as $\beta_c \sim \mathcal{O}(\varepsilon^2)$. The beam ion beta value is also ordered as $\beta_h \sim \mathcal{O}(\varepsilon^2)$. Here the beta value is the ratio between particle and magnetic pressures. We also make the ordering $\omega_{\zeta 0}/\omega_c \sim \mathcal{O}(\varepsilon^2)$, with ω_c the gyrofrequency of the energetic ions.

With the effects of resistivity ignored, the stability analysis of the internal kink mode is carried out by following the generalized variational principle [7,8]. The energy functional is given by

$$D(\omega) = \delta I + \delta W_{\rm MHD} + \delta W_k\,, \tag{1}$$

$$\delta I = -\frac{1}{2}\,\omega(\omega - \omega_{*i})\int d^3x\,\rho|\vec{\xi}|^2, \tag{2}$$

$$\delta W_{\rm MHD} = \frac{1}{2}\int d^3x \left[\begin{array}{c} \frac{\delta B_\perp^2}{\mu_0} + \frac{B^2}{\mu_0}|\nabla\cdot\vec{\xi}_\perp + 2\vec{\xi}_\perp\cdot\vec{\kappa}|^2 - j_\parallel(\vec{\xi}_\perp^*\times\vec{b})\cdot\delta\vec{B}_\perp \\ -2(\vec{\xi}_\perp\cdot\nabla p)\vec{\xi}_\perp^*\cdot\vec{\kappa} + \frac{5}{3}\,p_c|\nabla\cdot\vec{\xi}|^2 \end{array} \right], \tag{3}$$

$$\begin{aligned} \delta W_k &= \frac{1}{2}\int d^3x\,[(\delta p_h^\parallel - \delta p_h^\perp)\vec{\xi}_\perp^*\cdot\vec{\kappa} - \delta p_h^\perp\nabla\cdot\vec{\xi}_\perp^*] \\ &= \frac{1}{2}\int d^3x\,d^3v\,m_h\delta h^*\delta f\,, \end{aligned} \tag{4a}$$

$$\delta h = E(2 - 3\lambda B)\vec{\kappa}\cdot\vec{\xi}_\perp - E\lambda B\nabla\cdot\vec{\xi}_\perp\,, \tag{4b}$$

where ω_{*i} is the ion diamagnetic frequency [7]. $\delta\vec{B} = \nabla\times(\vec{\xi}\times\vec{B})$. The subscript $\perp$ ($\parallel$) denotes the component perpendicular (parallel) to the equilibrium magnetic field $\vec{B}$. $\vec{b} = \vec{B}/B$. $j_\parallel$ is the parallel equilibrium current density. $\vec{\xi}$ is the usual fluid displacement. $E = v^2/2$; $\lambda = \mu/E$, $\mu = v_\perp^2/2B$, with v the particle velocity. $\vec{\kappa}$ is the curvature of $\vec{B}$. δI ($\delta W_{\rm MHD}$) is the usual MHD inertial (potential) energy functional. $p = p_c + (p_h^\parallel + p_h^\perp)/2$, $[p_h^\parallel, p_h^\perp] = m_h\int d^3v\,EF[2(1-\lambda B), \lambda B]$. p_c is the core plasma pressure. F is the equilibrium distribution of the beam ions. δW_k is the nonadiabatic contribution of the energetic ions. m_h is the mass of the beam ion. $\vec{\xi}_\perp \sim \exp[-i(\omega t - \zeta + \theta)]$, with ζ and θ the toroidal and poloidal angles, respectively. The nonadiabatic part of the perturbed distribution of the hot component, δf, is given by the drift kinetic equation (DKE) [15]

$$(\omega - \dot{\zeta} + i\dot{\theta}\partial_\theta)\delta f = (\omega_* - \omega)\delta h\partial_E F\,, \tag{5}$$

with $\omega_* \equiv q\partial_r F/(\omega_c r\partial_E F)$. In writing the propagator we have dropped the effects of finite radial drift of the CEI, which is related to the crossing resonance inducing the fishbone instability in the tangential injection case [13] and is beyond the scope of this Letter.

The above equations form the basis of our stability analysis, and they are valid in the limit $\omega \to 0$ for the CEI, since the basic formalisms in Refs. [7,8] and Ref. [15], which lead to Eqs. (1)–(4) and Eq. (5), respectively, do not exclude the $\omega \to 0$ case for the CEI.

The usual minimization of $\delta W_{\rm MHD}$ gives [5–8]

$$\begin{aligned} \vec{\xi}_\perp &= \xi_0(\vec{e}_r - i\vec{e}_\theta)H(r_s - r)\exp[-i(\omega t - \zeta + \theta)], \\ 0 &= \nabla\cdot\vec{\xi}_\perp + 2\vec{\kappa}\cdot\vec{\xi}_\perp\,, \end{aligned} \tag{6}$$

with ξ_0 a constant. H is the Heaviside step function. $\vec{e}_r$ ($\vec{e}_\theta$) is the radial (poloidal) unit vector. The normalized energy functional is given by

$$\begin{aligned} \widehat{\delta W}_{\rm MHD} + \widehat{\delta I} &= \delta W_f - i[\omega(\omega - \omega_{*i})]^{1/2}/(\pi\omega_A)\,, \\ \delta W_f &= 3\varepsilon_s^2(1 - q_0)(13/144 - \beta_{ps}^2)\,, \end{aligned} \tag{7}$$

where $\beta_{ps} = -\varepsilon_s^{-2}(2\mu_0/B^2)\int_0^{r_s}(r/r_s)^2\,dr\,dp/dr$, and we have performed the normalization $\widehat{\delta W}_{\rm MHD} \equiv \delta W_{\rm MHD}/[V_s(B^2/\mu_0)(\xi_0/R)^2]$, with V_s the volume within the singular surface. $\varepsilon_s \equiv r_s/R_0$, $R = R_0[1 + \varepsilon_s(r/r_s)\cos\theta]$, $q_0 = q(r = 0)$.

With the well-established generalized variational principle, the stability analysis reduces to solving the DKE. We assume that $\lambda \approx 0$, $v_\parallel \approx \sigma v$, with σ the sign of $v_\parallel$ ($\sigma = +$ for the co-CEI, and $\sigma = -$ for the counter-CEI); this is reasonable for tangential injection case [13,14]. Then using Eq. (6) and $\vec{\kappa} = (\vec{e}_\theta\sin\theta - \vec{e}_r\cos\theta)/R$, writing $\delta h = \overline{\delta h}\exp[-i(\omega t - \zeta)]$, we found

$$\overline{\delta h} = -2E(\xi_0/R)H(r_s - r)\,; \tag{8}$$

$$\dot{\theta} = \sigma v/(qR), \qquad \dot{\zeta} = \sigma v/R\,. \tag{9}$$

Since all of the coefficients in Eq. (5) depend on θ only through the common factor $1/R = (1/R_0)[1 - \varepsilon_s(r/r_s)\cos\theta]$, δf should be θ independent to $\mathcal{O}(\varepsilon_s^0)$. To $\mathcal{O}(\varepsilon_s)$, we have $\delta f = \overline{\delta f}\exp[-i(\omega t - \zeta)]$,

$$\overline{\delta f} = \overline{\delta f}_0(r) + \varepsilon_s\overline{\delta f}_1^c(r)\cos\theta + \varepsilon_s\overline{\delta f}_1^s(r)\sin\theta\,. \tag{10}$$

Using Eqs. (8)–(10) and the small ε_s expansion of $1/R$, we obtained the lowest order DKE:

$$\left(\omega - \sigma\frac{v}{R_0}\right)\overline{\delta f}_0 = (\omega_* - \omega)\frac{\partial F}{\partial E}\frac{R}{R_0}\overline{\delta h}\,. \tag{11}$$

The solution to the DKE is readily found:

$$\delta f = \frac{\omega_* - \omega}{\omega - \sigma v/R_0}\frac{R}{R_0}\delta h\partial_E F + \mathcal{O}(\varepsilon_s)\,. \tag{12}$$

In contrast to Ref. [14] where a similar solution is given for $\omega \sim v/R$, Eq. (12) is also valid for $\omega \ll v/R$. δf found here is responsible for the toroidal circulation of the CEI ($\sigma v/R$ is the toroidal circulation frequency). The previous method in solving the DKE [8–10] is based on the small $\omega\tau_b$ expansion (τ_b is the bounce period of the TEI), while our solution for the CEI is based on the small ε_s expansion. To take into account the effect of toroidal circulation of the CEI, it is more convenient to use our small ε_s expansion than to use the small $\omega\tau_b$ expansion. Note that our method does not involve the bounce (transit) average, as is different from the small $\omega\tau_b$ expansion method. A more general but complicated method in solving the DKE can be found in Refs. [11,16]. We have verified that a similar solution to the DKE can be found for the CEI by using the formalism given in Refs. [11,16], provided that Eqs. (8) and (9) are used and the effect of the finite radial excursion of the CEI is dropped. It is not hard to understand why δf for the CEI is determined by their toroidal circulation. Since $d\zeta/dt$, $d\theta/dt$, and δh (the energy exchange between the energetic ions and the mode) are roughly independent of θ for the CEI, their poloidal motion is unimportant. So the dominant part of δf for the CEI is determined only by their toroidal circulation.

To proceed, we adopt the model slowing-down equilibrium distribution for a population of CEI formed by a purely co-CEI ($\lambda \approx 0, \sigma = +$) component and a purely counter-CEI ($\lambda \approx 0, \sigma = -$) component. $F = \Sigma_\sigma F^\sigma$,

$$F^\sigma = \frac{p_h^\sigma(r)}{\sqrt{2}\,\pi m_h B E_0^\sigma} E^{-3/2}\delta(\lambda)H(E_0^\sigma - E), \quad (13)$$

where Σ_σ denotes summation for $\sigma = +$ and $\sigma = -$. $m_h E_0^\sigma = m_h(v_0^\sigma)^2/2$ is the beam ion birth energy, and $p_h^\sigma(r) = \int d^3v\, m_h E F^\sigma$ is the pressure (energy density) of the CEI of each component. We assume that the ion mass is the same for the two components, but they can have different birth energy and pressure. With Eq. (13) and $d^3v = \Sigma_\sigma \sqrt{2}\,\pi d\lambda dE B E^{1/2}(1 - \lambda B)^{-1/2}$, substituting δh and δf into Eq. (4a) yields $\delta W_h \equiv \widehat{\delta W}_k$,

$$\delta W_h = \Sigma_\sigma \delta W_{h0}^\sigma[1 + \mathcal{O}(\omega R/v_0^\sigma)] \approx \Sigma_\sigma \delta W_{h0}^\sigma, \quad (14a)$$

$$\delta W_{h0}^\sigma \equiv \sigma \frac{2v_0^\sigma}{3\varepsilon_s^2 \omega_c R} \beta_{hs}^\sigma, \quad (14b)$$

$$\beta_{hs}^\sigma \equiv -(2\mu_0/B^2)\int_0^{r_s} dr\, q dp_h^\sigma(r)/dr, \quad (14c)$$

where we have dropped the $\mathcal{O}(\omega R/v_0^\sigma)$ term, since $\omega \ll v_0^\sigma/R$ for the current issue. We assume that $dp_h^\sigma(r)/dr < 0$. For the co-CEI ($\beta_{hs}^- = 0$), $\delta W_h = \delta W_{h0}^+ \times [1 + \mathcal{O}(\omega R/v_0^+)] \approx \delta W_{h0}^+$. Note that $\delta W_h(\omega \to 0) = \delta W_{h0}^+ > 0$ for the co-CEI. It is the finite positive value of $\delta W_h(\omega \to 0)$ that introduces the sawtooth stabilization by the TEI [10] and by the co-CEI. δW_{h0}^+ represents the nonadiabatic part of energy spent to displace the co-CEI, or an additional sink of the free energy. The mechanism of sawtooth stabilization by the co-CEI identified here is different from that by the TEI [10]; $\delta W_h(\omega \to 0) > 0$ for the co-CEI is due to the toroidal circulation of the co-CEI, while $\delta W_h(\omega \to 0) > 0$ for the TEI is due to the toroidal precession of the TEI. For the counter-CEI ($\beta_{hs}^+ = 0$), $\delta W_h = \delta W_{h0}^-[1 + \mathcal{O}(\omega R/v_0^-)] \approx \delta W_{h0}^-$. Note that $\delta W_h(\omega \to 0) = \delta W_{h0}^- < 0$ for the counter-CEI. $-\delta W_{h0}^-$ represents the nonadiabatic part of free energy or an additional source of free energy coming from the counter-CEI due to the effects of their toroidal circulation.

The dispersion relation reads

$$\delta W_f + \delta W_h - i[\omega(\omega - \omega_{*i})]^{1/2}/\pi\omega_A = 0, \quad (15)$$

which is valid for $\delta W_f + \delta W_h < 0$. When $|\delta W_f + \delta W_h| \gg |\omega_{*i}|/(\pi\omega_A)$, it gives the ideal growth rate

$$\gamma_I = -\pi\omega_A(\delta W_f + \delta W_h). \quad (16)$$

Note that $\delta W_h \sim \delta W_f \sim \mathcal{O}(\varepsilon_s^2)$. $\delta W_{h0}^+ > 0 > \delta W_{h0}^-$; the co-CEI (counter-CEI) are stabilizing (destabilizing).

When $\delta W_f + \delta W_h > 0$, Eq. (15) should be modified by including the effects of finite resistivity [3,10].

$$\delta W_h + \delta W_f - iG[\omega(\omega - \omega_{*i})]^{1/2}/\pi\omega_A = 0, \quad (17)$$

with $G = 8\Lambda^{-9/4}\Gamma[(\Lambda^{3/2} + 5)/4]/\Gamma[(\Lambda^{3/2} - 1)/4]$. $\Lambda = -i[\omega(\omega - \omega_{*i})(\omega - \omega_{*e})]^{1/3}/(s^2 S_M^{-1/3}\omega_A)$ is evaluated at r_s. ω_{*e} is the electron drift wave frequency [3]. $S_M = (\mu_0 r_s^2/\eta_\parallel)/[\sqrt{3}R/(V_A s)]$, with $\eta_\parallel$ the parallel resistivity. The stability conditions for the resistive branch of the mode are [10]

$$\begin{aligned} \delta W_f + \delta W_h > \delta W_{\text{crit}} \equiv s^3\omega_A^{1/2}/(\pi^2 S_M|\omega_{*i}|)^{1/2}, \\ |\omega_{*i}| > \gamma_{\eta k} \equiv s^2 S_M^{-1/3}\omega_A\,. \end{aligned} \quad (18)$$

For typical JT-60U experiments on sawtooth stabilization by the co-CEI [1,2] produced by 350 keV deuterium coinjection into the hydrogen plasma, the main parameters in the sawtooth-free period are $q_0 \approx 0.8$, $s \approx 0.4$, $\varepsilon_s \approx 1/12$, $R \approx 3.2$ m, $B = 3.5$ T, $\beta_{ps} \approx 0.5$, $\eta_\parallel \approx 1.2 \times 10^{-8}\ \Omega \cdot$ m, $\omega_A \approx 8 \times 10^6$/sec, $S_M \approx 10^7$, $|\omega_{*i}| \approx 1 \times 10^4$/sec; $\beta_{hs}^+ \approx 0.002$, $\omega_c R/v_0^+ \approx 93$; $\beta_{hs}^- \sim \beta_{hs}^+$, and $v_0^- \ll v_0^+$. We obtained $\delta W_f \approx -0.7 \times 10^{-3}$, $\delta W_h \approx 1.9 \times 10^{-3}$. $\delta W_f + \delta W_h \approx 1.2 \times 10^{-3} > 0$; the ideal branch of the internal kink mode is stabilized. Further we estimated that $\gamma_{\eta k} \approx 6 \times 10^3$/sec and $\delta W_{\text{crit}} \approx 2 \times 10^{-4}$. Therefore, Eq. (18) is satisfied, and the resistive branch is also stabilized. The termination of the sawtooth-free period observed in these experiments may be due to the further decreasing of q_0.

In conclusion, we have established a theoretical model of sawtooth stabilization by the co-CEI, and we have predicted sawtooth destabilization by the counter-CEI. The toroidal circulation of the co-CEI (counter-CEI) provides an additional sink (source) of the free energy and consequently stabilizes (destabilizes) the internal kink mode. Clearly, the effects of the CEI on sawtooth oscillations are different from the effects of the TEI in two aspects. First, the effects of the CEI are due to the toroidal circulation of the CEI, while the effects of the TEI are due to the toroidal precession of the TEI. Second, the effects of the TEI on sawtooth are only stabilizing, while the effects of the CEI on sawtooth can be either stabilizing (co-CEI case) or destabilizing (counter-CEI case). The key parameter for sawtooth stabilization by the co-CEI is $\delta W_{h0}^+ = +2v_0^+\beta_{hs}^+/3\varepsilon_s^2\omega_c R$. Since NNBI-produced fast ions generally have a higher velocity than PNBI (positive-ion-based NBI)-produced fast ions, it is more efficient to use NNBI than to use PNBI in stabilizing sawtooth by the co-CEI. For the balanced tangential neutral beam injection case, $\beta_{hs}^+ v_0^+ = \beta_{hs}^- v_0^-$, we conclude from Eq. (14) that $\delta W_h = 0$. The stabilizing effect of the co-CEI (measured by $\beta_{hs}^{+}v_0^{+}$) is canceled by the destabilizing effect of the counter-CEI (measured by $\beta_{hs}^{-}v_0^{-}$) for the balanced injection case ($\beta_{hs}^{+}v_0^{+} = \beta_{hs}^{-}v_0^{-}$).

While the stabilizing effect of the co-CEI has been corroborated by the recent tokamak experiments [1,2], the destabilizing effect of the counter-CEI is not experimentally known; there is a need for experiment to verify the destabilization prediction. It should be noted that in an isotropic plasma the overall kinetic contribution of the CEI can be ignored in comparing to the kinetic contribution of the TEI; however, if the CEI are unidirectional, their kinetic contribution can be competitive with other MHD effects. Finally, it is of interest to estimate how much neutral beam ($\sim$1 MeV deuterium coinjection) power, in a plasma with ITER-like parameters [17], would be needed to roughly double the ideal sawtooth threshold when $q_0 \approx 0.8$, $\varepsilon_s \approx 0.1$. The main parameters are $R = 8.14$ m, $a = 2.80$ m (the minor radius of the plasma boundary), $B = 5.68$ T, $V_p = 2000$ m^3 (the plasma volume), $\tau_s \sim 1$ sec (the slowing-down time of the energetic deuterium ion). Since the rate of energy transfer from the co-CEI to the core plasma is $V_p\langle p_h\rangle/\tau_s$, with $\langle p_h\rangle$ the volume averaged value of p_h, from energy balance we have $\langle p_h\rangle = \tau_s P_{\rm NBI}/V_p$, with $P_{\rm NBI}$ the injection power. Assuming $p_h(r) = p_{h0}[1 - (r/a)^2]^{\alpha_h}$, we have $\beta_{hs} = (1 + \alpha_h)\{1 - [1 - (r_s/a)^2]^{\alpha_h}\}(2\mu_0\langle p_h\rangle/B^2)$. Using Eqs. (7) and (14), we have

$$\delta W_f + \delta W_h = 3\varepsilon_s^2(1 - q_0)[(\beta_{ps}^{\rm crit})^2 - \beta_{ps}^2], \quad (19a)$$

$$(\beta_{ps}^{\rm crit})^2 = \frac{13}{144} + \frac{1}{3\varepsilon_s^2(1 - q_0)}\frac{2v_0}{3\varepsilon_s^2\omega_c R}\beta_{hs}, \quad (19b)$$

where $\beta_{ps}^{\rm crit}$ represents the ideal sawtooth threshold value of β_{ps}. Without the co-CEI ($\beta_{hs} = 0$), $\beta_{ps}^{\rm crit} = (13/144)^{1/2}$. In order to double the ideal sawtooth threshold [to have $\beta_{ps}^{\rm crit} = 2(13/144)^{1/2}$], assuming $\alpha_h = 7$, we found that $\sim$38 MW is needed. If we assume $\alpha_h = 6$, then $\sim$49 MW is needed. Note that the peaking factor of the profile of p_h is $p_{h0}/\langle p_h\rangle = (1 + \alpha_h)$. It is reasonable to assume $\alpha_h = 7$ or $\alpha_h = 6$. A brief discussion of the peaking factor of p_h can be found in Ref. [14]. Therefore, the stabilizing effect of the co-CEI is significant for ITER-like burning plasmas.

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