Effect of Solvation on Hole Motion in DNA

D. M. Basko* and E. M. Conwell

Department of Chemistry, University of Rochester, Rochester, New York 14627
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An excess charge on a DNA chain in solution interacts with polar solvent molecules and mobile counterions. We give the first theoretical estimate of the resulting hole self-localization energy and calculate the corresponding contribution to hole mobility on a DNA stack consisting of a single base pair repeated.

Many experiments on DNA are performed in a water solution. So-called physiological conditions, i.e., those in which DNA is placed in nature, correspond to a 0.1 M solution of NaCl. Under these conditions DNA, being an acid, donates protons to the solution (one proton per phosphate group), which results in a negative charge of the molecule ($-2e$ per lattice constant $a = 3.4$ Å). Effects due to interaction of this static charge with the environment (water and counterions Na^+ and Cl^-) were studied extensively in the past years (for a review, see [1]). In particular, the charge on the chain is subject to (i) dielectric screening by the surrounding water molecules, (ii) Debye screening by the mobile counterions with a characteristic distance ~ 10 Å (Debye screening length), and (iii) partial compensation by Na^+ ions that condense directly onto the DNA chain, an effect related to its one-dimensional nature [2].

We will study the effect of the environment on the motion of an excess charge carrier (to be specific, we consider holes) added to a DNA molecule. Each of the two DNA strands may be viewed as a chain, each site of which is one of the four bases: guanine (G), adenine (A), cytosine, and thymine, each corresponding to a certain value of the on-site potential for the hole. The overlap of electronic π orbitals of the neighboring bases tends to delocalize the carrier along the chain. Charge transport along DNA has attracted much attention, both experimental and theoretical [3], in connection with such problems as DNA damage and repair or potential applications in nanoeletronics.

Because the lowest on-site potential is that of G, in a random sequence of bases transport may involve tunneling between G's through other bases [4]. However, when a G is followed by a long bridge sequence of A's, after a hole tunnels through the first 3 A's it moves through many later A's with essentially no further attenuation [5]. Several mechanisms have been proposed for the hole transport through the bridge: incoherent hopping [6], fluctuation-induced hopping [7], bandlike transport [8], polaron drift [9]. We are aware of only one work addressing the problem of the solvation effect on hole transport [10], where classical molecular dynamics simulations were performed. However, polarization of the environment by the charge was not taken into account there. To construct an approximate theory of this effect is the task of this Letter.

The configuration of the solvent produced by the interaction with the static charge on phosphate groups, as mentioned above, on average produces a constant shift of the energy of the hole, same for all the bases. Obviously, this will not affect the motion of the hole. What one has to consider is the change in this configuration due to the presence of the hole and the corresponding feedback effect on the hole motion. In other words, describing the interaction of the hole with the polarizable environment, we do not need to consider the static polarization already present due to the static charge of the phosphates.

Considering the hole-induced polarization of the environment independently of the static polarization induced by phosphates implies several assumptions. First, water is assumed to be a linear dielectric. Second, the linearized version of the Poisson-Boltzmann equation for the ion-screened electrostatic potential in the electrolyte is assumed to be valid, i.e., the electrostatic potential felt by ions in solution is assumed to be small compared to temperature. Third, we assume that the presence of the hole does not affect the counterions condensed on the chain. Even if the hole is completely localized on one base, it introduces just a charge $+e$ to the unit cell in addition to $-2e$ on the phosphates. Actually, as we show later, the hole wave function is spread over 3–5 sites, which would result in the additional charge of $0.2e$–$0.3e$ per unit cell. It is unlikely that this small correction will change the configuration of condensed counterions.

We will now look at the hole stationary states to estimate the effect of the environment on hole self-localization. Let the hole charge density be $\rho(\mathbf{r})$. The energy of interaction with the environment is given by

$$E_{\text{env}} = \frac{1}{2} \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \rho(\mathbf{r})\left[G(\mathbf{r},\mathbf{r}') - \frac{1}{|\mathbf{r}-\mathbf{r}'|} \right]\rho(\mathbf{r}'), \tag{1}$$

where $G(\mathbf{r},\mathbf{r}')$ is the electrostatic Green's function in this environment:

$$\varphi(\mathbf{r}) = \int d^3\mathbf{r}'\, G(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}'), \tag{2}$$

with $\varphi(\mathbf{r})$ being the electrostatic potential. Physically, Eq. (1) represents the electrostatic energy of the charge in the presence of the environment with the energy of interaction of the charge with itself subtracted.

A practical way of describing the environment, adopted in most works, is to consider the DNA molecule to be placed inside a cavity C. Physically, the cavity is due to the presence of the sugar-phosphate backbone and the hydrophobicity of the DNA bases. Its characteristic size R is determined by the size of the helix: $R \sim 10$ Å. The space outside the cavity is filled with the solution with dielectric constant ε at temperature T, in which counterions are dissolved whose equilibrium densities (far from the DNA molecule) are $n_{\text{Na}} = n_{\text{Cl}} = n$. Neglecting the polarizability of the backbone and the bases, we assume the cavity to be empty. The Green's function can be found from the Poisson equation

$$\nabla^2 G(\mathbf{r}, \mathbf{r}') = 4\pi \delta(\mathbf{r} - \mathbf{r}'), \quad \mathbf{r} \in C,$$
$$[\nabla^2 - \kappa_D^2] G(\mathbf{r}, \mathbf{r}') = 0, \quad \text{otherwise,} \tag{3}$$

with the usual boundary conditions at the interface. The inverse Debye screening length κ_D is defined as $\kappa_D^2 \equiv 8\pi n e^2/(\varepsilon T)$, the presence of two types of ions being taken into account. We represent the hole wave function as a linear combination of molecular orbitals: $\psi(\mathbf{r}) = \sum_n \psi_n \phi_n(\mathbf{r} - \mathbf{r}_n)$, where ϕ_n is the orbital of the nth base and ψ_n is the probability amplitude for the hole on this base. Neglecting the orbital overlap for different bases [11], we obtain the corresponding charge density,

$$\rho(\mathbf{r}) = e|\psi(\mathbf{r})|^2 \approx e \sum_n |\psi_n|^2 \phi_n^2(\mathbf{r} - \mathbf{r}_n). \tag{4}$$

By substituting it into Eq. (1) and denoting the integrals over $\mathbf{r}, \mathbf{r}'$ for each pair of terms of the sum (4) by $g_{n-n'}$, we express the hole energy as

$$H(\psi_n, \psi_n^*) = H_0(\psi_n, \psi_n^*) + \frac{1}{2} \sum_{n,n'} g_{n-n'} |\psi_n|^2 |\psi_{n'}|^2. \tag{5}$$

Hereafter we restrict ourselves to the case where the DNA stack consists of the same base pair repeated. Then the first term H_0 corresponds to free hole motion—nearest-neighbor hopping with the transfer integral t_0:

$$H_0(\psi_n, \psi_n^*) = -t_0 \sum_n (\psi_n \psi_{n+1}^* + \psi_n^* \psi_{n+1}). \tag{6}$$

Hole stationary states can be found from the (nonlinear) Schrödinger equation: $\epsilon \psi_n = \partial H/\partial \psi_n^*$, obtained by varying the energy with respect to ψ_n^*.

To estimate the coupling in the hole effective Hamiltonian, we consider first just one base pair and a spherical cavity of radius R which will give us an estimate for the coefficient g_0. Let us put a charge e into the cavity. If we assume its distribution to be spherically symmetric, then the potential outside the cavity is is given by the spherically symmetric solution of Eq. (3) decreasing at infinity: $\varphi(r) = e_* \exp(-\kappa_D r)/r$, where e_* is the effective charge which includes also the charge at the interface $r = R$. The electric field inside the cavity is determined only by the charge inside the cavity, hence the potential

inside the sphere will differ from that in the absence of the environment only by a constant φ_0. The field at $r \to R$ is equal to $-e/R^2$ and $-e_*(1 + \kappa_D R)e^{-\kappa_D R}/R^2$ inside and outside the sphere, respectively. Using the continuity of the electric displacement we find e_* and obtain

$$\varphi_0 = \frac{e}{R} \left[\frac{1}{\varepsilon(\kappa_D R + 1)} - 1 \right]. \tag{7}$$

The typical characteristics of the solvent outside the cavity are the following: at $T = 298$ K the static dielectric constant of water is $\varepsilon = 78$ [12], the Debye radius for the 0.1 M solution of NaCl $1/\kappa_D \approx 9.6$ Å. For this value of ε the first term in the square brackets is negligible, whatever κ_D is. Thus for such dielectric contrast the dominant effect is interaction with water, not with counterions. The energy (1) is then equal to $-e^2/2R$, so that $g_0 = -e^2/R$. Taking $R = 10$ Å we get $|g_0|/2 = 0.72$ eV.

Neglect of the first term in Eq. (7) corresponds to setting the electric field outside the cavity equal to zero. Generally, for an electrostatic problem with such a large value of ε a dielectric is equivalent to a conductor [13]: the corresponding correction is of the order of $1/\varepsilon$. In this case it is sufficient to solve the Poisson equation,

$$-\nabla^2 G(\mathbf{r}, \mathbf{r}') = 4\pi \delta(\mathbf{r} - \mathbf{r}'), \qquad \mathbf{r} \in C, \tag{8}$$

requiring the tangential component of the electric field to vanish at the cavity surface.

Clearly, the spherical cavity overestimates the interaction energy because in reality some part of the boundary will be at a larger distance from the charge. Another possibility is to consider a cylindrical cavity of the radius R and to assume the charge to be concentrated on the axis of the cylinder at evenly spaced points $z_n = na$ corresponding to the bases. Taking advantage of the translational invariance along the cylinder axis, we make the Fourier transform

$$\rho(\mathbf{r}) = \delta(x)\delta(y) \int_{-\infty}^{\infty} \frac{dk}{2\pi} \rho_k e^{ikz}. \tag{9}$$

The corresponding Fourier component of the potential $\varphi_k(r_\perp)$ (where we denoted $r_\perp \equiv \sqrt{x^2 + y^2}$) satisfies the Laplace equation for $0 < r_\perp < R$ with the boundary conditions,

$$-\frac{\partial^2 \varphi_k}{\partial r_\perp^2} - \frac{1}{r_\perp} \frac{\partial \varphi_k}{\partial r_\perp} + k^2 \varphi_k = 0,$$
$$\frac{\partial \varphi_k}{\partial r_\perp} \bigg|_{r_\perp \to 0} \sim \frac{2\rho_k}{r_\perp}, \qquad \varphi_k(R) = 0. \tag{10}$$

The solution may be expressed in terms of the modified Bessel functions $I_0(|k|r_\perp)$ and $K_0(|k|r_\perp)$:

$$\varphi_k(r_\perp) = 2\rho_k \left[K_0(|k|r_\perp) - \frac{K_0(|k|R)}{I_0(|k|R)} I_0(|k|r_\perp) \right]. \tag{11}$$

The first term in the square brackets corresponds to the free space solution, while the second term is entirely due

to the boundary. Subtracting the self-interaction, we omit the first term and obtain the interaction energy

$$E_{\text{env}} = -\frac{1}{2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{K_0(|k|R)}{I_0(|k|R)} 2\rho_k \rho_{-k}. \qquad (12)$$

The corresponding coefficients in Eq. (5) are given by

$$g_n = -\frac{e^2}{R} v(na/R),$$
$$v(\xi) \equiv \frac{2}{\pi} \int_0^{\infty} \frac{K_0(q)}{I_0(q)} \cos q\xi \, dq. \qquad (13)$$

Setting $R = 3a = 10.2$ Å, we may evaluate the integral for different n (Table I). For large ξ the value of the integral is determined by the singularity $K_0(q) \sim -\ln q$ at $q \to 0$: $v(\xi \ll 1) \approx 1/|\xi|$ which gives 1% precision already at $\xi = 3$.

Looking at these numbers one may notice two things. First, the value of g_0 is only slightly less than that obtained from the spherically symmetric estimate. This suggests that both estimates are relatively reliable. Second, assuming the value of $t_0 \sim 0.1$–0.2 eV typically used for DNA [4,8], from Eq. (13) and Table I we see that the dependence of g_n on n is not very strong:

$$g_1 - g_0 \sim 0.02 e^2/R \sim 0.03 \text{ eV} \ll t_0,$$

due to the fact that $a < R$. This means that the kinetic term will not allow localizing the hole to one lattice site. The kinetic and interaction terms will balance each other when ψ_n is spread over some number l of sites, such that $g_l - g_0 \sim t_0$, which gives $l \sim 3$–5. If one takes into account coupling to the lattice displacements [9], which localize the charge within 3–5 sites, only g_0, $g_{\pm 1}$, and probably $g_{\pm 2}$ will effectively contribute, which means that (i) the shape of the hole wave function will not be strongly different from the one obtained without taking into account solvation effects, (ii) the latter increase the binding energy by $\sim |g_0|/2$.

These considerations are illustrated in Fig. 1, where we show the hole population profile $|\psi_n|^2$ obtained by taking into account coupling to the lattice displacements and to the environment separately and together. The parameters used are $t_0 = 0.2$ eV, $\alpha^2/K = 0.2$ eV (α being the hole-lattice coupling constant, and K being the elastic constant for the lattice [9]). The lattice contribution can be included in the hole Hamiltonian (5) by adding $-2\alpha^2/K$ to $g_{\pm 1}$ from Eq. (3). The binding energies corresponding to the curves (a), (b), and (c) are 0.057 eV (lattice only), 0.52 eV (environment only), and 0.62 eV (both), respectively. The shapes are not strongly different from each other and are weakly dependent on the parameters (for $t_0 = 0.1$ eV the population profile due to interaction with

the environment only is indistinguishable from the curve (c) and the binding energy is 0.56 eV). These results allow us to conclude that this stationary state is stable (as the binding energy is much larger than temperature) and has a polaronic nature.

To study the effect of the environment on the hole dynamics, one should adopt a model for the dynamics of the solution, namely, how the polarization of the environment responds to the hole motion. For the dielectric screening by the water molecules the relevant time scale is the orientational relaxation time ($\tau = 8.3$ ps at $T = 298$ K [12]). For the Debye-Hückel screening by counterions the characteristic time is related to the conductivity σ of the solution as $\tau = \varepsilon/(4\pi\sigma)$. For the 0.1 M solution of NaCl at $T = 298$ K, one has $\tau = 650$ ps [12], which is much longer than typical times for the hole transfer in experiments (10–100 ps). This gives the second argument for the dominant contribution of water: counterions are too slow to follow the hole motion.

To model the kinetics of water polarization we associate with each site n a collective degree of freedom Q_n, corresponding to the motion of water molecules around the nth base and having the sense of the screening charge induced in the solution. The equation of motion for this degree of freedom describes relaxation to the equilibrium value $Q_n^{(\text{eq})}$ [14]:

$$\dot{Q}_n = -(Q_n - Q_n^{(\text{eq})})/\tau, \qquad (14)$$

τ being the orientational relaxation time mentioned earlier, and $Q_n^{(\text{eq})}$ determined by the charge density $|\psi_n|^2$. The Hamiltonian for the coupling of this degree of freedom to the hole motion can be written from the following considerations. First, no kinetic energy should be associated with Q_n, as it is overdamped. Second, for the equilibrium displacements $Q_n^{(\text{eq})}$ to be finite, some "elastic constant" $\mathcal{K}$ should be introduced. Third, the coupling to the hole
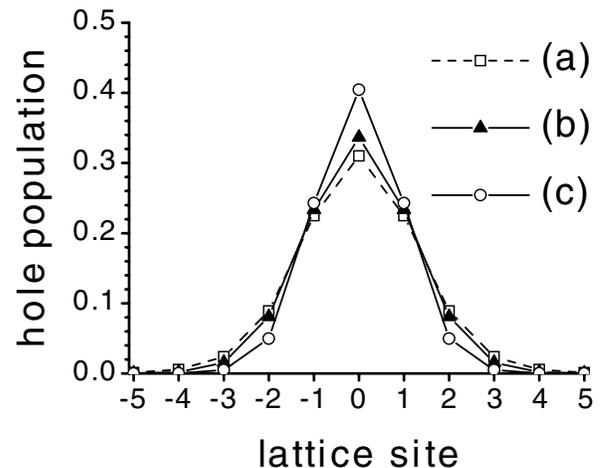
FIG. 1. Hole population $|\psi_n|^2$ for a stationary state formed by interaction with (a) lattice only, (b) environment only, (c) both. $\alpha^2/Kt_0 = 1.0$, $e^2/Rt_0 = 7.0$.

TABLE I. Calculated values $v(\xi)$.

ξ	0	0.33	0.67	1.0	1.33	1.67	2.0	2.33	2.67
$v(\xi)$	0.87	0.85	0.79	0.71	0.62	0.54	0.48	0.42	0.37

motion should be linear in Q_n and $|\psi_n|^2$ because both of them represent some charge, and the coupling is of Coulomb origin. This allows us to write the Hamiltonian,

$$H(\psi_n, \psi_n^*, Q_n) = H_0(\psi_n, \psi_n^*) + \sum_{n,n'} \lambda_{n-n'} Q_n |\psi_{n'}|^2$$
$$+ (\mathcal{K}/2) \sum_n Q_n^2, \qquad (15)$$

and the equation of motion for ψ_n: $i\hbar\dot{\psi}_n = \partial H/\partial \psi_n^*$.

To relate the values of the coupling $\lambda_{n-n'}$ and the elastic constant $\mathcal{K}$ to physical parameters we find the static equilibrium displacements from the Hamiltonian (15):

$$Q_n^{(eq)} = -\sum_{n'} \frac{\lambda_{n-n'}}{\mathcal{K}} |\psi_{n'}|^2. \qquad (16)$$

Substituting them into the Hamiltonian (15), we obtain an effective Hamiltonian for stationary states that should be identical to the Hamiltonian (5). Since Q_n has the sense of screening charge, we require also $\sum Q_n^{(eq)} = -\sum |\psi_n|^2$. As a result, we obtain two conditions for λ_n and $\mathcal{K}$:

$$g_n = -\sum_{n'} \lambda_{n'-n} \lambda_{n'}/\mathcal{K}, \qquad \sum_n \lambda_n/\mathcal{K} = 1. \qquad (17)$$

It is convenient to use Fourier transform,

$$f(k) \equiv \sum_n f_n e^{-ikn}, \qquad f_n = \int_{-\pi}^{\pi} \frac{dk}{2\pi} f(k) e^{ikn} \qquad (18)$$

(f_n being a generic function); these conditions can then be written as

$$\mathcal{K} = -g(0), \qquad \lambda(k)\lambda(-k) = g(0)g(k). \qquad (19)$$

Using this model we can estimate the mobility μ of the polaron on the chain. For this we note that the total energy of the system is not conserved, but instead changes in time due to the frictional force according to

$$\dot{E} = -\tau \mathcal{K} \sum_n \dot{Q}_n^2. \qquad (20)$$

Consider a stationary polaron of dimensionless width l moving with the velocity v along the chain, e.g., dragged by an applied electric field. Then the hole population has the form of a propagating pulse: $|\psi_n(t)|^2 = F(n - vt/a)$, with a being the lattice constant (3.4 Å), and $F(\xi)$ being a continuous function. If $v \ll la/\tau$, then Q_n will follow quasistatically: $Q_n(t) \approx Q_n^{(eq)}(t)$, the latter given by Eq. (16). On one hand, the dissipated power is given by the sum (20); on the other hand, it is equal to ev^2/μ. As a result, for μ we obtain

$$e/\mu = -(\tau/a^2) \sum_{n,n'} g_{n-n'} F'(n) F'(n'), \qquad (21)$$

where $F'(\xi) \equiv dF/d\xi$. This sum can be estimated as to the order of magnitude: $\mu/e \sim l^3 (a^2/\tau)(e^2/R)^{-1}$,

which for $l \sim 3$ and the cited parameters gives $\mu \sim 3 \times 10^{-3}$ cm^2/(V s). To evaluate the sum (21) we take $F(n) = |\psi_n|^2$ for the standing polaron [15], namely, curve (c) in Fig. 1, and define the derivative of the lattice function $F(n) = |\psi_n|^2$ through the Fourier transform $[F'(n) \leftrightarrow ikF(k)]$. The result is $\mu = 2.4 \times 10^{-3}$ cm^2/(V s).

In the opposite limit $v \gg la/\tau$ the solvent will not follow the hole; thus the dissipation will be negligible and high-field mobility will be determined by different effects, which are beyond the scope of this paper.

In conclusion, we have considered the effect of solvation on hole motion on a DNA chain consisting of the same base pair periodically repeated. Our results are (i) the most important interaction is that with polar water molecules, not with counterions; (ii) the self-localization energy due to this interaction is $\sim$0.5 eV; (iii) due to nonlocality of the interaction the hole does not self-localize to one lattice site, but is spread over 3–5 sites; (iv) interaction with water provides an upper bound for the low-field hole mobility along the chain: $\mu < 10^{-2}$–10^{-3} cm^2/(V s).

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*Electronic address: basko@chem.rochester.edu

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